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APPLICATION AND VALIDATION OF THE RESONANCE METHOD FOR EVALUATION OF CORONA DISCHARGE IGNITION VOLTAGES

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Abstract. This paper presents the application of the resonance method for the evaluation of the inception voltage U_0 for non-self-sustained discharges in dry air within a 1D axisymmetric geometry using the commercial software COMSOL Multiphysics. The model is first validated against a reference configuration from previous work. A parametric study is performed to assess the influence of electrode polarity, gas pressure, and electrode geometry on U_0. The results confirm the expected trends. Furthermore, the use of a local kinetics model for negative ions reveals the reduction in the model complexity while introducing only minor deviations in U_0, demonstrating its suitability for efficient calculations. In contrast, the omission of photoionization leads to expected strongly polarity-dependent effects. These findings show the applicability of the resonance method as a robust tool for U_0 evaluation and highlight the critical role of selected physical processes in discharge modeling.

Keywords: drift-diffusion-reaction model, local-field approximation, space-charge-free regime, inception voltage, resonance method, COMSOL Multiphysics.

1. Introduction

The determination of the inception voltage U_0 of gaseous discharges represents a fundamental problem in high-voltage engineering, with direct implications for the design and diagnostics of electrical insulation systems. In practical applications, the prediction of U_0 is commonly based on empirical criteria, such as integral ionization conditions or local electric field thresholds [1–3]. While these approaches are computationally efficient, they are typically limited to simplified physical assumptions and restricted ranges of applicability, particularly in the presence of strongly non-uniform electric fields or additional ionization mechanisms.

An alternative class of approaches is based on physically consistent plasma models formulated in terms of coupled drift–diffusion–reaction equations. Within this framework, the inception of a self-sustained discharge can be interpreted as a transition from a trivial (no-discharge) solution to a non-trivial solution with non-zero charged particle densities. Several numerical approaches have been proposed to identify this transition, including direct eigenvalue formulations and stability analysis of the trivial solution [4]. However, these methods may suffer from numerical sensitivity or increased computational complexity, especially when additional physical effects, such as photoionization, are taken into account.

A promising alternative is the so-called resonance method, originally proposed in [4], which reformulates the problem by introducing a small external ionization source term W and analyzing the response of the system in the stationary regime. In this approach, the inception voltage U_0 is identified as a resonance point, where the solution exhibits a transition from physically meaningful to non-physical behavior. This formulation avoids explicit eigenvalue computations while preserving the physical consistency of the model, which makes it particularly attractive for practical implementations in numerical solvers.

In this work, the resonance method is applied to a one-dimensional axisymmetric configuration of concentric electrodes (inner electrode R_1 and outer electrode R_2), following the framework introduced in [5]. The study focuses on systematic analysis of the inception voltage under varying operating conditions and model assumptions. In particular, the following aspects are investigated:

- the effect of voltage polarity on the predicted inception voltage U_0 over a range of pressures 68.75–200 kPa and inner electrode radii 50–148 µm,
- the impact of local kinetics of negative ions proposed in [6], evaluated through a comparison with the corresponding full model,
- and the role of photoionization in the determination of U_0.

The results are presented in the form of parametric dependencies of the inception voltage on pressure p and inner electrode radius R_1 for both positive and negative polarity. The comparison between simplified and full models provides insight into the validity of commonly used modeling assumptions and their influence on the predicted discharge behavior.

The presented study aims to demonstrate that the resonance method proposed in [4] provides a robust and physically consistent tool for the evaluation of inception conditions in non-uniform electric fields, while also offering sufficient flexibility to incorporate

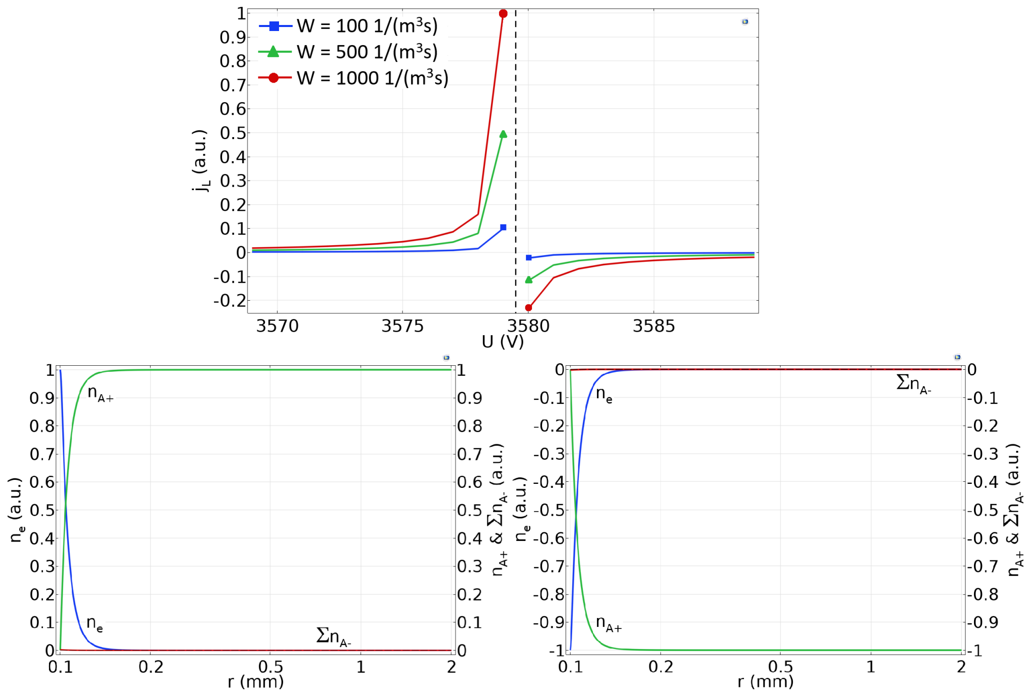

Figure 1. Current-voltage characteristic of a non-self-sustained discharge with the positive polarity in 1D axisymmetric geometry with a resonance point at $U_0 = 3579\,\mathrm{V}$ for various values of the external ionization source term W (top) and density distributions of the three particle species (n_e for electrons, $n_{\mathrm{A}+}$ for positive ions, and $\Sigma n_{\mathrm{A}-}$ for the sum of three negative ion species) for $U = U_0 - 0.1\,\mathrm{V}$ (bottom left) and $U = U_0$ (bottom right).

additional physical effects relevant to practical high-voltage applications.

2. Numerical model and the resonance method

2.1. Numerical model

The considered model describes a non-self-sustained discharge in air using a 1D axisymmetric drift–diffusion–reaction formulation for charged species coupled with the analytical formula for the electric field E in a coaxial geometry (the space-charge-free assumption is considered) within the scope of the local-field approximation (transport coefficients were obtained using the online solver BOLSIG+ [7] and depend only on the local reduced field).

The governing equations include transport equations for electrons, one positive ion specie (O_2^+), and three negative ion species (O^-, O_2^-, and O_3^-), together with source terms accounting for collisional ionization, attachment, recombination, and photoionization (the three-exponential Helmholtz model introduced in [8] is employed).

The complete model consists of a system of coupled partial differential equations describing the stationary form of the particle densities and the three-exponential Helmholtz photoionization model. In the following sections, reduced versions that employ a local approximation of the negative ion kinetics proposed in [6] and a model without photoionization (and with all three negative ion species) are also considered and evaluated.

The numerical model used in the present work was implemented in the commercial software COMSOL Multiphysics and its detailed description, including reaction schemes and transport coefficients, can be found in [4].

2.2. The resonance method

The inception voltage U_0 is determined using the resonance method originally proposed in [4], which is based on the identification of a singularity in the current–voltage characteristic of the non-self-sustained discharge by introducing independent source term W. In practice, U_0 is obtained by solving the stationary form of the governing equations for varying applied voltage U and locating the point at which the system response exhibits resonant behavior (transition from positive physical particle distributions to negative non-physical particle distributions), see Figure 1.

Further details of the resonance method and its theoretical background are provided in [4].

3. Results and discussions

3.1. Model validation

The implemented model and the associated resonance method were first validated against the reference configuration presented in [5]. The considered setup consists of a one-dimensional axisymmetric geometry of concentric electrodes with the radii $R_1 = 100\,\mu\mathrm{m}$ and $R_2 = 2\,\mathrm{mm}$, operating in air at pressure $p = 1\,\mathrm{atm}$

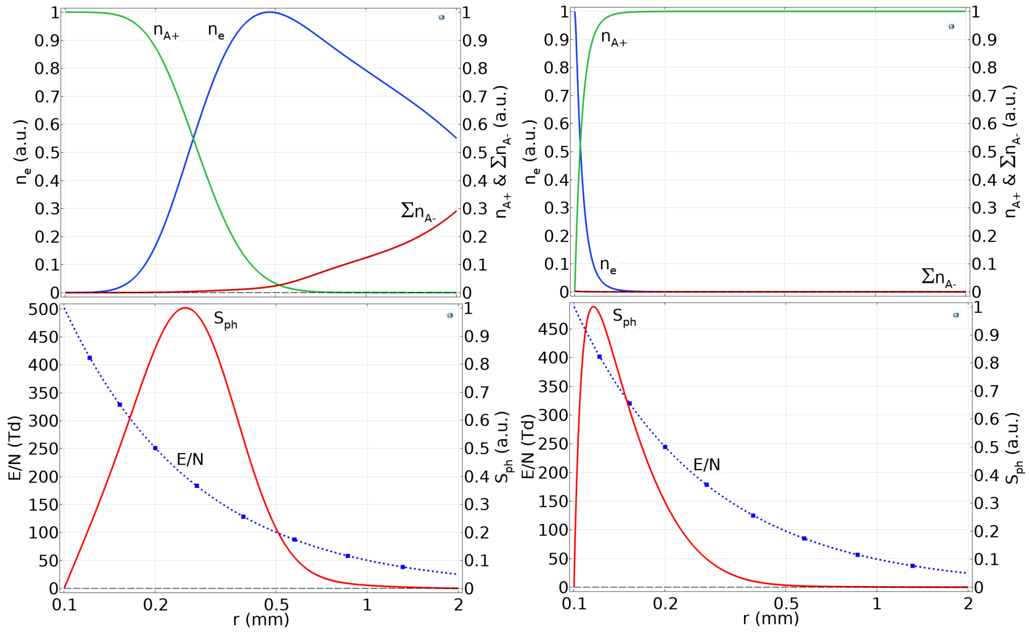

Figure 2. Density distributions of the three particle species (n_e for electrons, n_{A+} for positive ions, and Σn_{A-} for the sum of three negative ion species, top) and the distributions of the reduced electric field E/N and the photoionization rate S_{ph} (bottom) for the case with the inner electrode radius $R_1 = 100\,\mu$m, pressure $p = 1$ atm, voltage U just below the resonance point for the negative (left) and positive (right) polarity.

and temperature $T = 300$ K. The secondary electron emission coefficient was set to $\gamma = 10^{-4}$.

For this configuration, the reference value reported in [5] is $U_0 = -3686.7$ V, where the negative polarity corresponds to the inner electrode acting as the cathode. The present model yields $U_0 = -3677.8$ V, resulting in a deviation of less than 0.3 %. This level of agreement is considered sufficient to validate both the numerical implementation and the application of the resonance method.

The resulting spatial distributions of the electron density n_e, positive ion density n_{A+}, total negative ion density Σn_{A-}, reduced electric field E/N, and photoionization rate are shown in Figure 2 (left) for a voltage slightly below the inception threshold ($U = U_0 + 0.1$ V, where $U_0 < 0$). For comparison, the corresponding distributions for the positive polarity are presented in Figure 2 (right).

Note that in Figure 1 and Figure 2 the linear current j_L, the charged particle densities and the photoionization rates are expressed in a.u. (arbitrary unit) due to the proportionality with the external ionization source term W.

3.2. The negative polarity and discussion on stability

The observed particle distributions are governed by several key factors:

1. Electrode polarity, which determines the spatial separation of charged species. Positive ions accumulate near the cathode, while electrons and negative ions tend to accumulate near the anode.
2. Distribution of the reduced electric field, which is enhanced near the inner electrode and leads to increased ionization as well as more efficient drift of charged particles.
3. Differences in particle mobilities, with electrons responding significantly faster than ions.

As a result, the negative polarity case (Figure 2, left) is characterized by an increased concentration of positive ions in the vicinity of the inner cathode. Electrons are rapidly transported toward the anode, where reduced ionization and electron attachment processes lead to a decrease in electron density and a corresponding increase in the negative ion density. This results in a characteristic electron density profile with a maximum located slightly downstream of the cathode region.

The spatial distribution of photoionization reflects the interplay between the collisional ionization coefficient $\alpha(E/N)$ and the electron density n_e, with its maximum located in the region between the peak of the reduced electric field distribution and the peak of the electron density.

The described configuration is inherently prone to oscillatory behavior, as previously reported in [5], where it was analyzed using both (i) eigenvalue analysis of the linearized system and (ii) time-dependent simulations. Based on the present results, the mechanism leading to the onset of oscillations and the subsequent development of Trichel pulses can be interpreted as follows:

1. A small increase in the applied voltage ΔU (negative increment for negative polarity) leads to an

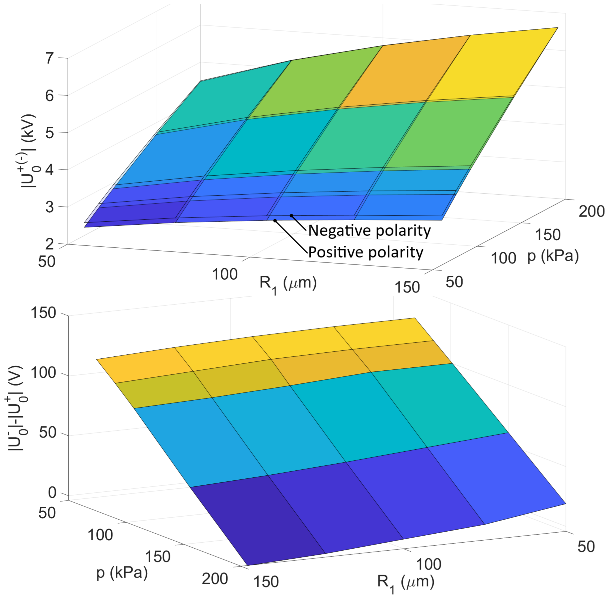

Figure 3. The inception voltage $|U_0|$ *for both polarities (top) and the inception voltage difference* $|U_0^-| - |U_0^+|$ *(bottom) for the pressure range* 68.75–200 kPa *and the inner electrode radius range* 50–148 μm.

immediate increase in the reduced electric field, most prominently in the ionization region near the cathode.

2. The elevated field enhances collisional ionization and consequently also the photoionization rate, increasing the densities of electrons and positive ions, while negative ions respond with a delay due to attachment kinetics, lower mobilities and lower drift in the lower field.

3. The increased space charge near the cathode enhances the local electric field, establishing a positive feedback loop between ionization and field amplification.

4. This positive feedback is counteracted by the delayed increase of the negative ion densities near the anode, driven by enhanced electron attachment, which reduces the electric field near the cathode.

5. The accumulation of negative ions near the anode introduces a negative feedback mechanism, modifying the field distribution and promoting the removal of negative ions from the gap.

6. Once the negative ion density is reduced, the positive feedback near the cathode is re-established, and the cycle repeats.

Photoionization plays a crucial role in this process by influencing the spatial localization of ionization sources, thereby facilitating the transition from harmonic oscillations to the strongly non-linear regime associated with Trichel pulses.

The above-described sequence assumes a negative voltage increment (i.e., an increase in the absolute value of the applied voltage). However, an analogous mechanism can be formulated for a positive voltage increment. In this case, the reduction of the reduced electric field near the cathode leads to enhanced electron attachment and increased accumulation of negative ions near the anode. This further suppresses the electric field in the cathode region, and the system evolution continues analogously through the feedback loop described in steps 5–6–1.

This cyclic behavior highlights the critical role of the negative ions in the overall discharge dynamics. In particular, the negative ions act as a stabilizing component that regulates the feedback between ionization and electric field redistribution. Consequently, their inclusion is essential for a physically consistent description of the stability loss and the transition to the Trichel pulse regime.

The proposed interpretation of the oscillatory behavior is consistent with the findings reported in [5] and will be further investigated in future work.

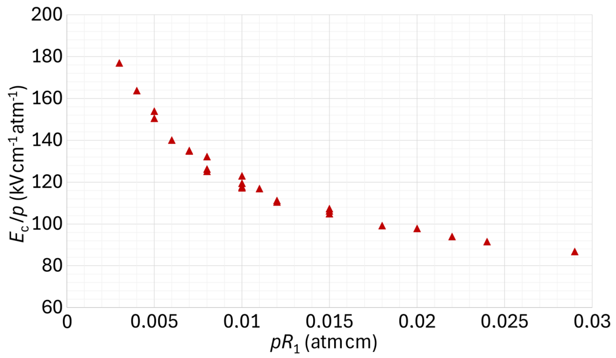

Figure 4. Reduced inception field on the surface of the positive inner electrode as a function of pR_1 *for the range from Figure 3.*

3.3. Parametric study: effect of polarity, pressure, and inner electrode radius

To demonstrate the applicability of the resonance method under varying physical conditions, a parametric study of the inception voltage U_0 was carried out with respect to the electrode polarity, gas pressure p (68.75–200 kPa), and inner electrode radius R_1 (50–148 µm). The obtained values are visualized in Figure 3 at the top.

For both polarities, U_0 increases with increasing pressure and field homogeneity, in agreement with established theoretical expectations.

Figure 4 shows the dependence of the reduced inception field E_c/p on pR_1 parameter for the range in Figure 3 for the positive polarity. The coincidence of E_c/p values for different p and R_1 values with the constant pR_1 product according to the similarity scaling can be seen. Furthermore, the results in Figure 4 match those in Figure 7 in [6] nearly exactly.

In Figure 3 at the bottom, it can be clearly seen that the negative polarity exhibits higher absolute values of U_0 in most cases. The inception voltage difference

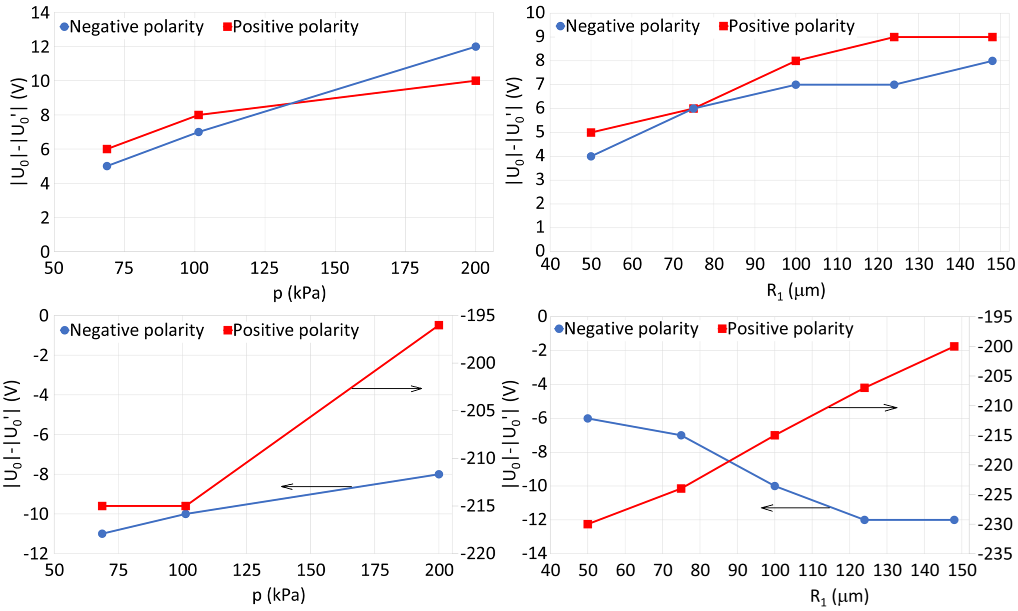

Figure 5. The inception voltage deviations (full model - reduced model) for different pressures and $R_1 = 100\,\mu$m (left) and different inner electrode radii and $p = 1\,$atm (right), considering the model with local kinetics of negative ions (top) and model without photoionization (bottom) for both polarities.

between the polarities decreases with increasing pressure and also with increasing field homogeneity due to the weakening of the photoionization rate at higher pressures (originating in collisional quenching of radiative states) and more homogeneous field distributions (originating in lower levels of the collisional ionization rate).

The maximum polarity-induced difference of 126 V was observed at the pressure of $p = 68.75\,$kPa and the inner electrode radius of $R_1 = 50\,\mu$m, corresponding to the lowest U_0 value within the studied parameter range. At a pressure of $p = 200\,$kPa, the difference changes sign as the field homogeneity increases and reaches $-4\,$V for conditions corresponding to the highest U_0 value within the studied parameter range.

3.4. Local kinetics of negative ion species and photoionization

In the final step, the resonance method was applied to two reduced model configurations:

1. a model employing local kinetics of negative ions together with photoionization, and
2. a model neglecting photoionization.

The results are expressed in terms of the difference between the full model $|\, U_0 \,|$ and the reduced model $|\, U_0' \,|$ for the parameter range defined in the previous section.

3.4.1. Local kinetics of negative ions

The use of the local approximation for negative ion kinetics proposed in [6] leads to a reduction of U_0 by at most 12 V (high pressure and high field homogeneity) and at least 4 V (low pressure and low field homogeneity) for both polarities, see the top part of Figure 5. The deviation slightly increases with pressure and field homogeneity but remains negligible from a practical standpoint.

These results indicate that the model of local kinetics of negative ions provides a substantial simplification of the governing system (reduction from 8 to 5 PDEs) while preserving the accuracy of the predicted inception voltage.

However, it should be noted that this reduced model is not capable of capturing the full non-linear dynamics discussed in the stability analysis section. Its applicability is therefore primarily limited to the cases where negative ions do not modify an external electric field, and its computational advantages are expected to become more significant in higher-dimensional configurations with a larger number of degrees of freedom.

3.4.2. Model without photoionization

For the negative polarity, neglecting photoionization leads to a maximum deviation of approximately $-12\,$V, indicating a minor influence on U_0 under the considered conditions (the lines with circles at the bottom part of Figure 5).

In contrast, the effect is significantly more pronounced for the positive polarity, see the lines with squares at the bottom part of Figure 5. Although the variation is relatively small, the deviation decreases with pressure from approximately $-215\,$V to $-196\,$V for $R_1 = 100\,\mu$m, which may be associated with enhanced collisional quenching of radiative states at higher pressures (analyzed in detail in [3, 6]). A similar trend is observed with increasing field homogeneity, where the deviation decreases from $-230\,$V to $-200\,$V for $p = 1\,$atm, which may be related to the weakening of local ionization near the inner electrode.

The presented results therefore confirm the increasing importance of photoionization in consistent discharge modeling and U_0 calculations, especially at low pressures and low levels of the field homogeneity.

4. Conclusions

The presented study demonstrates the applicability of the resonance method proposed in [4] for evaluating inception voltage U_0 in non-self-sustained discharges under varying physical conditions. The method was successfully validated against a reference configuration from previously published work [5] resulting in a deviation of less than 0.3 %.

The parametric analysis confirmed the expected dependence of U_0 on gas pressure (in the range of 68.75–200 kPa) and field homogeneity (inner electrode radius in the range of 50–148 µm), as well as systematic differences between positive polarity (inner electrode acting as an anode) and negative polarity (inner electrode acting as a cathode). The negative polarity exhibits higher absolute values of U_0 at $p = 68.75\,\mathrm{kPa}$ and $R_1 = 50\,\mathrm{\mu m}$ (with a difference of approximately 126 V). As the pressure and field homogeneity increase, the values of U_0 for the positive polarity begin to dominate (with a difference of approximately −4 V at $p = 200\,\mathrm{kPa}$ and $R_1 = 148\,\mathrm{\mu m}$).

A key outcome of this work is the assessment of model simplifications. The local approximation of negative ion kinetics proposed in [6] was shown to significantly reduce the complexity of the governing equations (reduction from 8 to 5 PDEs without considering the electrostatics PDE, which was calculated using the analytic formula in a coaxial geometry) while maintaining high accuracy in the prediction of U_0 (the highest deviation from the full model reached 12 V within our parametric study). This makes it a suitable approach for efficient simulations, particularly in higher-dimensional problems.

In contrast, photoionization was identified as a strongly polarity-dependent mechanism. While its influence on U_0 is negligible for the negative polarity within the space-charge-free approximation, it plays a crucial role for the positive polarity, where it significantly reduces inception voltage, especially at low pressures and low field homogeneity (deviations from the full model in the range of 196–230 V in the absolute values are given within our parametric study). This asymmetry highlights the importance of correctly capturing photoionization effects in discharge modeling.

In addition, a qualitative analysis of discharge stability was performed, providing a physically consistent interpretation of the transition from harmonic oscillations to Trichel pulse regime analyzed in [5]. The proposed mechanism highlights the strong coupling between ionization dynamics, delayed formation of negative ions, and photoionization processes, which together govern the perturbation-induced feedback loops that are independent of the perturbation polarity, responsible for instability development.

Although this interpretation is supported by the observed charged particle distributions, its rigorous validation requires dedicated time-dependent analysis and will be the subject of future work.

Overall, the results confirm that the resonance method provides a physically consistent and flexible framework for inception voltage evaluation, while also enabling systematic investigation of the role of individual physical processes. Future work will focus on extending the approach to more complex geometries and on a detailed analysis of discharge stability and transient behavior.

Acknowledgements

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