Propagation of extraordinary laser beam in cold magnetized plasma

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This article studies the evolution of spot size of an intense extraordinary laser beam in cold, transversely magnetized plasma. Due to the relativistic nonlinearity, the plasma dynamic is modified in the presence of transversely magnetic filed. In order to specify the evolution of the spot size of extraordinary laser beam, nonlinear current density is set up and the source dependent expansion method is used. It is shown that enhancing the external magnetic field decreases the spot-size of laser beam significantly, and thus the self-focusing effect becomes more important due to the extraordinary property of laser beam.

Keywords: spot size, extraordinary laser beam, transversely magnetized plasma, self-focusing.

1 INTRODUCTION

It is known that, a laser beam propagating in plasma with plasma frequency smaller than the laser frequency undergoes relativistic self-focusing as soon as its total power exceeds the critical values [1]. So, high power laser propagating through plasma can acquire a minimum spot size due to relativistic and ponderomotive self-focusing. When a laser pulse propagates through plasma embedded in a uniform magnetic field, the Lorentz force acting on plasma electrons introduces changes in relativistic mass and causes electron density perturbations, leading to modification in the propagation characteristics of the laser beam. It is revealed that transverse magnetization of plasma enhances the self-focusing property of the laser beam and the critical power is reduced due to the presence of the magnetic field [2, 3]. In the latest study on self-focusing of laser beam in magnetized plasmas, the extraordinary properties of laser wave has been ignored [2]. In fact, when laser propagates through plasma, a longitudinal electrostatic field is generated due to the panderomotive force acting on plasma electrons, and this makes the laser beam to be extraordinary. So, for an accurate investigation, we should take the extraordinary property of laser into account. In the present study, we analyze, the effect of the uniform external magnetic field on self-focusing property of an intense extraordinary laser pulse propagating in a cold, homogenous plasma. The magnetic field is perpendicular to the electric field and the direction of propagation of the radiation field. Nonlinear wave equation [4] is set up and the source dependent expansion method [5] is used to determine the evolution of the spot size of a laser beam having a Gaussian profile. The effect of transverse magnetization of plasma on the self-focusing property of the extraordinary laser beam is investigated.

2 NONLINEAR WAVE EQUATION

Consider a uniform plasma of electron density n_0. The plasma is embedded in a static magnetic field $B\hat{y}$. A high intensity extraordinary laser at (ω_0, k_0) propagates through it along $\hat{z}$, with electric field,

$$\boldsymbol{E} = E_0\left(\hat{x} + i\beta_0\hat{z}\right)e^{i\theta_0} \tag{1}$$

where, $\theta_0 = (k_0 z - \omega_0 t)$, $\beta_0 = \frac{\omega_c}{\omega_0}\frac{\omega_p^2}{\omega_0^2 - \omega_{UH}^2}$.

and ω_p, ω_c, ω_{UH}, are the plasma frequency, electron cyclotron frequency, and upper-hybrid frequency, respectively [6].

In cold plasma, the plasma electrons are initially at rest and relativistic effects are ignored in the zeroth order. The response of plasma electrons to the pump wave is governed by the equations of motion and continuity,

$$\frac{d}{dt}(\gamma\vec{v}) = -\frac{e}{m}\left[\vec{E} + \frac{\vec{v}\times\left(\vec{B}_0 + \overline{B}\right)}{c}\right] \tag{2}$$

$$\frac{\partial n}{dt} = -\frac{\partial}{\partial z}(nv_z) \tag{3}$$

Where $\vec{v} = v_x\hat{i} + v_z\hat{k}$ and γ is the relativistic factor. Assuming $v_x = v_x^{(1)} + v_x^{(2)} + v_x^{(3)}$, and expanding Eq.(2) in the mildly relativistic limit $\left(\gamma^{-1} < 1, \omega_c < \omega_0\right)$, we find the three orders of transverse electron velocities up to third order nonlinearity,

$$v_x^{(1)} = ca_0\left(\frac{\omega_0^2 + \beta\omega_0\omega_c}{\omega_0^2 - \omega_c^2}\right)\sin\theta_0 \tag{4}$$

$$v_x^{(2)} = -\frac{c^2 a_0^2 k_0 \omega_0 Q_2}{2\left(\omega_0^2 - \omega_c^2\right)^2\left(4\omega_0^2 - \omega_c^2\right)}\sin 2\theta_0 \tag{5}$$

$$v_x^{(3)} = ca_0^3\left[\frac{c^2 k_0^2 \omega_0}{4\left(\omega_0^2 - \omega_c^2\right)^4\left(4\omega_0^2 - \omega_c^2\right)}Q_3 - \frac{3}{8}\frac{\omega_0}{\left(\omega_0^2 - \omega_c^2\right)^4}Q_4\right]\sin\theta_0 \tag{6}$$

Here, $a_0\left(= eE/mc\omega_0\right)$ is the normalized potential vector. First order and second order electron density perturbations can be find by expanding Eq.(3),

$$n^{(1)} = -n_0 c k_0 a_0\left(\frac{\beta_0\omega_0 + \omega_c}{\omega_0^2 - \omega_c^2}\right)\cos\theta_0 \tag{7}$$

$$n^{(2)} = \frac{n_0 c^2 a_0^2 k_0^2 \omega_0 Q_1}{(\omega_0^2 - \omega_c^2)^2(4\omega_0^2 - \omega_c^2)}\cos 2\theta_0 \tag{8}$$

Parameters, Q_1, Q_2, Q_3, Q_4, are defined as,

$$Q_1 = 4\omega_0^2\omega_c^2 + \left(3\beta_0^2 - 1\right)\omega_0^4 + 5\beta_0\omega_0^3\omega_c + \beta_0\omega_0\omega_c^3,$$

$$Q_2 = \left(3\beta_0^2 - 1\right)\omega_0^3\omega_c + 5\beta_0\omega_0^2\omega_c^2 + 4\omega_0\omega_c^3 + \beta_0\omega_c^4,$$

$$Q_3 = 2\beta_0^2\omega_0^7 + \beta_0\left(6\beta_0^2 - 25\right)\omega_0^4\omega_c^3 + \beta_0\omega_c^7 + \left(26\beta_0^2 - 5\right)\omega_0^3\omega_c^4 - \left(30\beta_0^2 + 5\right)\omega_0^5\omega_c^2 + 27\beta_0\omega_0^2\omega_c^5 - \beta_0\left(6\beta_0^2 - 5\right)\omega_0^6\omega_c + 10\omega_0\omega_c^6,$$

$$Q_4 = \left(3\beta_0^2 + 1\right)\left(\omega_0^7 + 6\omega_0^5\omega_c^2 + \omega_0^3\omega_c^4\right) + 4\beta_0^2\left(\beta_0^2 + 3\right)\left(\omega_0^6\omega_c + \omega_0^4\omega_c^3\right).$$

The perturbed velocities and densities are used to obtain the transverse current density

$$J_x = -e(n_0 v_x^{(1)} + n_0 v_x^{(3)} + n^{(1)}v_x^{(2)} + n^{(2)}v_x^{(1)}) \tag{9}$$

The second and fourth terms are due to change in the relativistic mass correction and additional density perturbations, respectively, while the third term arises due to the lowest order longitudinal electron oscillations. We have also neglected the second harmonic terms in defining Eq.(9). Substituting the values of perturbed quantities in Eq.(9) we find the nonlinear current density,

$$J_x = -en_0 ca_0\left(\frac{\omega_0^2 + \beta_0\omega_0\omega_c}{\omega_0^2 - \omega_c^2} - N_0 a_0^2\right)\sin\theta_0$$

$$N_0 = \frac{c^2 k_0^2 \omega_0^2 Q_1\left(\omega_0 + \beta_0\omega_c\right)}{2(\omega_0^2 - \omega_c^2)^3(4\omega_0^2 - \omega_c^2)} - \frac{c^2 k_0^2 \omega_0 Q_2\left(\beta_0\omega_0 + \omega_c\right)}{4\left(\omega_0^2 - \omega_c^2\right)^3\left(4\omega_0^2 - \omega_c^2\right)} - \frac{c^2 k_0^2 \omega_0 Q_3}{4\left(\omega_0^2 - \omega_c^2\right)^4\left(4\omega_0^2 - \omega_c^2\right)} + \frac{3}{8}\frac{\omega_0 Q_4}{\left(\omega_0^2 - \omega_c^2\right)^4} \tag{10}$$

Jha, et al. [2] have ignored the extraordinary properties of the pump laser wave. It means that they have taken the z component of the laser electric field to be zero and found N_0 to be independent of β_0, while we see that for an extraordinary laser wave, we need to keep the terms including β_0. In fact, when a laser beam passes trough transversely magnetized plasma, a longitudinal component of laser electric field is generated and the laser becomes an extraordinary wave.

Now, nonlinear wave equation governing the propagation of extraordinary laser pulse in magnetized plasma is of the form,

$$\left(\nabla_\perp^2 + 2ik_0\frac{\partial}{\partial z}\right)a_0(r,z) = k_{p0}^2\left(\frac{\omega_0^2 + \beta_0\omega_0\omega_c}{\omega_0^2 - \omega_c^2} - N_0 a_0^2\right)a_0(r,z) \tag{11}$$

Where, k_{p0} is the initial wave number of the excited plasma wave. Since the total laser power is independent of z, i.e. $\left(a_s^2 r_s^2 = a_0^2 r_0^2\right)$, a_s is the normalized laser potential vector, and using the source dependent expansion (SDE), we obtain the differential equation describing the evolution of the laser spot r_s in magnetized plasma as,

$$\frac{\partial^2 r_s}{\partial z^2} = \frac{4}{k_0^2 r_s^3}\left(1 - \frac{k_{p0}^2 a_0^2 r_0^2}{8}N_0\right) \tag{12}$$

The first term on the right-hand side of Eq.(12) is due to the vacuum diffraction, while the second term arises from the nonlinear self-focusing effect. The solution of Eq.(12) is,

$$\frac{r_s^2}{r_0^2} = 1 + \left(1 - \frac{P}{P_{CM}}\right)\frac{z^2}{z_R^2} \tag{13}$$

Here, $P/P_{CM} = k_{p0}^2 a_0^2 r_0^2 N_0/8$, in which $P_{CM}\left(= 2\pi^2 c^5 m^2 / k_{p0}^2 \lambda_0^2 e^2 N_0\right)$ is the critical power for nonlinear self-focusing of a laser beam having a Gaussian profile in magnetized plasma, and Z_R is the Rayleigh length.

In fig. 1, we have plotted the variation of the normalized spot-size$\left(r_s/r_0\right)$ of a laser beam with, $I = 10^{17}\,W/cm^2$, $r_0 = 10\mu m$, $\lambda_0 = 1\mu m$, $a_0 = 0.25$ in a plasma with the initially plasma wave length $\lambda_p = 15\mu m$, and $\omega_p/\omega_0 = 0.5$, $\omega_c/\omega_0 = 0.2$. We see that the extraordinary laser beam is less diverged due to the magnetization of the plasma.

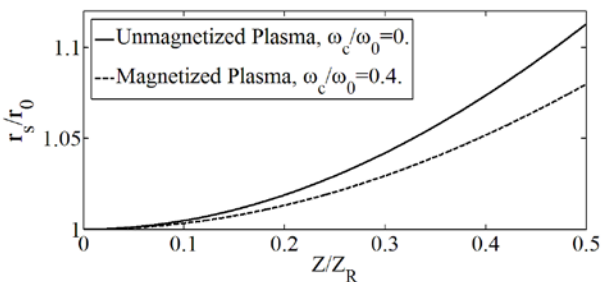

Fig.1: The variation of normalized laser spot-size against the normalized distance.

Fig. 2, shows the variation of the normalized spot-size against the normalized cyclotron frequency ω_c/ω_0 at $\omega_p/\omega_0 = 0.5$ for $a_0 = 0.25$ and $Z/Z_R = 0.4$. As it is shown, the spot-size is decreased, so the laser beam becomes more focused as the magnetic field increases.

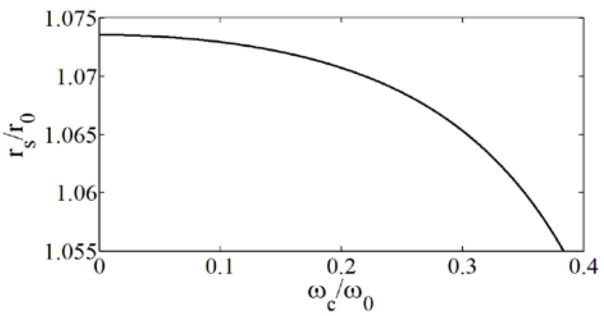

Fig.2: The variation of normalized laser spot-size against the normalized cyclotron frequency.

Thus, when we take the extraordinary property of laser beam into account, we see that the laser spot-size decreases significantly by increasing the external magnetic field. Thus, the self-focusing effect will be more considerable for extraordinary laser beam.

3 Results and conclusion

In the present paper, we introduced a new nonlinear wave equation governing the propagation of extraordinary laser wave trough magnetized plasma. Using the source dependent expansion method, we solved the nonlinear wave equation to investigate the effect of external magnetic field on the variation of laser spot-size. We obtained an accurate relation for the laser spot size which is different from that obtained by Jha, et al. [2]. We show that, the extraordinary property of the laser wave plays an important role in the propagation of laser in magnetized plasma. The present study reveals that the laser beam is less diverged due to the transverse magnetization of plasma. It was shown that increasing the external magnetic field decreases the laser spot size of extraordinary laser wave significantly.

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