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DYNAMIC BEHAVIOR OF A CABLE-STAYED FOOTBRIDGE DEPENDING ON THE CALCULATION ACCURACY

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Abstract. Footbridges are analyzed in terms of dynamic response for pedestrian comfort. This problem is solved mainly on light and at the same time rigid constructions, which are easily oscillated by pedestrian load. It is often solved on steel footbridges, but with the technological development of UHPC, we are capable nowadays to build relatively light and long span concrete footbridges. Behaviour of a thin concrete cross-section with its geometry is closer to steel structures, but at the same time we have to deal with rheological effects such as creep and shrinkage. There are many influences on the structure that can affect the dynamic behavior of the structure, including the non-linear behavior of the cable system. This paper presents an initial entry into the computational issues of more complex constructions that have more input influences.

Keywords: Dynamic characteristics, natural frequencies, footbridge, cable-stayed bridge, finite element method, UHPC, stiffness matrix, mass matrix.

1. Introduction

Dynamic analysis precedes the dynamic experiment in-situ, unfortunately construction companies are mostly forced to buy bridge dampers in advance, so calculation inaccuracy may lead to an uneconomical design and therefore an accurate model is required.

Critical freqency range in the vertical direction is between 1.25–2.3 Hz and for the horizontal natural frequences between 0.5–1.2 Hz. Footbridges with frequencies for vertical or longitudinal vibrations in range 2.5–4.5 Hz might be excited by resonanse by the 2$^{\text{nd}}$ harmonic of pedestrian load. In that case critical frequences range for vertical and longitudinal vibrations expands to 1.25–4.5 Hz. The frequencies are further analyzed more precisely, and the amount of oscillating mass in bridge engineering is also tracked. Dampers are designed into the locations, where the acceleration reaches critical values where the natural damping of the structure is insufficient. In practice, if there is an inaccuracy in the calculation, a damper can be designed in a place where it will not be needed [1].

2. Dynamic characteristics

The main dynamic characteristics that enter into the dynamic calculation of natural frequencies are stiffness matrix $[K]$ and mass matrix $[M]$. A two-dimensional model using the finite element method was chosen for monitoring changes in the behavior of the structure [2, 3].

2.1. Local stiffness matrix

The full stiffnes matrix of an element:

$$K_e = \frac{E}{L}\begin{pmatrix} A & 0 & 0 & -A & 0 & 0 \\ 0 & \frac{12I}{L^2} & \frac{6I}{L} & 0 & \frac{12I}{L^2} & \frac{6I}{L} \\ 0 & \frac{6I}{L} & 4I & 0 & -\frac{6I}{L} & 2I \\ -A & 0 & 0 & A & 0 & 0 \\ 0 & -\frac{12I}{L^2} & -\frac{6I}{L} & 0 & \frac{12I}{L^2} & -\frac{6I}{L} \\ 0 & \frac{6I}{L} & 2I & 0 & -\frac{6I}{L} & 4I \end{pmatrix}, \quad (1)$$

where

E is Young's modulus of the material [kPa],

I is the moment of inertia for the Y-axis in base of the cross-section [m^4],

A is the element area [m^2],

L is the lenght of the element [m].

This matrix belongs to the deck and the pilons, but the cables behaviour is different. The cables are non-linerar elements that can only be tensed and have zero bending stiffness. In this analysis, the effect of rope sag is neglected for simplification reasons.

To achieve the correct behavior of a cable-stayed structure, the geometric stiffness matrix of the element also needs to be included:

$$K_g = \frac{P}{30L}\begin{pmatrix} 0 & 0 & 0 & & 0 & 0 \\ 0 & 36 & 3L & 0 & -36 & 3L \\ 0 & 3L & 4L^2 & 0 & -3L & -L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & -3L & 0 & 36 & -3L \\ 0 & 3L & -L^2 & 0 & -3L & 4L^2 \end{pmatrix}, \quad (2)$$

where

P is the stressing/tension force in the element [kN].

The axial force in the member is denoted by P, with the positive axial force in this case representing pressure. Combining the element stiffness and geometrical stiffness we recive the local matrix of stiffness:

$$K_L = K_e + K_g. \tag{3}$$

2.2. Local mass matrix

Normally, cables are considered intangible, because they oscillate at low frequencies and thus introduce a large amount of low frequencies into the calculation, which usually unnecessarily interfere the global assessment. For the academic purposes, the cables in the numeric model are considered to have weight for realistical results.

The weight of the structure is expressed often using the diagonal mass matrix, which expresses that the mass is equaly divided to end nodes:

$$M_d = \frac{\eta L}{2} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

where

η is weight per unit length $[\mathrm{t\,m^{-1}}]$.

A more accurate option is to evenly distribute the mass along the element by consistent mass matrix:

$$M_k = \frac{\eta L}{420} \begin{pmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{pmatrix}. \tag{5}$$

In the past, it has been proven that the difference in the calculation is in the order of a few percent, so the diagonal weight matrix is considered an acceptable input, the error is negligible. Commercial softwars often use diagonal form, difference with the consistent one will also be compared in this analysis.

Increment to the weight matrices is a mass matrix with the influence of the bending moment:

$$M_i = \frac{\rho I}{30L} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & 3L & 0 & -36 & 3L \\ 0 & 3L & 4L^2 & 0 & -3L & -L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & -3L & 0 & 36 & -3L \\ 0 & 3L & -L^2 & 0 & -3L & 4L^2 \end{pmatrix}, \tag{6}$$

where

I is material moment of inertia.

The mass moment of inertia is the inertia in rotation around the center of the cross-section. In the vast majority of cases, this has minimal effect. This must be taken into account in the case of very high beams or extremely fast loads.

Inertia in the stiffness matrix corresponding to the rotation of the cross-section. But it is related both to the inertia resulting from the transverse movement and to the inertia resulting from the rotation of the cross-sections. Inertia, related to the member tip rotation parameter, which will come out in a consistent cross-section matrix. If a consistent inertia matrix is considered, then this term generally cannot be neglected.

Combining the element mass and the mass with the influence of the rotational bending moment we recive the local mass matrix:

$$M_L = M_{\mathrm{d/k}} + M_i. \tag{7}$$

2.3. Transformation to global matrices

Using the transformation matrix $[A]$ the local matrix is transformed into the global coordinate system, and we get the global matrix of stiffness:

$$A = \begin{pmatrix} c & s & 0 & 0 & 0 & 0 \\ s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

where

c is cosine of the angle the element subtends from the global system,

s is sine of the angle the element subtends from the global system.

$$\begin{aligned} K_G &= A^T K_L A, \\ M_G &= A^T M_L A, \end{aligned} \tag{9}$$

where

K_G is the global stiffness matrix,

M_G is the global mass matrix.

3. FEM model

To verify the effects of the imputs, an already tested structure was needed, as such cable-stayed footbridge in Čelakovice was chosen. The two-dimensional finite element model was parametrized in MatLab by the project documentation, which was avaible including the static evaluation [4].

3.1. Geometry

This is the footbridge with field spans of 21.5 m + 156 m + 21.5 m, pylons are 36 m high each with 14 pairs of cables (Figure 1).

The bridge deck is supported at the crossing points with pylons and bridge abutments on the shores of the river Labe. The deck is made of UHPC with

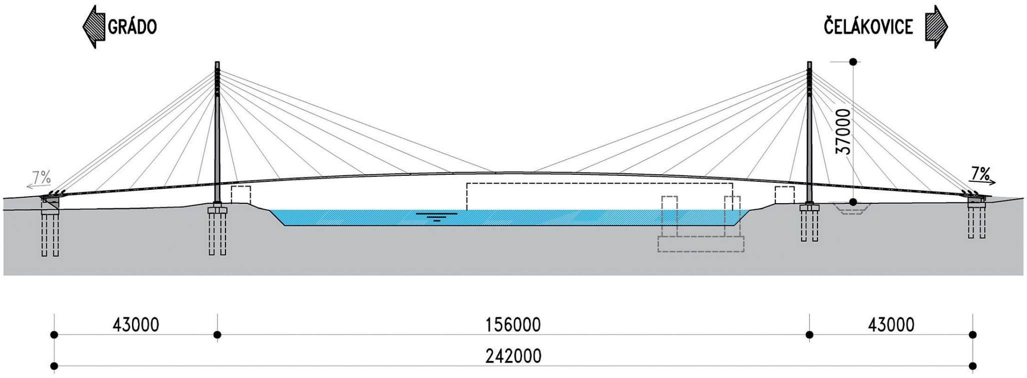

FIGURE 1. Sectional view.

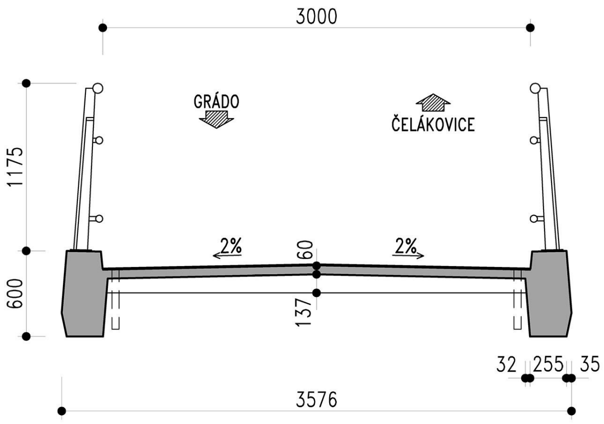

FIGURE 2. Cross section.

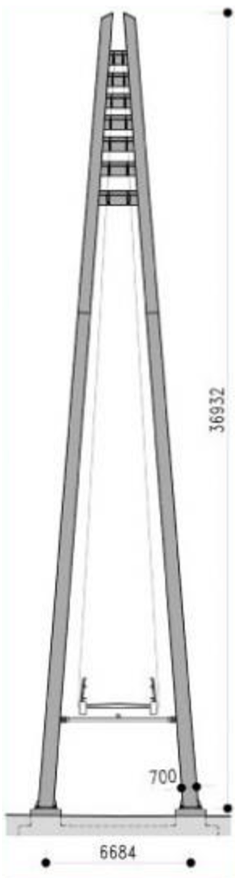

FIGURE 3. Pylon.

a strength C110/130 (Figure 2), the cables and pylons are made of steel (Figure 3).

The mathematical model is parameterized by dividing the construction into elements, where the end nodes at the beginning and end are assigned code numbers. The proposed 2D structure was designed in AutoCad (Figure 4).

3.2. CALCULATION

Elements were further divided so that material and physical characteristics could be assigned to them (Table 1). Each pair of cables was also assigned a prestressing force according to the static calculation (Table 2).

At the connection point of the cables between the pylon and the bridge plate, joints were defined. Where the bearings were supported, the bonds were loosened in the matrices so that the structure could move freely in the permitted directions. Numerically, this meant that at the support locations the rows and columns in the global matrices were zeroed/cleared because there is no unknown displacement but zero displacement/rotation.

For each finite element, the above-mentioned matrices were calculated in the local coordinate system, which were further transformed into the global coordinate system and global matrices K and M were formed. The natural shapes and natural frequences were calculated by direct solution by searching for eigenvalues on the diagonal, the correctness of the results was also verified by the method of inverse iterations. Another verification of the results was the modeling of the structure in the SCIA Engineer, which confirmed the correctness of the procedure (Figure 5).

Since the script is in MatLab (Figure 6), for final results the following command served the necessary outputs:

$$[V, D] = \mathrm{eig}(K, M), \tag{10}$$

where

V are the eigenform vectors,

D are eigenvalues – natural frequencies.

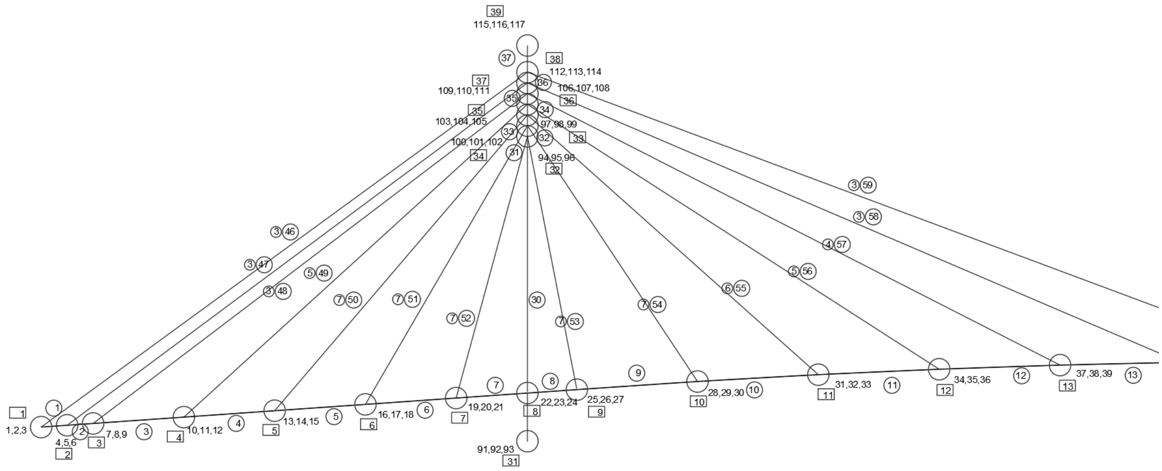

Figure 4. 2D drawing to determine code numbers on the lements.

Element type	E [MPa]	A [m^2]	I [m^4]	η [t m^{-1}]	ρ [kN m^{-3}]
Bridge deck	45 000	0.51368	0.012369	1.3356	26
Pylon	210 000	0.12478	0.0098726	0.9795	78.5
Cables	164464 – 165067	0.0053 – 0.00163	$1.12612 \times 10^{-8} - 1.0571 \times 10^{-7}$	0.0044 – 0.0136	82.53 – 83.70

Table 1. Material and geometric characteristics of the elements.

Cable number	Area [mm^2]	Area of doubled cable [mm^2]	Pre-stressing force [kN]
1 (101, 401)	266	532	75.6
2 (102, 402)	266	532	82.1
3 (103, 403)	266	532	87.1
4 (104 ,404)	521	1 042	133.6
5 (105, 405)	815	1 630	287.8
6 (106, 406)	815	1 630	287.8
7 (107, 407)	815	1 630	282.1
8 (201, 301)	266	532	90.7
9 (202, 302)	266	532	90.5
10 (203, 303)	416	832	117.8
11 (204, 304)	521	1 042	155.7
12 (205, 305)	639	1 278	185.1
13 (206, 306)	815	1 630	205.9
14 (207, 307)	815	1 630	276.8

Table 2. Cable parametres.

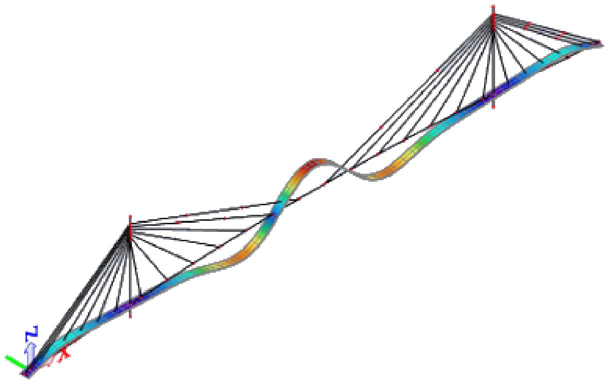

Figure 5. Numerical model verification in SCIA Engineer.

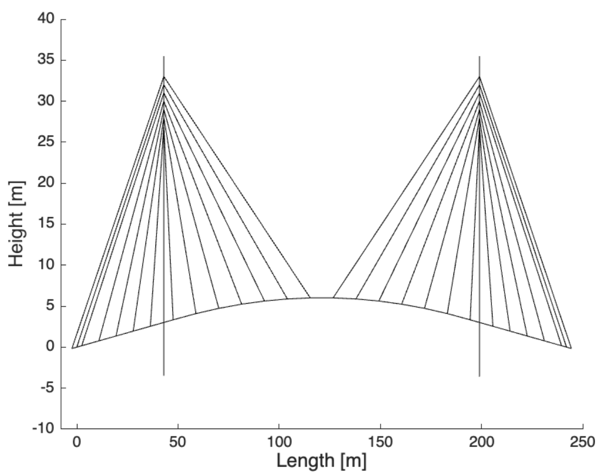

Figure 6. The shape of the structure ploted in MatLab.

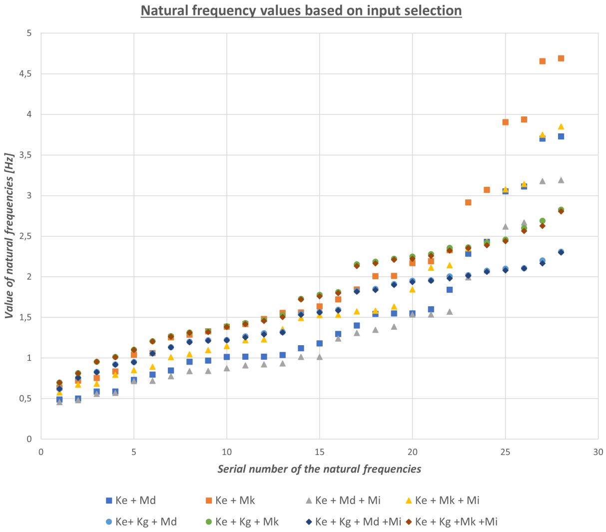

Figure 7. Comparison of calculation combinations.

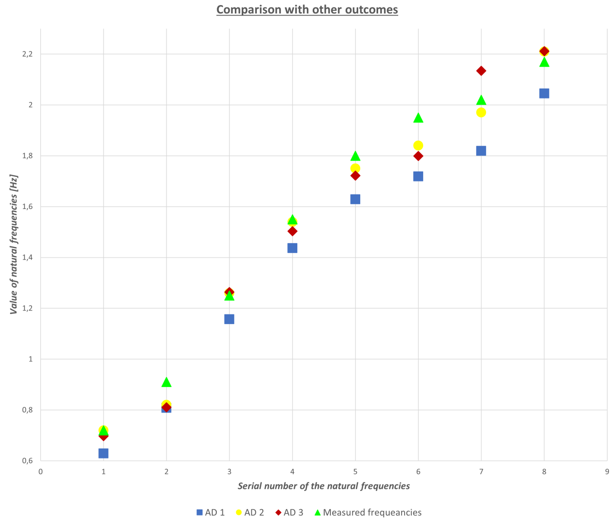

Figure 8. Comparison of vertical frequencies with other results on the same construction [5, 6].

4. Results

Various combinations of the stiffness and mass matrix were used in the dynamic calculation of the eigenshapes and eigenfrequencies. A comparison of individual combinations can be seen on the graph (Figure 7).

The biggest effect is noticeable at higher frequencies, where the curves move away from each other. As expected, the addition of the M_i global mass matrix has the least influence on the calculation, because it is a bridge deck with a low structural height. In the area around 1–2 Hz, which is observed in bridge structures, the difference between the results is up to a two-digit percentage number.

Furthermore, the results of this analysis were compared with the results of others who also calculated vertical frequencies on the same construction. The green curve is the actual measured frequencies on the structure (Figure 8), AD1 and AD2 are provided values from other mathematical models [5, 6], AD3 are the results from the most accurate form of this analysis, which is a combination of $K_e + K_g + M_k + M_i$.

The vertical frequencies are similar, but even if the results are based on the same project documentation, a difference of 5–10 % can be seen [5, 6].

5. Conclusion

The analysis helps for better understanding of the behaviour of cable-stayed bridges in terms of dynamics. In contrast with the commercial software, which are often black boxes, the hand-writed numerical code of a finite element model structure is much clearer in terms of changes in the influence on dynamic behavior. By testing different combinations of accuracy, it was proven that for such a complex construction, accuracy has a large effect on frequency results. It is important to continue to collect data from new UHPC constructions, for possible calibration of models, in order to achieve a correct prediction of the structure's behavior and avoid unnecessary design of dampers. The results confirm the sensitive behavior of such a complex non-linear construction, and with today's computer performance, where there is no need to save on computing memory, it is better to choose a more accurate form of calculation.

Acknowledgements

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References

[1] European Commission, Joint Research Centre, A. Goldack, et al. *Design of lightweight footbridges for human induced vibrations – Background document in support to the implementation, harmonization and further development of the Eurocodes*. Publications Office, 2009. `https://doi.org/doi/10.2788/33846`

[2] Z. Bittnar, P. Řeřicha. *Metoda konečných prvků v dynamice konstrukcí [In Czech]*. SNTL, Praha, 1981.

[3] M. Baťa, V. Plachý, F. Trávníček. *Dynamika stavebních konstrukcí [In Czech]*. SNTL, Praha, 1987.

[4] M. Kalný, J. Komanec, V. Kvasnička, et al. Lávka přes Labe v Čelákovicích – první nosná konstrukce z UHPC v ČR [In Czech; Footbridge over the Elbe river in Čelakovice – the first UHPC superstructure in the Czech Republic]. *Beton* (4):10–18, 2014. [2023-09-01]. `https://www.ebeton.cz/wp-content/uploads/2014-4-10_0.pdf`

[5] J. Štěpánek. *Dynamická analýza lávky pro pěší [In Czech; Dynamic analysis of the footbridge]*. Bachelor's thesis, Czech Technical University in Prague, Faculty of Civil Engineering, 2016.

[6] V. Šáňa. *Vibration of footbridges human-structure interaction*. Ph.D. thesis, Czech Technical University in Prague, Faculty of Civil Engineering, 2016.