SEMI-PROBABILISTIC ASSESSMENT OF EXISTING BRIDGE USING SIMPLIFIED METHODS FOR ESTIMATION OF VARIANCE

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Abstract. The paper is focused on assessment of existing prestressed concrete bridge by simplified methods for estimation of coefficient of variation. The bridge was selected in the framework of the European Project INTERREG AUSTRIA-CZECH REPUBLIC "ATCZ190 SAFEBRIDGE" focused on advanced numerical analysis of existing bridges represented by non-linear finite element model. The key ingredient in semi-probabilistic design and assessment of structures is an estimation of coefficient of variation (ECoV). Recently, correlation interval approach together with novel Eigen ECoV were proposed by authors of this paper and theoretically proved to be an efficient and accurate alternative to existing methods. This contribution is focused on practical application of Eigen ECoV on real example solved by NLFEM and its comparison with other existing simplified methods.

Keywords: Concrete structures, statistical analysis, Estimation of Coefficient of Variation.

1. Introduction

The development of computational methods for civil engineering has become more important than ever, since it is often necessary to employ advanced numerical methods for the design of new structures in order to fulfil the significantly increasing economical and safety requirements in the last decades. Moreover, there are a lot of structures, especially bridges, built in the last century, which must often be enhanced for higher loads assuming actual conditions of the structures. As a result of these industrial needs, researchers and civil engineers are more interested in advanced numerical methods to solve the mathematical models of structures – typically non-linear finite element method (NLFEM). Although NLFEM is a very accurate numerical method for solving differential equations, there is still a lack of knowledge of material characteristics (e.g. fracture energy), actual geometrical properties (e.g. position of reinforcement) and even mathematical models of some physical phenomena (e.g. fracture mechanics of quasi-brittle materials) collectively called uncertainties. As can be seen from the given examples, uncertainties play an important role, especially in the case of concrete structures. This lack of knowledge may generally lead to inaccurate results and even fatal failures despite the advanced numerical analysis performed by NLFEM.

In modern structural analysis, uncertainties are represented by random variables or vectors described by specific probability distribution, the structural system can then be seen as a mathematical function of a set of random parameters. Deterministic NLFEM numerical analysis of structures must thus be enriched by stochastic analysis [1]. The elementary task of stochastic analysis is to propagate uncertainties through a mathematical model in order to obtain statistical information of outputs.

In the semi-probabilistic approach for NLFEM [2], the structural resistance R is separated and the design value R_d that satisfies safety requirements is evaluated, as a simplification of the direct calculation of failure probability $p_f = P(Z(\mathbf{X}) < 0)$. The typical formula for the estimation of R_d, assuming a lognormal distribution of R, is

$$R_d = \mu_R \cdot exp(-\alpha_R \beta v_R), \qquad (1)$$

where μ_R is the mean value, v_R is the coefficient of variation (CoV) and α_R represents sensitivity factor derived from First Order Reliability Method (FORM) [3]; the recommended value is $\alpha_R = 0.8$ according to Eurocode [4]. The target reliability index is dependent on consequence classes (dependent on a type of structure), e.g. β for the ultimate limit state, moderate consequences of failure and a reference period of 50 years is set at $\beta = 3.8$ according to the Eurocode. Note that, from a probabilistic point of view, the whole process represents the estimation of a quantile satisfying the given safety requirements under the prescribed assumption of lognormal distribution.

2. Semi-probabilistic methods

There are several safety formats and semi-probabilistic methods for determination of a design value of response in normative documents or scientific papers, which are briefly described in the following paragraphs. These methods assume several simplifications and thus are less computationally demanding in comparison to the fully probabilistic approach. Main advantage of safety formats and semi-probabilistic methods is possibility of their usage without complex reliability knowledge.

2.1. PARTIAL SAFETY FACTORS

The commonly known method according to EN 1990 works with design values of input random variables and obtained result of NLFEM corresponds directly to the $\tilde{R}_d$. Design values of material parameters, extremely low quantiles, can be obtained from laboratory experiments or directly from EN. Herein, lognormal probability distribution of material parameters was assumed and design value is estimated as the following quantile:

$$X_d = \mu_X \, exp(-\alpha \, \beta \, v_X) \qquad (2)$$

where v_X is coefficient of variation (CoV), α represents sensitivity factor and β target reliability index. However, this approach leads to extremely low design values of input random variables which migh lead to unrealistic behavior of the numerical model. Moreover, this method does not estimate statistical moments of the resistance and therefore is not employed in this comparison.

2.2. ECoV *FIB* METHOD

Assuming Lognormal distribution of the response variable R, the task of ECoV methods is the estimation of the mean value and variance. The first presented method is ECoV according to *fib* Model Code 2010 [5–7], the coefficient of variation caused by uncertainty of input parameters v_R can be estimated by simplified formula for $v_R < 0.2$ as

$$v_R = \frac{1}{1.65} ln\left(\frac{R_m}{R_k}\right). \qquad (3)$$

The simplified formula is based on two numerical simulations - R_m using mean values of input random variables and R_k using characteristic values (5% percentile). The global resistance safety factor is then calculated as

$$\gamma_R = exp(\alpha_R \beta v_R). \qquad (4)$$

This method was adopted in the *fib* Model Code 2010, though design value R_d was later decreased by additional factor γ_{Rd}=1.06 to take model uncertainties into account:

$$R_d = \frac{R(f_{cm}, f_{ym}, a_{nom}, ...)}{\gamma_R \gamma_{Rd}}. \qquad (5)$$

Note that the strong assumption of R_k being equal to simulation with characteristic values is not generally applicable for non-linear functions, but it can be justified for engineering applications where the non-linearity of investigated function is not typically very high. The second significant limitation arises from a fact, that this method is special case of Taylor Series Expansion assuming full correlation among all input random variables [8].

2.3. TAYLOR SERIES EXPANSION

Classical method for a statistical analysis of function of random input vector is Taylor Series Expansion (TSE). The most significant advantage of ECoV based on TSE is its versatility and adaptability. It is common in engineering applications to use TSE truncated to linear terms and thus $\mu_R \approx R_m = r(\mu_{\mathbf{X}})$ and CoV is obtained as:

$$v_R \approx \frac{1}{R_m}\sqrt{\sum_{i=1}^{N}\left(\frac{\partial r(X)}{\partial X_i}\sigma_{Xi}\right)^2}. \qquad (6)$$

where the derivatives are evaluated at $\boldsymbol{\mu_X}$ for all N input random variables. The efficiency and accuracy of TSE depends on the number of used terms (truncation of infinite TSE) and the differencing scheme for the practical computation of derivatives. The simplest form is linear TSE with derivatives approximated by one-sided differencing as:

$$\frac{\partial r(X)}{\partial X_i} = \frac{R_m - R_{\mathbf{Xi\Delta}}}{\Delta_{X_i}}. \qquad (7)$$

where the response of mathematical model R_m is determined by a calculation with mean values, and $R_{\mathbf{Xi\Delta}}$ is the result of a model using mean values of input random variables and a value of the i-th random variable which has been reduced by Δ_{X_i}. For the sake of clarity, the difference is calculated as $\Delta_{X_i} = \mu_{X_i} - X_{i\Delta}$.

Of course, one can use various differencing schemes instead of Eq. 7 as originally proposed in [9] and illustrated in Fig. 1. Assuming linear TSE, one of the most promising advanced differencing schemes using $n_{sim} = 2N + 1$ simulations is

$$\frac{\partial r(X)}{\partial X_i} = \frac{3R_m - 4R_{\mathbf{Xi\frac{\Delta}{2}}} + R_{\mathbf{Xi\Delta}}}{\Delta_{X_i}}, \qquad (8)$$

where the middle additional term $R_{\mathbf{Xi\frac{\Delta}{2}}}$ is obtained via the evaluation of the original mathematical model with mean values and a reduced i-th variable $X_{i\frac{\Delta}{2}} = \mu_{X_i} - \Delta_{X_i}/2$.

The last aspect of adaptivity of TSE is represented by step size parameter c used for definition of the difference $\Delta_{X_i} = \mu_{X_i} - X_{i\Delta}$, where $X_{i\Delta} = F_i^{-1}\left(\Phi\left(-c\right)\right)$, where F_i^{-1} is an inverse cumulative distribution function of the i-th variable and Φ is the cumulative distribution function of the standardized Gaussian distribution. Schlune et al. [10] proposed step size parameter in dependence on target reliability index as $c = \left(\alpha_R \beta\right)/\sqrt{2}$. However, it brings additional computational burden when analyzing different limit states with different target β, since it is necessary to calculate $N + 1$ (Eq. 7) or $2N + 1$ (Eq. 8) simulations for each limit state. Therefore, it can be recommended to use $c = 1.645$ independently on type of investigated limit state, which is in compliance to ECoV according to fib Model Code 2010 [7].

Note that TSE is suitable for uncorrelated input random variables, since the differencing schemes evaluate the influence of each input variable independently. If one needs to include specific correlation coefficient, one can use following formula:

$$Var_{RT_l} \approx \sum_{i=1}^{N} \left(\frac{\partial r(X)}{\partial X_i} \right)^2 \sigma_{X_i}^2 + \tag{9}$$

$$+ \sum_{\substack{i,j=1,\ldots,N \\ i \neq j}} \rho_{i,j} \sigma_{X_i} \sigma_{X_j} \frac{\partial r(X)}{\partial X_i} \frac{\partial r(X)}{\partial X_j}. \tag{10}$$

where ρ is the correlation coefficient. However, it is often necessary to use advanced differencing scheme or higher order of TSE (such as quadratic depicted in Fig. 1) for accurate estimation of variance in case of correlated input random variables [9].

2.4. Eigen ECoV Method
The recently proposed Eigen ECoV [8] is derived directly from TSE. However, there is an assumption of fully correlated input random variables similarly to ECoV according to fib Model Code 2010 and thus number of simulations is dramatically reduced – 3 independent on N. Eigen ECoV is based on idea of projection of input random vector on 1D eigen distribution Θ with variance equal to the first eigenvalue of input covariance matrix $\sigma_\Theta^2 = \sum \sigma_{X_i}^2 = \lambda_1$ and mean value is simply obtained as:

$$\mu_\Theta = \sqrt{\sum_{i=1}^{N} (\mu_{X_i})^2}. \tag{11}$$

In the original proposal, there are three levels of Eigen ECoV corresponding to TSE methodology consisting of three increasing levels of complexity and accuracy [9]. The most promising Eigen ECoV formula for estimation of v_R offering a balance between the efficiency and accuracy (derived directly from Eq. 8) is in the following form:

$$v_R \approx \frac{3R_m - 4R_{\Theta \frac{\Delta}{2}} + R_{\Theta \Delta}}{\Delta_\Theta} \cdot \frac{\sqrt{\lambda_1}}{R_m}, \tag{12}$$

where a simulation $R_{\Theta \Delta} = r(\mathbf{X}_{\Theta \Delta})$ with coordinates of input realization $\mathbf{X}_{\Theta \Delta} = (X_{1\Delta}, \ldots, X_{N\Delta})$ and $R_{\Theta \frac{\Delta}{2}} = r\left(\mathbf{X}_{\Theta \frac{\Delta}{2}}\right)$ with coordinates $\mathbf{X}_{\Theta \frac{\Delta}{2}} = \left(X_{1\frac{\Delta}{2}}, \ldots, X_{N\frac{\Delta}{2}}\right)$ are depicted together with illustration of Eigen ECoV method in Fig. 1 For the sake of clarity, the input vectors consisting of reduced values of input random variables are $X_{i\Delta} = F_i^{-1}(\Phi(-c))$ and the intermediate coordinates are as follows:

$$X_{i\frac{\Delta}{2}} = \mu_{X_i} - \frac{\mu_{X_i} - X_{i\Delta}}{2} = \mu_{X_i} - \frac{\Delta_{X_i}}{2}. \tag{13}$$

Finally, the Δ_Θ represents distance between μ_Θ and desired quantile $F_\Theta^{-1}(\Phi(-c))$. It is generally obtained

directly from the inverse cumulative distribution function of lognormal distribution with corresponding statistical moments of eigen distribution Θ or one can use the following approximation suitable for calculation without software:

$$\Delta_\Theta = \mu_\Theta - \mu_\Theta \cdot exp\left(-c \cdot \frac{\sqrt{\lambda_1}}{\mu_\Theta}\right) \tag{14}$$

The Eigen ECoV combines versatility and adaptability of TSE via various differencing schemes and step size parameter c together with efficiency of ECoV according to fib Model Code 2010. Note that, more theoretical details can be found in the original proposal of Eigen ECoV including additional formulas based on another differencing schemes or higher TSE, which is suitable for input variables with high CoV [8].

3. Post-tensioned Concrete Bridge
The described semi-probabilistic methods are employed for probabilistic assessment of an existing concrete bridge. This study significantly extends the obtained results from the original simplified case-study [17]. The bridge was selected in the framework of the European Project INTERREG AUSTRIA-CZECH REPUBLIC "ATCZ190 SAFEBRIDGE" focused on advanced numerical analysis of existing bridges. The bridge consists of three spans constructed from 16 bridge girders KA-61 in transverse direction. The crucial part of the bridge for assessment is the mid-span: 19.98 m long with total width 16.60 m. The geometry of a typical bridge girder KA-61 is created according to an original documentation describing also positions of reinforcement and tendons. The drawing together with the simplified cross-section is depicted in Fig. 2.

From structural point of view, it was necessary to create numerical model of the whole bridge span. The reason is that although the structure is symmetric, the individual bridge girders are not transversely prestressed, which leads to the different deflection of each girder in dependence on their distance to the loading position. In order to create numerical model reflecting their real connection conditions, the girders are connected by reinforcement according to original documentation together with a concrete mixture between single girders.

3.1. Finite Element Model
The cross-sections of girders KA-61 were simplified to regular shapes in order to reduce number of finite elements and to obtain regular mesh, see Fig. 2. Boundary conditions are assumed to be as a simply supported beam with elastic blocks as supports. The geometry of elastic blocks and positions of loading plates are modeled according to bridge documentation and a national annex of Eurocode for load-bearing capacity of road bridges by exclusive loading (by six-axial truck).
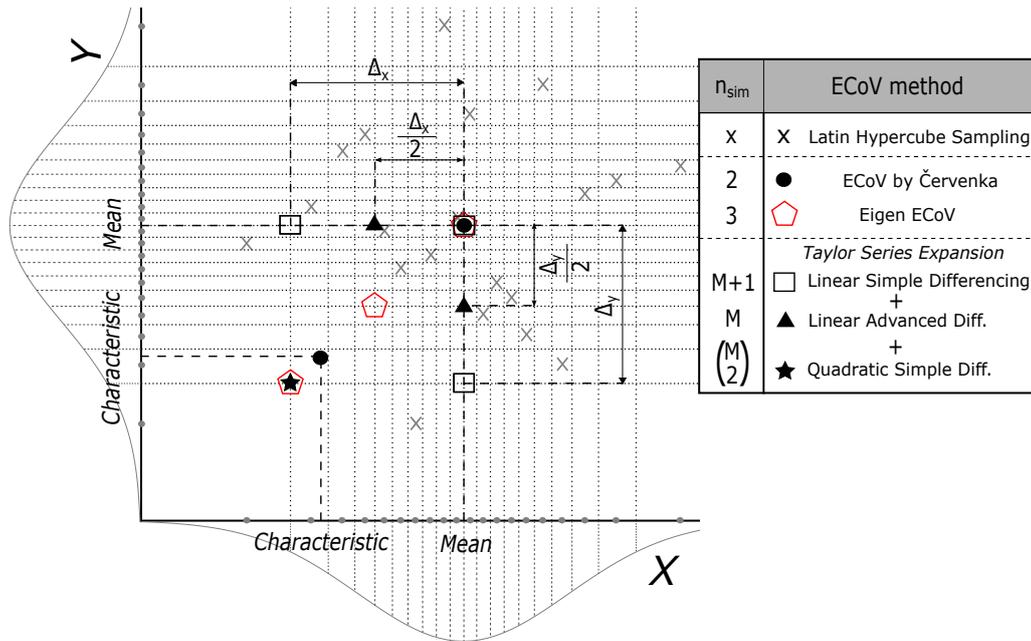
FIGURE 1. Representation of semi-probabilistic methods in 2D example.
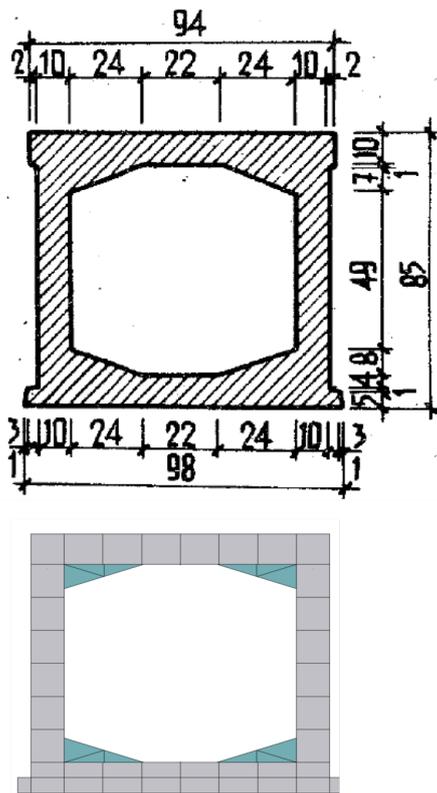
FIGURE 2. Cross-section of a singe bridge girder KA-61.

The non-linear finite element model is created in software ATENA Science including theory of non-linear fracture mechanics [11]. In order to reflect complex behavior of the bridge, the numerical model contains three construction phases as illustrated in Fig. 3:

(1.) prestressing of bridge girders and simultaneous application of the self-weight;

(2.) activating of the pavement and concrete among girders connecting bridge girders;

(3.) application of a load by a six-axial truck.

The major part of NLFEM is represented by 13,000 elements of hexahedra type and triangular 'PRISM' elements in the blue-colored parts of the cross-section (see Fig. 2). Hexahedra elements lead to better numerical stability of simulation and leads to easier construction of mesh compatible between two volumes connected by fixed contact, i.e. nodes of elements in both connected sub-volumes have same coordinates. Another advantage of brick elements is that the structured mesh constructed from brick elements leads to a significantly lower number of finite elements in comparison to tetrahedra elements. Fracture-mechanical behavior of concrete is described by a non-linear mathematical model [11]. Reinforcement together with tendons are modeled as discrete 1D elements with positions, diameters and shape according to the original documentation.

The numerical model is further analysed in order to investigate three limit states of the bridge:

(1.) the ultimate limit state (ULS) (peak of a load-deflection diagram);

(2.) the first occurrence of cracks in bridge girders (Cracking);

(3.) the serviceability limit state of decompression defined according to Eurocode (SLS).

Note that obtained results are further reduced by dynamic amplification factor $\delta = 1.4$ in order to reflect that results are from static analysis.
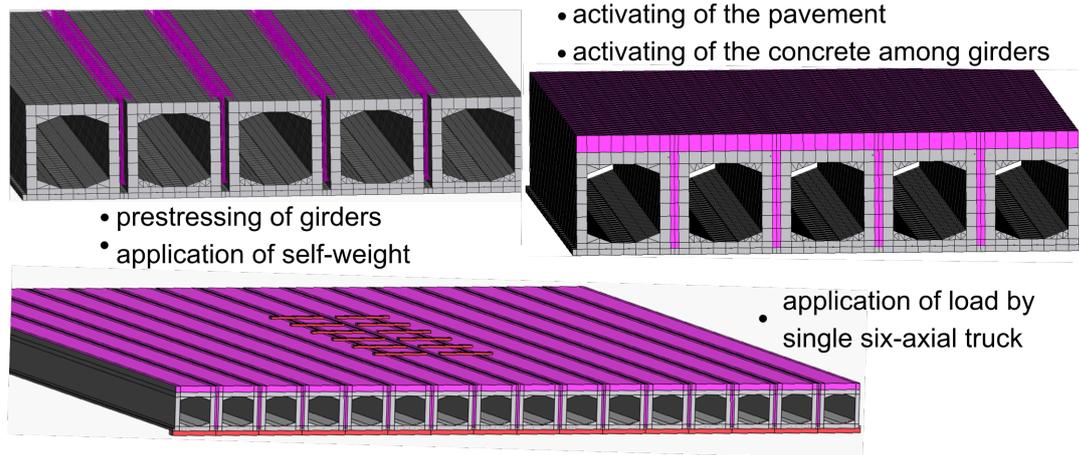
FIGURE 3. Three construction phases of the bridge represented by NLFEM.

3.2. STOCHASTIC MODEL

The stochastic model contains 4 random material parameters of a concrete C50/60: Young's modulus E, compressive strength of concrete f_c, tensile strength of concrete f_{ct} and fracture energy G_f. Characteristic values of E, f_{ct}, G_f were determined from f_c according to formulas implemented in the fib Model Code 2010 [7] (G_f, E) and prEN 1992-1-1: 2021 (f_{ct}). The last random variable P represents prestressing losses with CoV according to JCSS: Probabilistic Model Code [12]. Note that prestressing force itself is assumed to be deterministic, since it was measured and controlled during prestressing process. The stochastic model is summarized in Tab. 1.

The bias factor for determination of mean value and CoV of f_c were taken from prEN 1992-1-1: 2021 (Annex A) for adjustment of partial factors for materials. CoV of f_{ct} was assumed according to prEN 1992-1-1 and bias factor is identical to f_c. Note that there is lack of information about G_f in literature and thus it is assumed to be described by identical bias and CoV as f_{ct}. Note that values in Tab. 1 determined according to prEN 1992-1-1: 2021 (Annex A) reflect also geometrical uncertainties by additional amplification of material uncertainty.

Var.	Mean	CoV [%]	Distrib.	Units
f_c	56	16	Lognormal	$[MPa]$
f_{ct}	3.64	22	Lognormal	$[MPa]$
E	36	16	Lognormal	$[GPa]$
G_f	195	22	Lognormal	$[Jm^2]$
P	20	30	Normal	$[\%]$

TABLE 1. Stochastic model of the numerical example.

In this first numerical study, there is an assumption of uncorrelated random variables. Though the correlation might play crucial role in case of concrete structures.

3.3. NUMERICAL RESULTS

The design values of resistance R_d for each limit state in tons are determined as a quantile of Lognormal distribution with identified statistical moments, target reliability indices are $\beta_{ULS} = 3.8$, $\beta_{crack} = 3.8$ and $\beta_{SLS} = 1.5$ according to EN 1990 [4]. Note that β_{crack} is not assumed to be serviceability limit state, since a cracking of the post-tensioned bridge leads to corrosion of tendons and ultimately collapse of the structure. Moreover, corroded tendons can not be replaced. Obtained design values are additionally reduced by global safety factor reflecting model uncertainties $\gamma_{R_d} = 1.06$. Obtained statistical moments together with determined R_d can be found in Tab. 2.

Note that the reference solution obtained by Latin Hypercube Sampling (LHS) [13, 14] is based on 30 numerical simulations and one simulation takes approx. 24 hours. From the obtained results, it can be seen that all presented semi-probabilistic methods are in agreement with reference solution, though there are some differences leading to lower R_d (approx. 10%) in comparison to LHS. However, one can see that ECoV methods are both on conservative side while TSE leads to accurate estimation near the reference results obtained by LHS.

4. DISCUSSION AND FURTHER RESEARCH

The obtained results summarized in Tab.2 are in good agreement among all presented methods. This can be explained by relative low non-linearity of this example failing in bending. In case of shear failure, this difference could be significantly higher as presented in previous work of the authors of this paper [15, 16, 18].

Note that the results of simplified methods also show good agreement with their theoretical behavior, since the ECoV according to fib and Eigen ECoV assume fully correlated input random variables (in order to reduce number of simulations to 2 resp. 3) and thus their estimation should be conservative in

Limit state:	ULS			Cracking			SLS		
Method	μ	v_R	R_d	μ	v_R	R_d	μ	v_R	R_d
ECoV *fib*	485	0.098	340	399	0.107	272	154	0.153	121
Eigen ECoV	485	0.075	364	399	0.097	280	154	0.153	121
TSE	485	0.066	375	399	0.085	290	154	0.132	124
LHS (Reference)	480	0.061	376	396	0.074	298	154	0.142	123

TABLE 2. Obtained statistical moments and corresponding design values of resistance in tons for each limit state.

comparison to TSE and LHS. The conservative estimation is caused by a fact, that high correlation among input random variables typically leads to higher variance of R. If one needs to obtain accurate results without assumption of full correlation among input random variables, it is necessary to perform higher number of simulations by TSE (at least $N+1$) or LHS (tens-hundreds). For the correct comparison, further work will be focused on reference solution assuming fully correlated variables and also realistic correlation matrix. The two limit cases – uncorrelated input random variables and fully correlated random variables – then define the interval of design values affected only by assumed correlation [8]. Unfortunately, there is not any recommendation about correlation in codes and other relevant documents and thus this vague information might lead to significant differences in R_d and further research should be also focused on this aspect as already discussed in the previous general pilot comparison of existing semi-probabilistic methods [19].

5. CONCLUSIONS

The paper presented simplified probabilistic assessment of the existing bridge by selected semi-probabilistic methods for estimation of coefficient of variation. The bridge is represented by highly computationally demanding NLFEM reflecting theory of nonlinear fracture mechanics of concrete. The stochastic model contains 5 random variables representing concrete characteristics and prestressing losses. The comparison of selected methods included also recently proposed Eigen ECoV and corresponding Taylor Series Expansion. From the obtained results, it can be concluded that all selected methods lead to accurate estimation of design value of resistance for selected limit states. Moreover results also corresponds to theoretical behavior of the methods with respect to assumed correlation among input random variables.

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