doi:10.14311/ppt.2024.3.45
Plasma Physics and Technology **11**(3):45–49, 2024

THE EFFECT OF THE EXTERNAL MAGNETIC FIELD ON THE POWER OF THE LASER PASSING THROUGH THE MAGNETIZED PLASMA AND THE CALCULATION OF THE RADIUS OF CONVERGENCE AND THE INVESTIGATION OF THE CONDITION OF THE BEAM'S FILAMENTATION IN THE MEDIUM

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Abstract. In this article, the power of a laser passing through a magnetized plasma medium which is affected by an external magnetic field is calculated.The change in the intensity of the magnetic field and its rotation around the axis of laser beam propagation in the plasma, on the power of the passing beam is investigated.This work is done based on the indirect effect of the magnetic field on the density distribution of electrons in the plasma, which we can consider the density changes as changes in the refractive index of the medium, so the laser power in this case is different from the case where there is no magnetic field. Also, the condition of beam filamentation has been researched as one of the third-order nonlinear phenomena, and the convergence radius has been calculated under these conditions.

Keywords: magnetized plasma, density distribution, filamentation.

1. Introduction

The high power of the laser causes it to change the density distribution of electrons [1, 2] during the interaction with the plasma and create inhomogeneity in the medium [3]. Also, the creation of non-linear phenomena due to the interaction of the laser with the medium is one of the obvious results of this type of interactions. One of the non-linear phenomena that can be mentioned is the focusing of the laser beam inside the medium [4] which is created under certain conditions and the laser beam is focused and the medium acts like a converging mirror for the beam [5, 6]. In this paper, we consider a density magnetized plasma that is affected by an external magnetic field and interacts with a strong laser pulse. Then, based on the electron motion transfer equation in the plasma, we obtain its density and as a result, we obtain the transmission power and beam focal condition based on the changes of the external magnetic field and its rotation. Despite numerous studies on the interaction of lasers with various mediums, particularly plasma medium1 the effects of such interactions on nonlinear phenomena such as light condensation and light filamentation have been less studied, with more focus on particle acceleration and density changes in the medium [6]. We studied the nonlinear phenomena created in a plasma medium considering external factors. High-order nonlinear phenomena created by the interaction of high-intensity laser beams with the medium may be affected by external factors. Therefore, we considered magnetic fields as an external factor and investigated its effects on the third-order nonlinear phenomenon, i.e. light filamentation [7]. Given the numerous applications of nonlinear phenomena [8], studying the influential conditions on the output of these interactions can play a significant role in accurately understanding these phenomena.

2. Basic equations

The electron motion transfer equation is as follows:

$$mn_{\mathrm{e}}\frac{\partial Ve}{\partial t} = -n_{\mathrm{e}}e(E + Ve \times B) + \bigtriangledown P - n_{\mathrm{e}}F_{\mathrm{pe}}. \quad (1)$$

In Eq (1), P is the gas pressure in the plasma, B is the external magnetic field and $F_{\mathrm{pe}} = \frac{-e^2 \bigtriangledown |Ey|^2}{m\omega^2}$ is the ponderomotive force [9]. Also, we ignore the changes in the speed of electrons (we assume that the wave created in the medium is static) [10], as a result, we do not take into account the changes in its speed, as well as the acceptable condition of the characteristic length. Let's also use the Debye wavelength $10^{-5}(\mathrm{cm}) \succ 10^{-6}(\mathrm{cm})$, so the changes in the density of electrons will be as follows:

$$n_{\mathrm{e}}(z) = n_{\mathrm{e}0}(z) \exp\left(\frac{-e^2 \bigtriangledown |E_{\mathrm{y}}|^2 + ev_{\mathrm{e}}|B|\sin\theta}{kt}\right). \quad (2)$$

According to Eq (2), $\omega_0 = \frac{4\pi e^2}{m} n_{\mathrm{e}0}$. The dielectric coefficient of plasma is defined as follows:

$$\varepsilon = 1 - \frac{\omega_0^2}{\omega^2} \exp\left(\frac{-e^2 \bigtriangledown |E_{\mathrm{y}}|^2 + ev_{\mathrm{e}}|B|\sin\theta}{kt}\right). \quad (3)$$

The electric field is obtained using Maxwell's equations. The resulting equation is a non-linear equation that can be solved numerically

$$\frac{\mathrm{d}^2 E}{\mathrm{d}z^2} + \frac{\omega}{c}\varepsilon E = 0. \quad (4)$$

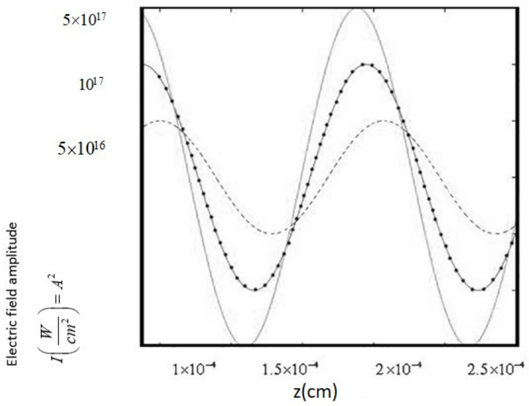

Figure 1. Electric field inside the plasma for different intensities. (solid line 5×10^{17} W/cm^2, dotted line 10^{17} W/cm^2, dashed line 5×10^{16} W/cm^2).

Figure 1 shows the electric field inside the plasma for different intensities. The maximum electric field strength passing through a plasma medium decreases with a forward shift, in other words, it can be said that with an increase in the electric field strength proportional to the square of the amplitude, more space of the plasma medium is affected.

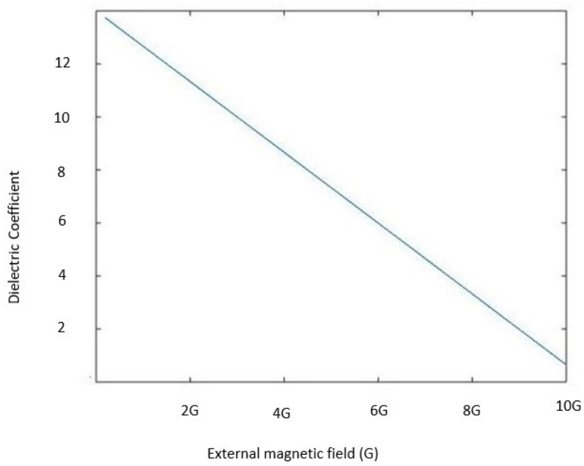

Figure 2. Dielectric changes of the plasma in the presence of an external magnetic field.

Figure 2 shows the dielectric changes of the plasma in the presence of an external magnetic field. The dielectric constant, or permittivity, varies for different mediums and depends on factors such as temperature, pressure, and chemical composition of the medium [11]. For a laboratory plasma medium, the dielectric constant may range from 1 to 10 or even higher. Therefore we have considered this constant in the laboratory range between 1 to 14 which is drawn for the area involved in the magnetic field [4, 12]. It is possible that the changes in a very small area change linearly and in larger areas exponentially. We consider the electron temperature to be 10 keV, the gas pressure inside the plasma is considered to be the full gas

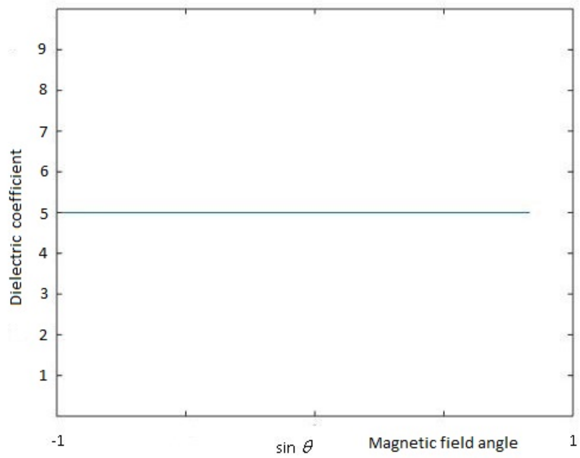

Figure 3. Dielectric changes based on the rotation of the external magnetic field.

pressure, and the laser intensity is 3.5×10^{19} W/m^2. We considered and choose the frequency of the electric field from the order 10^{15} rad/s [13]. As evident from the graph, this constant decrease with the increase of the external magnetic field. The magnetic field redistributes the ions in the medium and causes a new arrangement in the structure of the medium. As a result, the orientation of the ions and the force exerted on them by the magnetic field will undergo expected changes.

Figure 3 shows the dielectric changes based on the rotation of the external magnetic field. The rotation of the magnetic field around the emission of the laser beam has no effect on the dielectric coefficient of the environment. The relation (3) shows that the dielectric coefficient of the plasma changes with the application of the magnetic field. Since the density distribution of electrons changes under the influence of the external magnetic field and the interaction of the laser pulse with the environment, as a result, the inhomogeneity of the environment changes the dielectric coefficient of the environment and provides the conditions for third-order nonlinear processes. The refractive index depends on the intensity of the incident pulse [12] as a result of non-linear phenomena such as the focusing of the light beam, so first we will obtain the power of the laser pulse under such conditions and check the condition of focusing. If we consider the interaction electric field as a Gaussian field, $E_{\mathrm{y}}^2 = \frac{E_{0y}^2}{2|n|} \exp\left(-\frac{r^2}{r_0^2}\right)$ Eq (2) will be as follows:

$$n_{\mathrm{e}}(z) = n_{0z} \exp\left[\frac{1}{kT}\left(\frac{-e^2\left(1+|n|^2 E_{\mathrm{y}}^2\right)}{m\omega^2 16\pi |n| r_{01}}\sqrt{2}\exp\left(-\frac{1}{2}\right) + e v_{\mathrm{e}}|B|\sin\theta\right)\right]. \tag{5}$$

Since the self-focusing phenomenon is one of the characteristics of the nonlinear medium of the third order, so we will continue to investigate the general reflection conditions inside the medium under the influence of the nonlinear force involved in the interaction of density changes in two areas with different refractive index near the axis of the beam and it is observed in the area far from the beam axis. In the conditions of total reflection, the difference in the refractive index and the initial angle of the beam are used. If we consider θ_0 the radiation angle [13], we have:

$$\sin\left(\frac{\pi}{2}-\theta_0\right)=\frac{|n|}{|n_0|} \tag{6}$$

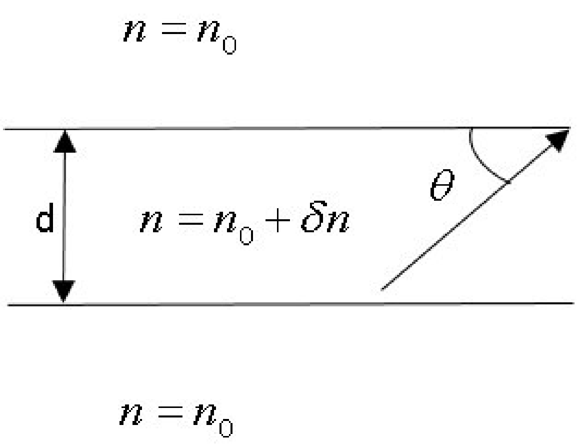

Figure 4. General reflection of the medium.

Figure 4 shows the general reflection of the medium, which can be calculated using the Snell-Descartes theorem for two medium with different refractive indices.

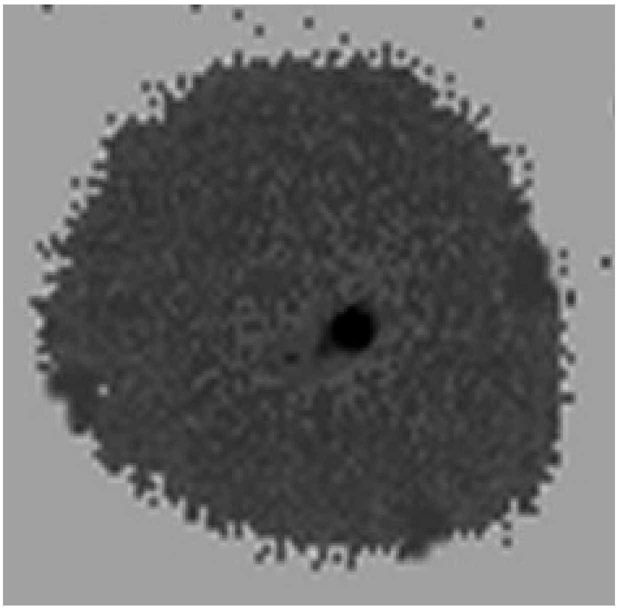

Figure 5. Laser beam convergence in a plasma medium.

Figure 5 illustrates the phenomenon of laser beam convergence in a plasma medium. The intensity of the beam is higher at the center of the page, indicating the density of the beams in this region.

Using the Taylor expansion for the density around the field diffusion axis, we have:

$$n_{r0}=n+\partial n=n+\frac{\partial n}{\partial r}r_0, \tag{7}$$

$$\cos\theta_0=\frac{n}{n+\frac{\partial n}{\partial r}r_0}=1-\frac{1}{2}\theta_0^2\approx 1-\frac{\partial n}{n}\to\theta_0^2=\frac{2\partial n}{n}, \tag{8}$$

$$\sin\left(\frac{\pi}{2}-\theta_0\right)=1-\frac{\partial n}{n}. \tag{9}$$

Using Eq (2), will be as follows:

$$\sin\theta\approx\exp\left[\frac{1}{kT}\left(\frac{-e^2(1+|n|^2E_y^2)}{16m\omega^2|n|r_{01}}\sqrt{2}\exp\left(-\frac{1}{2}\right)+ev_e|B|\sin\theta\right)\right]. \tag{10}$$

Checking the diffraction condition for the convergence of this diffraction angle is a hypothetical angle to determine the condition.

$$\sin\theta_{\mathrm{diff}}\approx\frac{\lambda}{\pi(2r_0)}=\frac{\pi c}{2\omega r_0}. \tag{11}$$

From Eq (10), the radius of the beam becomes as follows $r_0\geqslant\frac{\pi c}{2\omega\sin\theta}$. It turns out that the result will be as follows:

$$r_0\geqslant\frac{\pi c}{2\omega\left(\frac{\frac{-e^2\left(1+|n|^2E_y^2\right)}{16m\omega^2|n|r_{01}}\sqrt{2}\exp\left(-\frac{1}{2}\right)+ev_e|B|\sin\theta}{kt}\right)^{\frac{1}{2}}}. \tag{12}$$

Therefore, the minimum radius of convergence according to equation (12) is defined as follows:

$$r_0=\frac{\pi c}{2\omega\left(\frac{\frac{-e^2\left(1+|n|^2E_y^2\right)}{16m\omega^2|n|r_{01}}\sqrt{2}\exp\left(-\frac{1}{2}\right)+ev_e|B|\sin\theta}{kt}\right)^{\frac{1}{2}}}. \tag{13}$$

By parametrizing the Eq (13), an explicit form of the radius of convergence can be defined. The parameters controlling the convergence radius for the laser beam for the plasma medium magnetized by the magnetic field with significant intensity, so we can rewrite the Eq (13) as follows:

$$r_0=\frac{\pi c}{\omega\left(\frac{\frac{-e^2\left(1+|n|^2E_y^2\right)}{m\omega^2|n|r}+ev_e|B|\sin\theta}{kt}\right)^{\frac{1}{2}}}. \tag{14}$$

The relationship between the radius of convergence shows the conditions affecting the interaction of the beam with the plasma. A high intensity magnetic

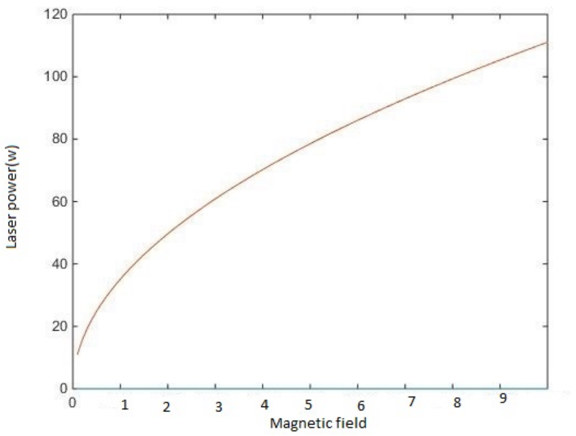

Figure 6. Change of laser power in terms of changes in the external magnetic field.

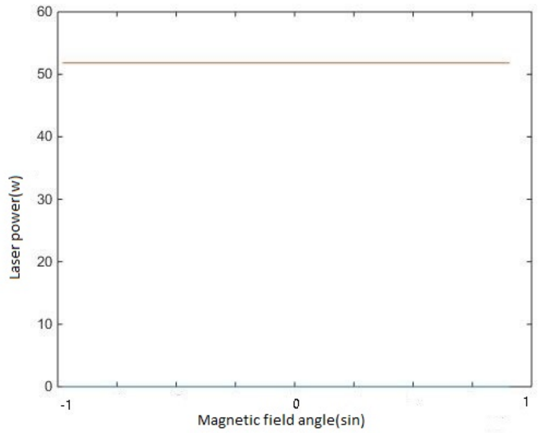

Figure 7. Changes in the power of the beam inside the plasma based on changes in the direction of the magnetic field.

field can change the radius of convergence. Now let's calculate the radiation power inside the plasma under the conditions of influencing factors. In general, the beam power is written as follows $p = \frac{\pi r_0 I}{2}$. Therefore, using equation (12), the beam power is obtained as follows:

$$p = \frac{\pi r_0 I}{\omega\left(\frac{\frac{-e^2\left(1+|n|^2 E_y^2\right)}{16 m\omega^2 |n| r_{01}}\sqrt{2}\exp\left(-\frac{1}{2}\right)+e v_e |B| \sin\theta}{kt}\right)^{\frac{1}{2}}}. \quad (15)$$

Figure 6 shows the change of laser power in terms of changes in the external magnetic field. It increases with the increase of the magnetic field in the condition that the magnetic field is perpendicular to the speed of the electrons.

Figure 7 shows the diagram of changes in the power of the beam inside the plasma based on changes in the direction of the magnetic field, according to the diagram shown that the rotation of the field around the emission axis does not cause a change in the laser power.

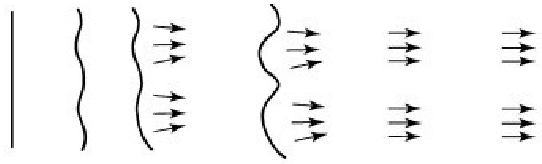

Figure 8. Illustration of laser beam breakup by the growth of wavefront perturbtion.

According to the critical power which is defined as $P_{\mathrm{cr}} \approx \frac{\pi\lambda^2}{8 n_0 n_2}$, we can compare the obtained power with the critical power of the filamentation condition. If the power of the beam inside the plasma is greater than the critical power, the filamentation process will occur [14, 15]

$$p \succ p_{\mathrm{cr}}, \quad (16)$$

$$\frac{\pi r_0 I}{\omega\left(\frac{\frac{-e^2\left(1+|n|^2 E_y^2\right)}{16 m\omega^2 |n| r_{01}}\sqrt{2}\exp\left(-\frac{1}{2}\right)+e v_e |B| \sin\theta}{kt}\right)^{\frac{1}{2}}} \succ \frac{\pi\lambda^2}{8 n_0 n_2}. \quad (17)$$

3. Conclusion

In this article, the laser power passing through a magnetized plasma which is affected by an external magnetic field was calculated and its effects on the laser passing power were investigated. Also, the laser beam stringing condition was compared based on the critical power and the convergence radius was also calculated. The graphs drawn based on the considered data showed the rotation of the field. Magnetization around the emission axis has no effect on the transmitted beam power, but the intensity of the magnetic field can affect the transmitted power by disrupting the density distribution and thus change the stringing conditions. By comparing the critical power with the obtained power, the phenomenon of light filamentation in the plasma can be controlled and this phenomenon can be created by changing the effective parameters. Similarly, with a change in the direction of the magnetic field, the changes occur in the opposite direction, and it is possible to define a frequency for alternating the field direction in two opposite directions in short time intervals.

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