EVALUATION OF SIMPLIFIED MECHANICAL POWER AND DISSIPATED ENERGY CALCULATIONS IN PHYSICAL RESPIRATORY MODELS WITH TISSUE AND AIRWAY RESISTANCE

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Mathematical derivations of simplified mechanical energy equations

We consider volume-controlled ventilation with constant inspiratory flow, linear compliance and linear airway flow resistance, and negligible inertance. Let VT be tidal volume (liters), P_{aw} airway pressure (cmH_2O), T total time (s), T_i inspiratory time (s), T_e expiratory time (s), RR respiratory rate (min^{-1}), Q flow ($L \cdot min^{-1}$), C_{rs} respiratory-system compliance ($L \cdot cmH_2O^{-1}$), R_{aw} airway flow resistance ($cmH_2O \cdot s \cdot L^{-1}$), $PEEP$ the end-expiratory pressure (cmH_2O), P_{peak} peak pressure (end-inspiration, before the inspiratory hold) in cmH_2O, and P_{plat} plateau pressure (during end-inspiratory hold) in cmH_2O. A conversion factor of 0.098 was used to convert cmH_2O to Pa. E corresponds to the delivered mechanical energy during the inspiratory phase of the respiratory cycle, expressed in joules. Mechanical power is calculated by multiplying E by the respiratory rate, yielding units of $J \cdot min^{-1}$.

Gattinoni's energy equation (2)

Starting from the equation of motion, at any given time, the pressure (P_{aw}) in the whole respiratory system is equal to:

$$P_{\mathrm{aw}}(t) = \frac{VT(t)}{C_{\mathrm{rs}}} + R_{\mathrm{aw}} \cdot Q(t) + PEEP.$$

Integrating over volume during inspiration with constant flow yields:

$$E = 0.098 \cdot \left(\frac{VT^2}{2 \cdot C_{\mathrm{rs}}} + R_{\mathrm{aw}} \cdot \frac{VT^2}{T_{\mathrm{i}}} + PEEP \cdot VT\right).$$

This produces equation (2) in the main text.

Comprehensive equation using P_{peak} and P_{plat} (3)

Knowing that airway resistance is given by

$$R_{\mathrm{aw}} = \frac{P_{\mathrm{peak}} - P_{\mathrm{plat}}}{Q},$$

Equation (2) of the main manuscript can be rewritten to separate the elastic, resistive, and $PEEP$ components:

$$E = \frac{VT^2}{2 \cdot C_{\mathrm{rs}}} + \frac{P_{\mathrm{peak}} - P_{\mathrm{plat}}}{Q} \cdot \frac{VT^2}{T_{\mathrm{i}}} + PEEP \cdot VT.$$

Since

$$C_{\mathrm{rs}} = \frac{VT}{P_{\mathrm{aw}}}$$

and

$$P_{\text{aw}} = P_{\text{plat}} - PEEP,$$

the expression becomes

$$E = \frac{VT \cdot (P_{\text{plat}} - PEEP)}{2} + (P_{\text{peak}} - P_{\text{plat}}) \cdot VT + PEEP \cdot VT.$$

After regrouping terms, this simplifies to:

$$E = 0.098 \cdot VT \cdot \left[(P_{\text{peak}} - 0.5 \cdot \left(P_{\text{plat}} - PEEP\right)\right],$$

which is the Comprehensive equation (3) used when both P_{peak} and P_{plat} are available.

Dynamic equation with no inspiratory hold (4)

If P_{plat} is unavailable, it can be approximated by P_{peak} in the Comprehensive equation (3). This requires only P_{peak} and *PEEP* and no need for inspiratory hold to derive equation (4).

Surrogate equation with fixed resistance (5)

Comprehensive equation (3) was used to derive Surrogate equation (5):

$$E = 0.098 \cdot VT \cdot \left[(P_{\text{peak}} - 0.5 \cdot \left(P_{\text{plat}} - PEEP\right)\right],$$

where the part in the square brackets was replaced by the average of the endpoints of the inspiratory pressure to get the unadjusted equation:

$$E_{\text{unadjusted}} = 0.098 \cdot VT \cdot \left[0.5 \cdot \left(P_{\text{peak}} + PEEP\right)\right].$$

Approximating 0.098 ≈ 0.10 gives

$$E_{\text{unadjusted}} = VT \cdot \left(\frac{P_{\text{peak}} + PEEP}{20}\right).$$

The unadjusted equation underestimates by half the resistive component. The bias is

$$E_{\text{bias}} = VT \cdot \left(\frac{R_{\text{aw}} \cdot Q}{20}\right).$$

R_{aw} was replaced with a fixed constant of 10 $\text{cmH}_2\text{O} \cdot \text{s} \cdot \text{L}^{-1}$ and Q was converted to $\text{L} \cdot \text{s}^{-1}$:

$$E_{\text{bias}} = VT \cdot \left(\frac{10 \cdot \frac{Q}{60}}{20}\right)$$

Then, the correct Surrogate equation (5) is

$$E = E_{\text{unadjusted}} + E_{\text{bias}} = VT \cdot \left(\frac{P_{\text{peak}} + PEEP + \left(\frac{Q}{6}\right)}{20}\right).$$

Mean-airway-pressure equation (6)

The mechanical energy delivered during inspiratory phase corresponds to the area under the pressure curve (A_{insp}) on a pressure-time graph, multiplied by inspiratory flow. Then

$$A_{\mathrm{insp}} = \frac{E}{Q} = \frac{E \cdot T_{\mathrm{i}}}{VT}.$$

Mean airway pressure is the average pressure during the entire respiratory cycle. P_{mean} can be calculated using the following equation:

$$P_{\mathrm{mean}} \cdot T = A_{\mathrm{insp}} + A_{\mathrm{decay}} + PEEP \cdot T_{\mathrm{e}},$$

where A_{decay} is the area of pressure decay from P_{peak} to *PEEP*. Owing to its relatively small contribution, this part of the equation was excluded. A_{insp} was then substituted using Comprehensive equation (3) and divided by Q.

$$P_{\mathrm{mean}} \cdot T = \frac{VT \cdot \left[\left(P_{\mathrm{peak}} - 0.5 \cdot \left(P_{\mathrm{plat}} - PEEP\right)\right] \cdot T_{\mathrm{i}}}{VT} + PEEP \cdot T_{\mathrm{e}}$$

$$P_{\mathrm{peak}} - 0.5 \cdot \left(P_{\mathrm{plat}} - PEEP\right) \cdot T_{\mathrm{i}} \;\; = P_{\mathrm{mean}} \cdot T - PEEP \cdot T_{\mathrm{e}}$$

$$P_{\mathrm{peak}} - 0.5 \cdot \left(P_{\mathrm{plat}} - PEEP\right) \;\; = \frac{P_{\mathrm{mean}} \cdot T - PEEP \cdot T_{\mathrm{e}}}{T_{\mathrm{i}}}$$

$$P_{\mathrm{peak}} - 0.5 \cdot \left(P_{\mathrm{plat}} - PEEP\right) \;\; = P_{\mathrm{mean}} + (P_{\mathrm{mean}} - PEEP) \cdot \frac{T_{\mathrm{e}}}{T_{\mathrm{i}}}$$

By multiplying by the tidal volume and the conversion factor, we obtain eqution 6.

$$E = 0.098 \cdot VT \cdot \left((P_{\mathrm{mean}}) + \frac{T_{\mathrm{e}}}{T_{\mathrm{i}}} (P_{\mathrm{mean}} - PEEP) \right)$$

Geometrical calculations of mechanical energy

For completeness, we provide the equations used to calculate mechanical and dissipated energy by the geometric method. The resulting values were obtained by numerical integration of the pressure–volume loop, with energy expressed for pressures measured at the airway opening with *PEEP* (E_{aw}), at the airway opening without *PEEP* (E_D), and at the artificial lung level without *PEEP* (E_L). The respective equations are listed below.

$$I_{\mathrm{insp}} = \{\, i\colon VT_{i+1} > VT_i \,\},$$

where I_{insp} is the set of indices i that belong to the inspiratory part of the cycle.

$$E_{\mathrm{aw}} = 0.098 \cdot \textstyle\sum_{i \in I_{\mathrm{insp}}} \left[0.5 \cdot (P_{\mathrm{aw}_i} + P_{\mathrm{aw}_{i+1}}) \cdot (VT_{i+1} - VT_i)\right],$$

where E_{aw} corresponds to the delivered mechanical energy during the inspiratory phase of the respiratory cycle in J, P_{aw} represents the measured pressure at a given time in cmH_2O at the airway opening, *VT* is the measured volume at a given time in L and i denotes the number of a sample in the inspiratory phase.

$$E_{\mathrm{D}} = 0.098 \cdot \textstyle\sum_{i \in I_{\mathrm{insp}}} \left[0.5 \cdot ((P_{\mathrm{aw}_i} + P_{\mathrm{aw}_{i+1}}) - 2 \cdot PEEP) \cdot (VT_{i+1} - VT_i)\right],$$

where E_D corresponds to the delivered mechanical energy during the inspiratory phase of the respiratory cycle without *PEEP* in J, P_{aw} represents the measured pressure at a given time in cmH_2O at the airway opening, *PEEP* is the positive end-expiratory pressure in cmH_2O, *VT* is the measured volume at a given time in L and i denotes the number of a sample in the inspiratory phase.

$$E_{\mathrm{L}} = 0.098 \cdot \sum_{i \in I_{\mathrm{insp}}} \left[0.5 \cdot \left(\left(P_{\mathrm{L}_i} + P_{\mathrm{L}_{i+1}}\right) - 2 \cdot PEEP\right) \cdot \left(VT_{i+1} - VT_i\right)\right],$$

where E_{L} corresponds to the delivered mechanical energy during the inspiratory phase of the respiratory cycle at the artificial lung level in J, P_{L} represents the measured pressure at the artificial lung level at a given time in cmH_2O, VT is the measured volume at a given time in L and i denotes the number of a sample in the inspiratory phase.

Dissipated energy was obtained by calculating the difference between the total energy during inspiration and the total energy during expiration. In practice, this meant first summing the energy associated with the inspiratory part of the PV loop, then summing the energy associated with the expiratory part, and finally subtracting the latter from the former. The resulting value corresponds to the hysteresis area of the PV loop.