RELIABILITY OF EXISTING REINFORCED CONCRETE SLABS EXPOSED TO PUNCHING SHEAR

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Abstract. Selected standardised models for the verification of punching shear in reinforced concrete structures are applied for the probabilistic assessment of their reliability level. It appears that the models given in EN 1992-1-1 and prEN 1992-1-1 lead to more realistic estimates of the reliability level of existing reinforced concrete members with respect to punching shear than the models recommended in some national codes. The controlled perimeter has significant influence on the results and should be harmonized in prescriptive documents.

Keywords: Assessment of existing structures, Eurocodes, national codes, partial factors, probabilistic models, punching, reinforced concrete slab, reliability.

1. INTRODUCTION

Construction works are designed using provisions specified in national, European or international standards. The Eurocodes allow the national selection of Nationally Determined Parameters (NDPs) including alternative design approaches, load combinations, partial factors and other safety elements. The actual reliability of a designed structure depends on applied national standards or the NDPs recommended in the National Annexes to Eurocodes. Presently, the 2nd generation of Eurocodes is nearly finished where some NDPs have been removed, some provisions simplified or better clarified and selected theoretical models updated. The National Annexes should be newly prepared with guidance on selecting NDPs in the CEN member countries.

The load-bearing capacity, serviceability or durability of a particular structure designed in accordance with original national standards or nationally implemented Eurocodes can be expected to be within a broad range. The actual structural resistance depends not only on used theoretical models and selected safety elements (including partial factors), but also on prescriptive rules recommended in applied standards, structural detailing (e.g. requirements on the reinforcement concrete cover, reinforcement ratio) and limiting factors (e.g. deflection, crack width).

It was shown that the reliability of reinforced concrete (RC) members designed to the ultimate limit state of punching shear according to prescriptive documents might have a considerable scatter and should be further harmonised [1,2]. The design of slab-column connection is the most critical for reliability of flat slabs as the localised concentration of shear stresses may lead to the progressive collapse due to punching [3].

This paper is focused on the evaluation of the design procedures for punching shear given in the currently valid Eurocodes, newly prepared Eurocodes and in original Czech national standard. The prescriptive procedures for punching shear are also applied in case studies for the reliability verification of existing RC slabs in residential houses in Prague designed according to the original Czech codes. Furthermore, the appearance of cracking and deformations in partition walls and RC slabs designed to the Czech standards precipitated the need to verify the reliability of existing buildings according to the presently valid Eurocodes.

2. DESIGN PROCEDURES FOR PUNCHING SHEAR VERIFICATION

2.1. INTRODUCTION

Various prescriptive documents provide approaches to reliability verifications with respect to punching shear of newly designed or existing concrete structures. There are significant differences in the models as some are empirical while the others are based on physical models. The theoretical models recommended in the Eurocodes EN 1992-1-1 [4] and prEN 1992-1-1 [5] (hereafter abbreviated as “EN 1992” and “prEN 1992”) are analysed and the results compared with those based on the relationship given in the original Czech national code CSN 73 1201 [6]. Such a comparison is particularly useful for the assessment of existing structures as it indicates whether slabs designed according to old standards are to be expected to have insufficient reliability.

The punching shear resistance of RC flat slabs without shear reinforcement is mainly influenced by the concrete compressive strength, tensile flexural reinforcement ratio, size and geometry of the column and the slab depth [1,2]. The influence of the basic variables on obtained reliability levels is further investigated. This provides the background information for surveys of existing structures that should be mainly...
focused on updating information about the variables primarily affecting reliability level.

2.2. EN 1992-1-1

Eurocode EN 1992 [4] provides the following expression for estimating the punching shear resistance of selected cross section (concrete contribution):

\[ V_{Rcd} = u d (C_{Rd,c} k (100 \rho f_{ck}) + k_1 \sigma_{cp}), \]

\[ (1) \]

where \( C_{Rd,c} \) is 0.18/\( \gamma_c \), \( k_1 = 0.1 \), \( \rho = (\rho_s \rho_y)^{0.5} \) with the reinforcement ratios \( \rho_s \) and \( \rho_y \) related to the bonded steel in tension in \( x \)- and \( y \)-directions respectively, \( u \) denotes the critical section, and \( d \) is the effective depth of the slab. The critical section \( u = 2d \) is to be verified.

In case where shear reinforcement is required, the punching shear resistance is a sum of concrete and reinforcement contributions [4]:

\[ V_{Rd} = 0.75 V_{Rcd} + 1.5d/s_r A_{sw} f_{ywd,sf} (1/(\sin \alpha) \sin \alpha) \]

\[ (2) \]

where \( A_{sw} \) is the total area of shear reinforcement placed around the column that crosses the critical inclined crack (considering approximately the reinforcement placed between 0.5d and 1.5d from the face of the column), \( s_r \) is the radial spacing of perimeters of shear reinforcement, \( f_{ywd,sf} \) is the effective design strength of the punching shear reinforcement, \( d \) is the mean of the effective depths in the orthogonal directions, \( \alpha \) is the angle between the shear reinforcement and the plane of the slab.

Adjacent to the column, punching shear resistance is limited to a maximum of [4]:

\[ V_{Ed} = \frac{\beta V_{Ed}}{u_0 d} \leq V_{Rd} = 0.5 v_{fzd} \]

\[ (3) \]

where \( u_0 \) is the length of perimeter for an inner column. The control perimeter \( u_{out} \) at which shear reinforcement is not required is given as [4]:

\[ u_{out} = \frac{\beta V_{Ed}}{V_{Rcd} d} \]

\[ (4) \]

The outermost perimeter of shear reinforcement should be placed at a distance not greater than \( kd \) but within \( u_{out} \).

2.3. PrEN 1992-1-1

The recently developed version of the Eurocode prEN 1992 [5] provides the following expression for estimating the punching shear strength of a concrete slab:

\[ \tau_{Rcd} = 0.6/\gamma_s k_{pb} (100 \rho_1 f_{ck} d_{dy}/d_v)^{1/3} \]

\[ \leq 0.6/\gamma_s f_{ck}^{3/5} \]

\[ (5) \]

where \( d_{dy} \) is a size parameter describing the failure zone roughness, which depends on the concrete type and its aggregate properties (newly introduced in the model), \( k_{pb} \) is the punching shear gradient enhancement coefficient, \( d_v \) is the resisting effective depth of the slab, and \( \gamma_s \) is the partial factor for shear and punching resistance without shear reinforcement.

The design shear stress \( \tau_{Ed} \) can be calculated as:

\[ \tau_{Ed} = \beta \nu_{Ed}/(b_{0.5} d_v), \]

\[ (6) \]

where \( V_{Ed} \) is the design shear force at the relevant control perimeter, \( \beta \) is the coefficient accounting for concentrations of the shear forces and \( b_{0.5} \) is the length of the control perimeter. The following reliability condition is to be checked:

\[ \tau_{Ed} \leq \tau_{Rcd} \]

\[ (7) \]

In the case where the reliability condition is not met then the punching shear reinforcement should be provided, and a further control perimeter where shear reinforcement is no longer required shall be checked.

Where shear reinforcement is required, it should be calculated as:

\[ \tau_{Rcs,d} = \eta_c \tau_{Rcs,d} + \eta_p \rho_w f_{ywd}. \]

\[ (8) \]

where \( \eta_c = \tau_{Rcd}/\tau_{Ed} \), \( \eta_p = d_v/(150 \varphi c) + (15d_{dy}/d_v)^{1.5}(\eta_c k_{pb})^{-1.5} \) and \( \rho_w \) is the shear reinforcement ratio at the investigated control perimeter, \( \rho_w = A_{sw}/s_r s_t \) where \( s_r \) is the radial spacing of shear reinforcement and \( s_t \) is the average tangential spacing of perimeters of shear reinforcement measured at the investigated control perimeter.

The punching shear resistance shall be limited to a maximum of \( \tau_{R,d,max} = \eta_{sys} \tau_{Rcs,d} \) where a coefficient \( \eta_{sys} \) accounts for the performance of punching shear reinforcing systems.

The outer control perimeter at which shear reinforcement is not required should be calculated as \( b_{0.5, out} = b_{0.5}(d_v/d_{dy, out})^1/(\eta_c)^{0.5} \).

2.4. Czech National Code CSN 73 1201

The Czech national standard CSN 73 1201 [6] gives the following expression for estimating design value of punching resistance of a concrete section given as:

\[ V_{Rcd} = 0.5 \times 0.42 h \kappa_s \kappa_h \kappa_n \gamma_b f_{ctd} u, \]

\[ (9) \]

where \( h \) is the slab depth; \( \kappa_s \) is the coefficient of amount of reinforcement (for common slabs \( \kappa_s = 1 \)); \( \kappa_h \) is the coefficient for the slab depth, for slab depth from 0.15 m to 0.3 m the coefficient is \( \kappa_h = 1.2 \); \( \kappa_n \) is the coefficient for normal force \( N \) (for \( N = 0 \) is \( \kappa_n = 1 \)); \( \gamma_b \) is the performance factor accounting amount of reinforcement in a cross-section, and effects of axial force, repeated loading, and concrete aging (here considered as \( \gamma_b = 1 \)); and \( f_{ctd} \) is the design value of the tensile concrete strength. The critical section \( u = 0.5 h \) should be verified.

Where shear reinforcement is required, the following expression is given for internal force given for unity length of the critical perimeter transmitted at the shear reinforcement limit:
is the distribution function of standardised normal variable. The probability of failure $F_p$ may be expressed by the reliability index $\beta = -\Phi^{-1}(p_F)$ where $\Phi$ is the distribution function of standardised normal variable. The probability of failure $p_F$ and reliability index $\beta$ are the specified (target) values that should not be exceeded during a considered reference period $t$.

The reliability differentiation of structures given in EN 1990 (see also ISO 13822 for existing structures) is based on three different levels of failure consequences (with respect to three classes CC1 to CC3) for verification of the ultimate limit states. For common structures the target reliability index $\beta_t = 3.8$ (or equivalently $p_t = 7.24 \times 10^{-5}$) is recommended for a 50-year reference period and medium failure consequences (CC2).

The reliability analysis of a structural member for the ultimate or serviceability limit states can be determined through the probability $p_F$ of the action effects $E(X)$ randomly exceeding the structural resistance $R(X)$ according to the following relationship:

$$p_F = P\{[\theta_R R(X) - \theta_E E(X)] < 0\}$$

where $X$ is the vector of basic variables; $\theta_R$ and $\theta_E$ are the model uncertainties of the resistance and action effects.

The knowledge of the reliability level of existing structures designed according to the national standards can be applied for optimization of design procedures. The structural reliability with respect to the ultimate limit state of punching shear is analysed in the following focusing on an example of RC slabs in typical existing residential buildings built in the Czech Republic in 2000s.

### 4. Probabilistic Analysis - Case Study

An important step in any reliability analysis is the specification of probabilistic models for the basic variables. The probabilistic models of resistance parameters and actions are based on the JCSS Probabilistic Model Code [13], following here simplifications adopted in [7, 8] to provide generic models. The models are normalised by characteristic values of the basic variables (Table 1). The permanent action is described by normal distribution (N), variable actions by Gumbel distribution (GUM), concrete and steel strengths by lognormal distribution (LN).

The model for the permanent actions covers self-weight of the concrete slabs and floor layers. A rather conservative value of coefficient of variation, 10\%, is

### Table 1. Probabilistic models of basic variables [7, 8].

<table>
<thead>
<tr>
<th>Basic variable</th>
<th>Distr.</th>
<th>Units</th>
<th>Char.</th>
<th>$\mu_X$</th>
<th>$\sigma_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-weight and permanent actions</td>
<td>N</td>
<td>MN/m$^2$</td>
<td>$G_k$</td>
<td>$G_k$</td>
<td>10%</td>
</tr>
<tr>
<td>Imposed - residential building (50 y.)</td>
<td>GUM</td>
<td>MN/m$^2$</td>
<td>$Q_k$</td>
<td>0.6 $Q_k$</td>
<td>35%</td>
</tr>
<tr>
<td>Slab depth</td>
<td>N</td>
<td>m</td>
<td>$h$</td>
<td>$h$</td>
<td>0.01*</td>
</tr>
<tr>
<td>Effective depth</td>
<td>N</td>
<td>m</td>
<td>$d$</td>
<td>$d$</td>
<td>0.01*</td>
</tr>
<tr>
<td>Concrete resistance</td>
<td>LN</td>
<td>MPa</td>
<td>$R_k$</td>
<td>$R_k + 2 \sigma_X$</td>
<td>15%</td>
</tr>
<tr>
<td>Steel resistance</td>
<td>LN</td>
<td>MPa</td>
<td>$R_k$</td>
<td>$R_k + 2 \sigma_X$</td>
<td>8%</td>
</tr>
<tr>
<td>Load effect model uncertainty</td>
<td>LN</td>
<td>–</td>
<td>$\theta_E$</td>
<td>1</td>
<td>5%</td>
</tr>
<tr>
<td>Resistance model uncertainty</td>
<td>LN</td>
<td>–</td>
<td>$\theta_R$</td>
<td>1</td>
<td>10%</td>
</tr>
</tbody>
</table>

* standard deviation.
considered as the study should be indicative for a population of the concrete slabs. For a particular existing slab, uncertainties in the permanent action can be reduced by measurements of thickness of floor layers and specification of volume densities. Similarly, the models for the resistance parameters - material and geometrical properties - can be updated by in-situ measurements and tests.

It is noted that the resistance model uncertainty in Table 1 represents a rather generic model, considered to allow for comparisons between various resistance models; for more details see discussion in Section 5.

The concrete slab depth of 0.25 m was originally designed according to the original Czech standards with concrete C25/35 and steel S400. However, after about 15 years considerable cracking appeared in partition walls of the residential buildings (Figure 1), leading to the concerns about reliability of the structures.

The inspection of load-bearing structures revealed that the actual depth of the existing slabs is 0.22 m only instead of 0.25 m assumed in the design. The shear reinforcement was unsatisfactory, failing to fulfill the prescriptive structural measures. For the verification of reliability of existing structures, the nationally implemented ISO 13822 and currently valid standards for structural design (Eurocodes) are applied (the original Czech standard for the design of RC structures has informative character only after the implementation of the Eurocodes in 2010 in the Czech Republic).

Selected results of the probabilistic analyses of slab punching shear are illustrated in Figure 2 for the existing slab thickness of 0.22 m. For the three punching shear resistance models under consideration (originally applied CSN 73 1201 [6] for the design of reinforced slabs, denoted as CSN, EN 1992 (EC2) and prEN 1992 (prEC2) in Figure 2), the reliability index $\beta$ increases with increasing shear reinforcement area $A_s$ (100\% indicates shear reinforcement area needed to achieve...
When conducting reliability analyses, the partial factors are not applied in the models provided in Sections 2.2, 2.3, and 2.4; the characteristic values are replaced by random values of the respective basic variables.

It appears that the application of the original Czech standard leads to overestimation of the resistance of the concrete slab. Application of the present Eurocode EN 1992 and prEN 1992 for the verification of the slab for punching shear leads to a lower reliability level, therefore the shear reinforcement should be sooner added in the structural design.

Variation of the reliability index $\beta$ versus the percentage of required reinforcement area $A_s$ for two classes of reinforcement is illustrated in Figure 3. Two classes of steel reinforcement are considered here: S400 (as applied in the original design) and S500 (as widely used in the present practice). The actual depth of existing slab 0.22 m is considered. The newly conducted material tests reveal that the actual concrete compressive strength corresponds to C35/45 rather than to C25/30 as assumed in the design; the difference is partly attributed to concrete aging (hardening) and partly to execution where concrete of higher quality might have been supplied to comply with the quality control criteria.

Considering presently valid EN 1992, the reliability level of the slab depth of 0.22 m and concrete strength C35/40 MPa is slightly lower, $\beta = 3.6$, in comparison to the target level. However, this may still be considered acceptable for existing residential buildings; see discussion in Section 5. In case that prEN1992 is applied, the reliability level of existing slab ($\beta = 4$) fulfils the target reliability level $\beta = 3.8$ recommended in Eurocodes and also in ISO 13822 for the assessment of existing structures. The latter standard indicates $\beta = 3.8$ for medium consequence of failure and the reference period considered as "a minimum standard period for safety (e.g. 50 years)"; both these specifications apply for the structures under investigation.

For a higher class of reinforcement (S500 instead of S400), the reliability of slab considerably increases - for the reference level of 100% from 3.6 to 4.5).

Sensitivity analyses of the basic variables entering the models under consideration indicate that apart from loads, structural reliability is significantly affected by uncertainties in yield strength of reinforcement, spacing of shear reinforcement, and by resistance model uncertainties; see Figure 4 as an example for the EN 1992-1-1 model. Note that these observations largely depend on a reinforcement ratio and dimensions of the slab - the sensitivity factor of concrete compressive strength would increase for lower reinforcement ratios; it would increase for effective depth of thin slabs.

The controlled perimeter is deemed to be also an
important random basic variable. Yet as is common practice [13], it is considered here as deterministic since associated uncertainties are deemed to be covered by the resistance model uncertainty. The fact that length of the controlled perimeter has been not harmonized in prescriptive documents indicates large uncertainties in specification of this parameter.

5. DISCUSSION

5.1. NOTES ON SELECTED PROBABILISTIC MODELS
To be representative for a range of the existing slabs under consideration, Table 1 provides the generic probabilistic models that should be updated in assessment of a particular slab. It is expected that based on measurements and tests, uncertainties in the following basic variables may be reduced:

- Coefficient of the permanent action could be decreased to about $V_G \approx 5\%$.
- Uncertainty in slab depth, considered here as $V_d \approx 4.5\%$, could be nearly entirely eliminated; the nearly complete draft of fib Model Code 2020 [15] assumes $V_d \approx 1\%$ for in-situ measurements of geometry. Yet it is emphasized that uncertainties in the measurements and possible variability of effective depth along the controlled perimeter (depending on actual position of reinforcement) should be carefully considered when specifying $V_d$-value.

5.2. RESISTANCE MODEL UNCERTAINTY
Resistance model uncertainty, $\theta_R$, should be specified for each model individually [16][18]. While the statistical information on $\theta_R$-characteristics for the CSN and prEN 1992 models is missing, the detailed studies focused on the performance of the EN 1992 model [13][20] indicated slightly conservative bias, $\mu_{\theta_R} = 1.07$ (1.14-1.17), and reasonable scatter, $V_{\theta_R} = 18\%$ (12\% - 18\%), for slabs with (without) shear reinforcement. The bias $\mu_{\theta_R}$ $> 1$ would slightly increase the reliability levels in Figure 2 and Figure 3 while considering $V_{\theta_R} = 18\%$ (in comparison to 10\% given in Table 1) would lead to a drop of the reliability level.

As a first approximation it is considered here that similar $\theta_R$-characteristics apply for the prEN 1992 model and the reliability levels obtained in the numerical study are thus deemed to be representative for both EN 1992 and prEN 1992 models. As the prEN 1992 model newly accounts for failure zone roughness and punching shear gradient effects, it may indeed provide a higher level of approximation than the presently valid EN 1992. CSN provides a more empirical-based model with a number of input parameters, depending on case-specific information model uncertainty may have different bias and coefficient of variation in comparison to the EN 1992 model. Uncertainty in the prEN 1992 model and perhaps also in the CSN model should be analysed within further research. The database of test results provided in [19] could be utilised in this regard.

The sensitivity analysis indicates the importance of resistance model uncertainty. A considerable improvement may be achieved by using a non-linear finite element analysis (NLFEA) for which $V_{\theta_R}$ may be significantly reduced. Cervenka et al. [21] derived for validated NLFEA models $V_{\theta_R} \approx 8\%$ for punching and generally for ‘all failure modes’, with biases close to unity.

6. TARGET RELIABILITY FOR ASSESSMENT
ISO 13822 assumes the same target reliability level for the design of new structures and upgrading of existing structures. However, for assessment the standard stipulates the possibility to adjust reliability targets by optimizing the total cost to a remaining working life. Some national standards (Austria, Czech Republic or Germany) explicitly or implicitly assume the same target reliability for new and existing structures, some documents (AASHTO Manual for Bridge Evaluation, 2011; CAN/CSA-S6-06:2006 Canadian Highway Bridges Design Code or NEN 8700:2011 Assessment of existing structures in case of reconstruction and disapproval - basic rules) provide less strict criteria for existing structures; for instance NEN applicable to buildings gives $\beta_{t,\text{ex},50y} = 3.3$ for medium failure consequences (CC2). The latter is in agreement with the principle that target levels decrease with increasing cost of reliability measures as recognised e.g. in ISO 2394:2015. Following the ISO standard, *fib* MC 2020 proposes $\beta_{t,\text{ex},1y} = 3.3$.

Holicky et al. [22] proposed the methodology on how to recalculate the target reliability for various reference periods using the concept of an interdependency interval, $k$. When annual failure events are nearly statistically independent (such as for steel members exposed dominating wind or snow loads), $k$ is close to 1y. For situations where uncertainties in time-independent variables are dominating (such as for masonry structures exposed to dominating permanent actions), $k$ becomes close to a reference period. Punching of a RC slab exposed to the imposed load is a somehow intermediate situation that might be characterised by $k \approx 10\,\text{y.}$ (as considered ‘on average’ for RC structures in *fib* MC 2020). Considering this, $\beta_{t,\text{ex},50y} = 2.8 - 2.9$ is estimated. The 50-year $\beta = 3.6$ obtained in Section 4 seems to be acceptable for the existing slab when considering the recalculated $\beta_{t,\text{ex},50y}$-values. For more details see [23][24].

7. CONCLUSIONS
Deterministic methods commonly used for the reliability verification of structures fail to provide deeper insights into structural reliability. The probabilistic methods make it possible to quantify and consistently treat uncertainties in the basic variables and to analyse
relative importance of a basic variable by sensitivity analysis.

Reliability of existing reinforced concrete slabs originally designed according to the Czech standards is verified considering the presently valid EN 1992 and the final draft of prEN 1992 using probabilistic methods. It appears that the model given in recently developed prEN 1992 leads to more realistic estimates of the reliability level of the slabs with respect to punching than the model previously given in the Czech standard. It appears that the use of the EN 1992 or prEN 1992 models in reliability assessments existing reinforced concrete structures exposed to punching shear improves estimating structural resistance and may help to avoid inadequate structural interventions.

Sensitivity analysis of the basic variables indicate that the dominating resistance parameter is model uncertainty; uncertainty in the prEN 1992 model should be analysed within further research. The controlled perimeter is expected to have also significant influence on the results and should be harmonized in prescriptive documents and their National Annexes [4][6][13]; detailed analyses are however necessary.

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