

Why a New Code for Novae Evolution and Mass Transfer in Binaries?

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Abstract

One of the most interesting problems in Cataclysmic Variables is the long time scale evolution. This problem appears in long time evolution, which is also very important in the search for the progenitor of SN Ia.

The classical approach to overcome this problem in the simulation of novae evolution is to assume: (1) A constant in time, rate of mass transfer. (2) The mass transfer rate that does not vary throughout the life time of the nova, even when many eruptions are considered.

Here we show that these assumptions are valid only for a single thermonuclear flash and such a calculation cannot be the basis for extrapolation of the behavior over many flashes. In particular, such calculation cannot be used to predict under what conditions an accreting WD may reach the Chandrasekhar mass and collapse.

We report on a new code to attack this problem. The basic idea is to create two parallel processes, one calculating the mass losing star and the other the accreting white dwarf. The two processes communicate continuously with each other and follow the time depended mass loss.

Keywords: nova - modeling thermonuclear runaways.

1 Introduction

The classical prediction or identification of a SN Ia progenitor as a WD in a compact binary system is usually based on the calculation of a single thermonuclear flash or at most few and how much mass the WD gains or loses in such a flash. Extrapolations of the behavior of binary systems based on single or few thermonuclear flashes are not expected to be reliable. Prialnik and Kovetz (1995) were the first to simulate numerically a rather long series of thermonuclear runaway, up to a 1000. These authors solved in this way the question of the initial conditions assuming that after so many flashes, the WD converges to periodic behavior. Idan et al (2013) carried a similar calculation for a high accretion rate and the results were not similar, nor were they strictly periodic. In both calculations the rate of accretion was constant in time and many flashes were calculated. However, the behavior of the mass losing star and its response to mass loss are not uniform in time so that the assumption of constant mass loss (and accretion at a constant rate on the WD) is not justified.

For a star of radius R undergoing mass loss

$$\frac{dR}{dt} = \left(\frac{\partial R}{\partial t}\right)_{ev} + \frac{\partial R}{\partial m} \dot{m}. \quad (1)$$

The index ev means change of R due to normal secular

stellar evolution. One usually assumes that

$$\left(\frac{\partial R}{\partial t}\right)_{ev} \ll \frac{\partial R}{\partial m} \dot{m}. \quad (2)$$

But then $R - R_{Roche} \neq \text{constant}$ and clearly, the expression for mass loss due to Roche lobe overflow yields a non constant rate of mass loss, as this expression depends on $R(t)$.

Note that if we assume that $dR/dt = 0$, namely an equilibrium or a steady state and we neglect the time variability of R_{Roche} , then

$$\dot{m} = - \left(\frac{\partial R}{\partial t}\right)_{ev} / \frac{\partial R}{\partial m} \ll 1, \quad (3)$$

in units of solar radius per solar mass. Hence this expression does not yield observable values and does not imply that $\dot{m}(t) = \text{Const}$. Hence, this assumption is unacceptable.

Webbink (1977) evaluated the mass derivative of the Roche radius and obtained:

$$\left(\frac{\partial \ln R_{Roche}}{\partial \ln M}\right)_{\bar{M}, J} = f(q) \quad (4)$$

where $q = \ln(M + 0.005M_*) - 0.5 \ln P - \ln(P + 10^{16})$ has to do with the mass shell division of the calculation. Webbink also assumed that

$$A = \left(\frac{d \ln R}{d \ln M}\right)_t \approx \frac{d \ln R}{d \ln M} \ll \frac{d \ln R_{Roche}}{d \ln M} \quad (5)$$

where A is the adiabatic constant and the inequality holds for all \dot{m} .

2 Mass Loss Rate

There are several empirical expressions for the mass loss. Webbink (1977) for example, assumed that

$$\dot{m} = -\lambda \left(\frac{R - R_{Roche}}{R} \right)^2 \text{ with } \lambda = \text{const} \quad (6)$$

provided $R - R_{Roche} > 0$, while Ritter (1988) wrote that

$$\dot{m} = \dot{m}_0 \exp \left(\frac{R - R_{Roche}}{h_p} \right) \text{ for } R - R_{Roche} > 0. \quad (7)$$

Here m_0 is a constant to be evaluated from the geometry of the Roche lobe while H_p is the pressure scale height near the L_3 point. It is clear that if these expressions are valid, then a change caused by a change in the radius of the star affects the accretion rate (and the nova long time evolution).

3 Time Scale Involved

The reaction of the donor star depends on three time scales. (a) The Kelvin-Helmholtz-Ritter time scale given by:

$$\tau_{KHR} = \frac{GM_{donor}^2}{RL_{donor}} \left(\frac{\Delta m}{M_{donor}} \right), \quad (8)$$

where Δm is the mass affected by the mass loss perturbation. (b) The accretion time scale

$$\tau_{acc} = \frac{\Delta m_{flash}}{\dot{m}} \quad (9)$$

where Δm_{flash} is the accreted mass at which the thermonuclear runaway takes place (of the order of $10^{-5} M_\odot$) and (c) the dynamic time scale is given by:

$$\tau_{dyn} = \sqrt{\frac{3}{4\pi G \bar{\rho}}}. \quad (10)$$

Here $\bar{\rho}$ is the mean density of the star. Only m_{flash} , the mass at which the nuclear flash occurs, depends (slightly) on the mass of the WD. The dynamic time depends on the entire star and τ_{KHR} depends on the outermost mass-shell involved. The interplay between these three time scales controls the phenomenon and it varies with the rate of mass loss. Consequently, we calculated the hydrodynamic and thermodynamic evolution of the star under the condition that the accretion rate is given (by the parameters of the binary system). As the present calculation is carried out irrespective of the mass of the accretor, we cannot evaluate the accretion rate but have to impose it as given.

4 The Dynamic Behavior of the Donor

Our goal is to investigate the dynamic response of the donor star to mass loss. We assume spherical symmetry and solve the full hydrodynamic equations of the donor and evaluated the requested derivatives. In Figure 1 we see how dR/dM behaves in the case of low accretion rate ($10^{-10} M_\odot/\text{yr}$). The donor is a Main Sequence star of $1.25 M_\odot$.

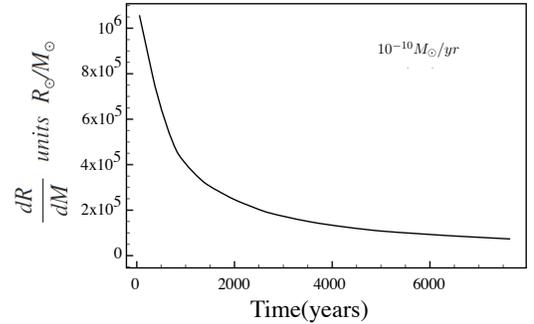


Figure 1: The derivative $\frac{dR}{dM}$ in units of M_\odot/τ_{KHR} for an imposed mass loss of $10^{-10} M_\odot/\text{yr}$. In this case $\tau_{flash} \approx 10^5 \text{yr}$.

We realize that neither dR/dt nor $(\partial R/\partial m)\dot{m}$ are constant and one should not expect the accretion rate to be constant (in time) either. Even during the period of mass building for a single flash, the accretion rate is not constant.

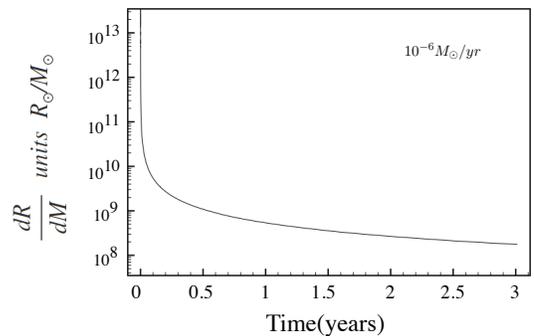


Figure 2: The derivative dR/dM in units of M_\odot/τ_{KHR} for an imposed mass loss of $10^{-6} M_\odot/\text{yr}$. In this case $\tau_{flash} \approx 10 \text{yr}$.

The results for all accretion rates are collected and summarized in Figure 3.

We see that in all cases, irrespective of the accretion rate, the time dependence of the derivative is given by:

$$\frac{dR}{dM} = \frac{14.07}{t^{2.35} + \epsilon(\dot{m})} \text{ for all } \dot{m}, \quad (11)$$

where $\epsilon(\dot{m})$ is a constant in time which depends on \dot{m} .

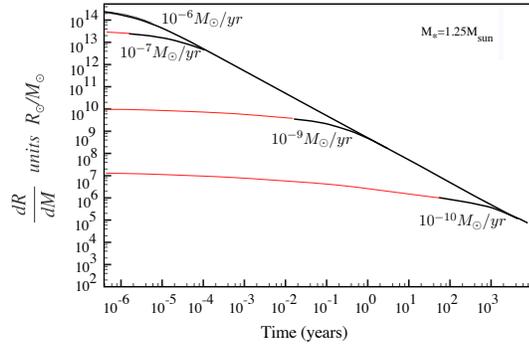


Figure 3: The derivative dR/dM in units of M_\odot/τ_{KHR} for all accretion rates calculated here (for a MS star of mass $1M_\odot$).

Moreover, time dependence of the derivatives tend for sufficiently long times, to an asymptote. We find that:

$$\text{for } t \gg \epsilon(\dot{m}) \quad \frac{dR}{dM} \rightarrow \frac{14.07}{t^{2.35}}, \quad (12)$$

namely, all results (for a given mass of the donor) converge for long times to an asymptote. We do not know at the moment how this asymptote changes with the mass of the donor.

At the same time we can write for the Roche lobe radius (Eggleton, 1984) that:

$$R_{Roche} \approx \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})} \text{ where } q = \frac{M_{wd} + \dot{m}t}{M_d - \dot{m}t} \quad (13)$$

assuming conservative mass loss. Hence for sufficiently small t (at the beginning) we have that:

$$\left| \frac{dR_{Roche}}{dM} \right| \ll \left| \frac{dR_{donor}}{dM} \right| \quad (14)$$

The thermodynamic state is shown in Figure 4

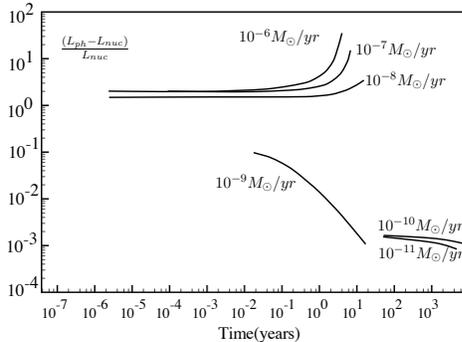


Figure 4: The time to reach thermodynamic equilibrium.

We see that stars with mass loss rate smaller than $10^{-9}M_\odot$ tend to thermal equilibrium. The timescales

to reach the thermal equilibrium vary. Accretion rates higher than $10^{-9}M_\odot$ diverge, namely they become unstable and runaway. The rate of accretion drives the star out of thermal equilibrium to be never restored. This fact should be taken into account in evaluating the mass loss rate from the donor.

5 Conclusions

First conclusion: It is not justified to assume that τ_{KHR} is negligible. Second, polytropic estimates are nice and simple, but wrong (Motl et al.2002) The mass loss is not constant in time. The mass loss does not start suddenly and reaches the assumed value gradually.

There is no fixed period between eruptions. The system can have n eruptions with an almost constant time interval and then pause and let the donor recover on a Kelvin-Helmholtz-Ritter time scale of its envelope. During this time the WD may relax to a new state.

The accretion rate changes in time and affects the evolution of the nova. A nova calculation must include the evolution of the donor and the accretion rate. An important element in the evolution of nova is the time variability of the accretion rate.

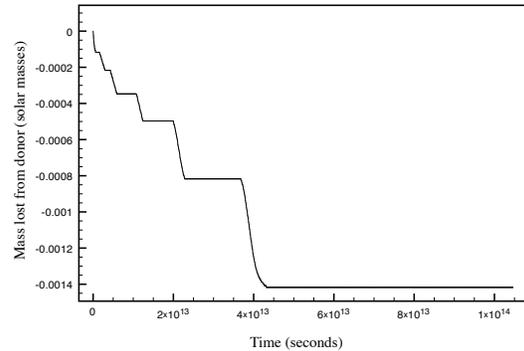


Figure 5: The time depended mass loss from the donor.

6 Discussion and Conclusions

A significant part of present day interest in nova includes the cases that may become progenitors of SN Ia. In this case the behavior of the binary system is followed through a single or few thermonuclear runaways and then the result is extrapolated over 6-7 orders of magnitudes. The mass accreted or lost is of the order of $10^{-7}M_\odot$. The initial mass of the WD is of the order of $1M_\odot$ and if a conservative mass transfer is assumed this means a huge extrapolation in the behavior of the accretor and the the donor.

We conclude that the evolution of the two stars must be followed simultaneously. We developed a code which

does just that. Two processors are created, each devoted to a star. Thus the donor star is calculated on one CPU and the accretor on a second CPU. The two processes, which may run on different computers or on a computer with more than one CPU, communicate with one another via an open gate. The communication can take place at fixed time intervals or whenever the conditions on one star deviate significantly and an update is due.

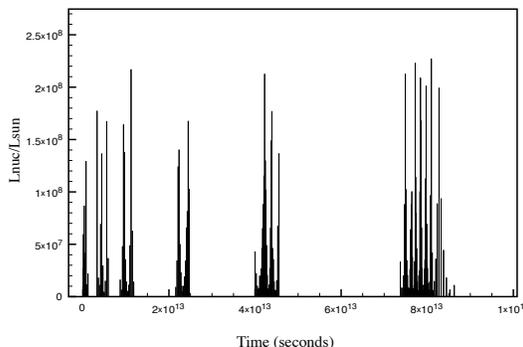


Figure 6: The flashes of nuclear energy as a function of time.

The code is in the debugging phase but the results seem to justify the claims present here. We find that the accretion rate is not constant in time and it stops when the radius of the donor shrinks below the Roche lobe radius. In Figure 5 we show one such example. The periods of no accretion appear as horizontal sections (no change in mass lost). The period of mass loss appear as decreasing lines. Note that these parts of the

curve are straight lines at the beginning but not later. Hence the rate of mass loss changes even during accretion. The recovery time depends on the mass loss rate and the τ_{KHR} of the donor.

The resulting nuclear flushes are shown in Figure 6. We see that the picture of flashes at a constant rate is correct only of a couple of flashes and the flashes come in groups. The time between the groups, the relaxation time of the donor increases gradually.

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