

ON SAMPLING BASED METHODS FOR THE DUBINS TRAVELING SALESMAN PROBLEM WITH NEIGHBORHOODS

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ABSTRACT. In this paper, we address the problem of path planning to visit a set of regions by Dubins vehicle, which is also known as the Dubins Traveling Salesman Problem Neighborhoods (DTSPN). We propose a modification of the existing sampling-based approach to determine increasing number of samples per goal region and thus improve the solution quality if a more computational time is available. The proposed modification of the sampling-based algorithm has been compared with performance of existing approaches for the DTSPN and results of the quality of the found solutions and the required computational time are presented in the paper.

KEYWORDS: dubins vehicle, dubins maneuver, DTSP, DTSPN.

1. INTRODUCTION

Path planning for a non-holonomic vehicle is a fundamental problem of surveillance mission where an unmanned aerial vehicle (such as a fixed-wing) is requested to visit a given set of locations. The basic model of such a vehicle with the limited turning radius is called the Dubins vehicle [1] for which the combinatorial problem of finding optimal sequence of visits to the locations is known as the *Dubins Traveling Salesman Problem* (DTSP) [2].

In this paper, we consider a generalized problem of the DTSP where the particular waypoints to be visited can be selected from a set of possible locations. Due to the similarity of this problem with the so-called *Traveling Salesman Problem with Neighborhoods* [3], the problem is called as the *Dubins Traveling Salesman Problem with Neighborhoods* (DTSPN) [4].

The DTSPN is a suitable problem formulation to address surveillance missions with unmanned aerial vehicles, where it is required to take a camera snapshot (or other type of measurement) of each target location. Such a snapshot can be acquired from some distance of the target location and thus it is not necessary to visit the location exactly. It is rather a more suitable to select the waypoints in such a way that all locations are covered while the total cost of the final path is minimal.

There are several approaches to address the DTSP and also the DTSPN in the literature [2, 4–6] including our work [7]. Therefore, in this paper, we provide an overview of the existing approaches to address the DTSPN and compare their performance according to the trade-off between the solution quality and computational requirements. In particular, we focus on the sampling-based algorithm [4] which is able to provide high quality solutions for very high number of samples. However, more samples increase the computational burden, and therefore, the algorithm is not directly suitable for real-time applications. On the other hand,

we can get a quick estimation of the solution quality using only few samples. This has motivated us to modify the original algorithm [4] to provide a first solution quickly that is then improved if a computational time is left.

The paper is organized as follows. A brief introduction to the problem background is presented in the next section. The addressed problem is formally introduced in Section 3. A brief overview of the existing approaches to solve the DTSPN is provided in Section 4. The proposed modification of the sampling-based approach is presented in Section 5 together with the analysis of its computational complexity. Evaluation results of the comparison of the existing approaches with the proposed modification are reported in Section 6. Finally concluding remarks are denoted to Section 7.

2. RELATED WORK

The problem of curvature-constrained path planning has been studied years ago and the fundamental work has been published in 1957 by Dubins. In [1], he proved that the optimal path between two configurations $q_1, q_2 \in SE(2)$ consists of only straight line segments and segments with the minimum turning radius. He also showed that only 6 maneuvers can be optimal, which can be further divided into two main types:

- CCC type: LRL, RLR;
- CSC type: LSL, LSR, RSL, RSR;

where C stands for a circle turn (R – right, L – left) and S for a straight segment.

Even though the optimal path for Dubins vehicle between two configurations is known and it is straightforward to compute, this is not sufficient to directly solve the DTSP. It is because the orientation of the vehicle at the waypoints is not known and thus it must be determined together with the optimal sequence of

visits to the waypoints. Moreover, the optimal orientation depends on the sequence and vice-versa, which make the problem difficult to address.

Probably the simplest approach to address the DTSP is the *Alternating Algorithm* (AA) proposed in [2]. In this approach, headings are established in the way that even edges are connected by straight line segments and odd edges are connected by Dubins maneuvers. In addition, the authors show that the length of the optimal solution of the DTSP can be bounded by $L_{TSP}\kappa\lceil n/2\rceil\pi\rho$, where ρ is minimum turning radius, L_{TSP} is the length of the optimal solution of the Euclidean TSP, n is the number of the goals, and $\kappa < 2.658$.

Based on the similar idea, authors of [5] proposed a receding horizon algorithm called the look-ahead (LA) approach. The heading is determined with respect to the next point in the sequence. This algorithm investigates each point only once, and therefore, the LA approach is very fast and suitable for real-time application while the authors reported better results than the AA.

The optimal solution of the Dubins planning to visit a given sequence of waypoints that are at the distance longer than 4ρ is presented in [8]. The approach is based on the convex optimization; however, the optimization needs to be solved several times because of possible alternation of the maneuvers directions. The authors bound the number of possible combinations to 2^{n-2} for n waypoints.

The DTSPN is a generalization of the DTSP, where each goal is extended by a neighborhood (goal region). As the DTSP is known to be NP-hard [6], also its generalization the DTSPN is NP-hard.

3. PROBLEM DEFINITION

The addressed problem is motivated by surveillance missions with fixed-wing aerial vehicles, which are nonholonomic vehicles due to their kinodynamic constraints. These vehicles are often modeled as the Dubins vehicle [1], which can go only forward at a constant speed and has a minimum turning radius ρ . The configuration space of such a vehicle can be represented as $SE(2)$, where each configuration $q \in SE(2)$ is a triplet (x, y, θ) , where (x, y) is the vehicle position in a plane and $\theta \in \mathbb{S}^1$ is the orientation of the vehicle. The mathematical model of the Dubins vehicle can be formulated as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{u}{\rho} \end{bmatrix}, \quad |u| \leq 1, \quad (1)$$

where v is the vehicle forward velocity, ρ is the minimum turning radius, and u is the control input, $u > 0$.

Now, we can introduce the DTSPN [9] a more formally. Let $\mathcal{R} = \{R_1, \dots, R_n\}$ be a set of n regions $R_i \subset \mathbb{R}^2$ that are requested to be visited by the Dubins vehicle and let $\Sigma = (\sigma_1, \dots, \sigma_n)$ be an ordered permutation of $\{1, \dots, n\}$. We define a projection

from $SE(2)$ to \mathbb{R}^2 , i.e., $\mathcal{P}(q) = (x, y)$, and let q_i be an element of $SE(2)$ whose projection lies in R_i .

The DTSPN is path planning problem where the Dubins vehicle has to visit each region R_i while satisfying the kinodynamic constraints of (1). Every optimal path has to intersect each goal region R_i in at least one configuration q_i . Hence, we can treat the DTSPN as an optimization problem over all possible permutations Σ and configurations (q_1, \dots, q_n) as follows:

Problem 3.1 (DTSPN).

$$\begin{aligned} \text{minimize}_{\Sigma, q} \quad & \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) + \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) \\ \text{subject to} \quad & \mathcal{P}(q_i) \in R_i, \quad i = 1, \dots, n \end{aligned}$$

where $\mathcal{L}(q_i, q_j)$ is the Dubins distance between q_i and q_j .

In this paper, we are focused on the problem with regions R_i that are mutually exclusive. Hence, we can define the minimum distance constraint D_K such that for all $i, j \in \{1, 2, \dots, n\}, i \neq j, \forall p_i \in R_i, \forall p_j \in R_j$:

$$\|p_i - p_j\| > K\rho. \quad (2)$$

In particular, we are focused on the problem for which the minimum distance D_K is equal to 4ρ , which is denoted as the D_4 constraint in the rest of this paper. Problem instances of the DTSPN with the D_4 constraint have special properties that are shown and used in [7] to find high quality solutions. In this paper, we also consider D_4 problems and compare the existing solutions with the sampling based methods. An overview of the existing methods is provided in the next section.

4. EXISTING METHODS

In literature, we can find several different approaches to address the DTSPN. The existing approaches can be divided into three main classes: 1) the decoupled methods; 2) sampling-based methods; 3) and evolutionary methods. Representatives of each particular class are briefly described in the following subsections.

4.1. DECOUPLED METHODS

A decoupled method addresses the DTSPN by decomposition of the problem into two sub-problems. First, the permutation of the visits to the requested areas (locations) is found independently on the Dubins path planning. The second sub-problem is to find a particular visiting configuration of each location. In this way, the original DTSPN is transformed to the DTSP with point locations to visit.

In [10], the authors address the DTSPN by solving the Euclidean TSP where cities represents centers of the regions to be visited. Once such a permutation is found, the final solution is constructed using the AA [2].

Authors of [7] also consider a solution the ETSP to estimate the permutation of the visits. But instead of simple AA, the proposed local iterative optimization (LIO) is used to find a significantly better solution.

4.2. SAMPLING-BASED METHODS

Sampling-based methods [4, 11] are another class of existing approaches to address the DTSPN. Similarly to the decoupled methods, also sampling-based methods are based on existing approaches for the TSP variants; however, in this case a single representative of each region is not considered as in [10] but each region is rather sampled by a particular number of random samples (including the orientation of the vehicle). The samples can be determined in the whole region or only on its boundary. Then, the samples of different regions are connected by the Dubins maneuver to form a roadmap. Such a roadmap is considered as an instance of the Generalized Asymmetric TSP (GATSP). If the GATSP is solved to the optimum, the sampling-based methods are resolution complete [11].

Since the optimal solution of the GATSP can be computationally very demanding, existing heuristic for the TSP can be a more suitable option. The generated GATSP can be solved by its transformation to the Asymmetric TSP (ATSP) using the Noon-Bean transformation [12]. Then, the transformed ATSP can be solved by existing algorithms, e.g., the Lin-Kernighan heuristic algorithm [13].

4.3. EVOLUTIONARY METHODS

The last class of existing approaches are genetic programming based algorithms. The general idea of these methods is to encode a solution by a chromosome in which the goal permutation and the configurations of the visits are stored. Similarly to another approaches, only configurations on the boundary of the regions are considered in these evolutionary methods.

Probably the first evolutionary approach to the DTSPN was published in [14]. The authors adapted the *Ordered Crossover* operator (OX) and added two new mutation operators (*orientation shift* and *position shift*) to optimize visiting configurations.

Relatively recently, the genetic approach from [14] was modified into a memetic algorithm in [15]. The authors used similar operators and added a local improvement of individuals in the population by optimizing the position of visiting points. The authors used AA as heuristic algorithm to speed up the algorithm.

5. MODIFICATION OF EXISTING SAMPLING-BASED ALGORITHM

The main issue of sampling based algorithms for the DTSPN is that they require a fixed number of samples, which need to be established in advance. We propose an iterative schema to avoid this issue and set the number of samples progressively, which provides first solutions very quickly. Moreover, if there is some

additional computational time, the algorithm can iteratively improve the solution by adding more samples, which results to obtain a first solution quickly that is further improved.

In the basic sampling-based algorithm, a roadmap with $(n \cdot m)^2$ edges is created where n is the number of regions to be visited and m is the number of samples per region. For each edge one Dubins maneuver is constructed. Since the Noon-Bean transformation does not change the number of vertices, the overall time complexity of generating an instance of the ATSP can be bounded by $\mathcal{O}((n \cdot m)^2)$.

A solution of the generated ATSP can be found by the available LKH solver. According to the author of the LKH, the time complexity of the LKH is approximately $\mathcal{O}(n_{ATSP}^{2.2})$ [16]. Hence, the expected total time complexity is $\mathcal{O}((n \cdot m)^{2.2})$. The time complexity to solve the ATSP is greater than a roadmap generation, and therefore, we do not need to explicitly consider the construction of the roadmap in the estimation of the required computational time based on the number of samples.

Based on the relation of the required number of operations on the number of samples, we can establish a sequence of the increasing numbers of the samples needed to find an initial solution quickly and improve its quality later. The sampling-based algorithm works with the constant number of samples given a priori, we can run the sampling based algorithm repeatedly with an increasing number of samples. Since the time complexity of the algorithm is nearly quadratic, an inverse function for the number of samples m_k according to the number k of the particular iteration can be defined as:

$$m_k \approx \sqrt{2^k}. \quad (3)$$

After rounding the m_k to an integer value, it gives us the following series of the numbers:

$$M = \{1, 2, 3, 4, 6, 8, 11, 16, 23, 32, 45, 64, \dots\}. \quad (4)$$

Figure 1 provides influence of the required computa-

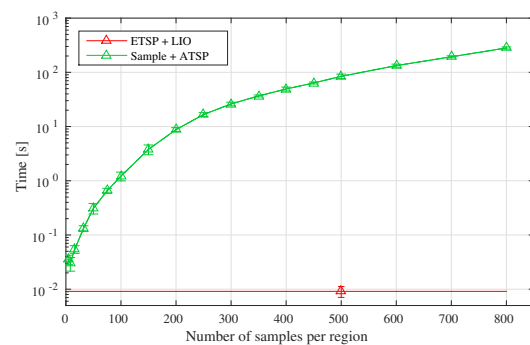


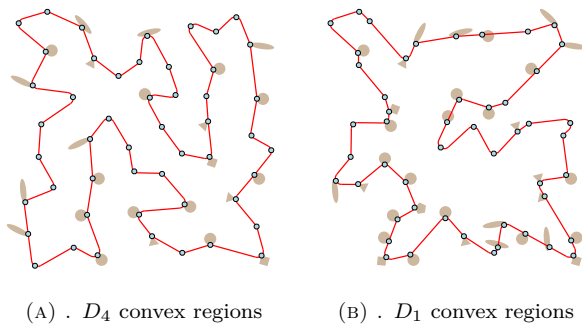
FIGURE 1. The average required computational time (from 20 trials) for the instance of the DTSPN with 4 circle regions and D_4 constraint.

tional time on the number of samples per each region

to visit. We plot the required computational time on the number of samples per each region to visit. In this case, a simple problem with 4 regions and D_4 constraint is considered and the computational time is compared with the ETSP+LIO algorithm proposed in [7].

6. RESULTS

The performance of the proposed modification of the sampling based algorithm has been evaluated in a series of scenarios. The evaluated instances of the DTSPN were generated randomly for convex regions satisfying the D_4 constraint. Several shapes of the regions have been considered: points, disks with the radius equal to ρ , ellipses with the semi-axis 2ρ and 0.5ρ , and convex polygons with up to 6 vertices created from a circle with the radius ρ . A relatively high and uniform density of the regions for the D_4 constraint has been considered by generating centers of the regions inside a bounding box with the side $6\sqrt{n}\rho$, where n is the number of the regions to be visited by the Dubins vehicles. An example of the examined problems is depicted in Figure 2.



(A) . D_4 convex regions (B) . D_1 convex regions

FIGURE 2. Examples of the randomly generated instances of the DTSPN.

The examined algorithms are denoted as follows. The newly proposed modification of the sampling-based algorithm is denoted **Sample+ATSP**. The algorithm is compared it with the evolutionary approaches [14] and [15] denoted as **Genetic** and **Memetic**, respectively. In addition, we implemented three variants of the decoupled approach based on the solution of the ETSP utilized as the heuristic estimation of the permutation of the visits to the regions. The first decoupled approach is denoted as **ETSPN+AA** and it is based on the Alternating Algorithm (AA) [2]. The second method is denoted as the **ETSP+LIO** [7] and is based on local optimization of position and heading of the waypoints. The last method is derived from the **ETSP+LIO**, but only local iterative optimization of the heading is considered, this method is denoted as the **ETSPN+HoLIO**.

The quality of found solutions has been evaluated regarding a dedicated computational time for problems with $n = 20$ and $n = 40$ regions with the D_4

constraint. The quality is measured as the ratio of the path length found by the particular algorithm to the solution provided by the ETSP+LIO algorithm, similarly as in [7]. All instances were generated randomly, and therefore, 50 trials have been solved for each problem and the algorithm variants. The achieved results are depicted in Figure 3 for the 20 regions and in Figure 4 for the 40 goal regions.

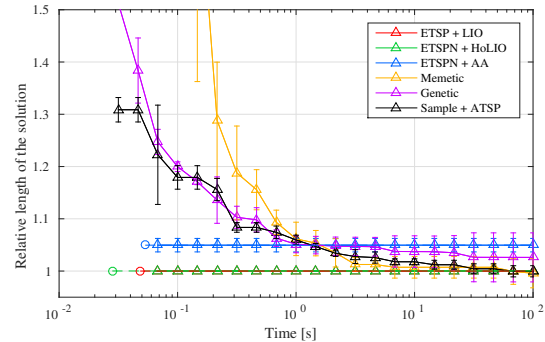


FIGURE 3. Average ratio of the tour length (from 50 trials) according to the ETSP+LIO solution for the DTSPN with $n=20$ convex regions. Plots start from the time when the first solution is available.

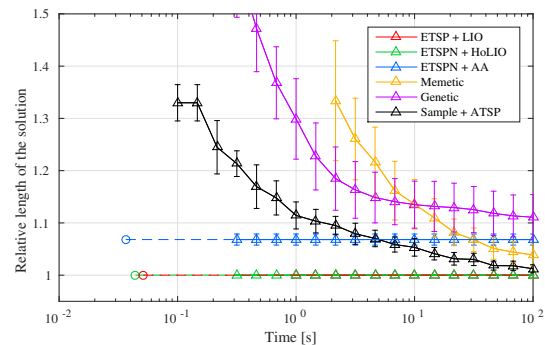


FIGURE 4. Average ratio of the tour length (from 50 trials) according to the ETSP+LIO solution for the DTSPN with $n=40$ convex regions. Plots start from the time when the first solution is available.

All the algorithms have been implemented in C++ and tested on a single core of the Intel Core i5-M480 CPU running at 2.67 GHz. The processor was accompanied with 4 GB RAM.

6.1. DISCUSSION

The presented results show that the proposed modification of the sampling-based algorithm can be considered as a meaningful alternative to the evolutionary based algorithms. In the case of the high number of regions to visit ($n = 40$), the modified sampling-based algorithm has even superior results to the both **Genetic** and **Memetic** algorithms, while it is less computationally demanding.

In the results depicted in Figures 3 and 4, the ETSP+LIO algorithm achieved the best results among

the evaluated algorithms. This is caused by the fact, that all instances of the DTSPN were generated with non-overlapping regions with the D_4 constraint and the ETSP+LIO has been designed on top of the properties of the optimal solution of such a problem and thus it directly searches for good solutions [7].

7. CONCLUSIONS

In this paper, we proposed a modification of the sampling-based algorithm for the DTSPN. The proposed algorithm repeatably executes the original sampling-based algorithm with an increasing number of the samples per each region. By this modification, the newly developed algorithm provides the first solution very quickly that can be further improve if more time is available. This makes the algorithm suitable in situations where a solution has to be found quickly and where the maximum time that can be dedicated for the computation is not known in advance.

We also compared the modified algorithm with other existing approaches on the randomly generated instances of the DTSPN with the D_4 constraint. The modified algorithm provides better results than the implemented evolutionary algorithms while it is less computationally demanding.

A further comparison of the algorithms' performance in the instances of the DTSPN where regions to visit are closer or overlapping is a subject of our future work.

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