INSTABILITY OF DUST-LOWER-HYBRID MODE IN IRRADIATED STREAMING DUSTY PLASMA WITH DUST CHARGE FLUCTUATION

M. S. MUNIR^{a, b}, M. K. Islam^{$b, *$}, M. A. H. Talukder^{a, b}, M. Salahuddin^a

^a Department of Physics, Jahangirnagar University, Savar, Dhaka1342, Bangladesh

^b Plasma Physics Division, Atomic Energy Centre, Dhaka-1000, Bangladesh

∗ khairulislam@yahoo.com

Abstract. A theoretical investigation of the photoelectric effect through dust charge fluctuation on the low frequency dust-lower-hybrid (DLH) mode has been done using fluid model of plasma. In this study collisional effects between charged and neutral particles and lighter particles streaming along the both external magnetic and electric fields are considered. It is assumed that dust grains are negatively charged. It has been observed that the DLH mode becomes unstable significantly due to photoelectric effect compared to the streaming and collisional effects.

Keywords: Photoelectric effect, dust lower hybrid mode, instability, dust charge fluctuation, irradiated dusty plasma.

1. Introduction

Dusty plasma consists of electrons, ions, highly charged $(Z_d \sim 10^1 - 10^5)$ and relatively massive $(m_d/m_+ \sim 10^6 - 10^{12})$ dust grains and neutral particles [\[1\]](#page-5-0). In such plasma, micron or sub-micron sized dust grains can be charged by absorbing electrons and ions of the plasma. Since the velocity of electrons is much more than that of ions, they readily sit on the surface of the dust grains and make them negatively charged. On the other hand, charging due to photoelectron emission, thermionic emission, secondary emission, etc. can be significant and dust grains may become positively charged [\[2\]](#page-5-1).

The existence and mobility of charged dust grains might revise the plasma modes that already exist or introduce new space and time scales as a result in novel modes, their instabilities, and related phenomena [\[1\]](#page-5-0). There have been several studies done on dust modes in dusty plasma, including theoretical predictions $[3-9]$ $[3-9]$ and experimental findings $[10-12]$ $[10-12]$. Dust charge fluctuation (DCF) occurs when the conditions in the plasma near the dust grain are changed due to a variety of reasons, such as the wave motion. Considering the dust charge as a time dependable variable, an interesting result of the damping of the electrostatic modes has been investigated [\[5](#page-5-6)[–7,](#page-5-7) [13,](#page-5-8) [14\]](#page-5-9). All these studies are done in the absence of radiation. On the other hand, radiation is invariably present in the naturally occurring dusty plasmas [\[15\]](#page-5-10). In addition, radiation is applied in laboratory dusty plasma for specific purposes $[2, 16]$ $[2, 16]$ $[2, 16]$. M. K. Islam et al. is studied the photoelectric effect through the DCF on dust modes using Vlasov kinetic model and shown that the high frequency plasma wave can be unstable due to the photoelectric effect in streaming and irradiated un-magnetized dusty plasma with positively charged dust grains [\[17\]](#page-5-12). Recently, M. S. Munir et al.

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theorized that the electrostatic low frequency dust acoustic mode could become unstable in an irradiated, un-magnetized dusty plasma [\[3\]](#page-5-2). M. A. H. Talukder et al. have presented that in un-magnetized dusty plasma, dust-ion-acoustic mode can be excited significantly due to photoelectric effect through DCF [\[4\]](#page-5-13). Magnetic field should be noted in irradiated dusty plasma since they are always present in naturally occurring dusty plasmas or are applied externally in laboratory dusty plasmas [\[6–](#page-5-14)[8,](#page-5-15) [10\]](#page-5-4). So far, instabilities of the dust modes in irradiated magnetized dusty plasma should be pointed out.

In this paper, the photoelectric effect through DCF on the low frequency electrostatic dust lower hybrid (DLH) mode in magneto dusty plasma using fluid model of plasma has been investigated theoretically. In this study, we consider magnetic and zero order electric fields are in the same direction. We also assumed that lighter particles streaming along the external zero order electric field and collision between neutral and charged particles of the dusty plasma. Dust grains are charged negatively. This is because in equilibrium plasma, we have considered that the photoelectric work function of the dust grain material is higher than the incident photon energy as well as the photon flux is much less than the electron flux to the dust grain.

The rest of manuscript is organized as follows: In Section [2](#page-1-0) derivation of the general dispersion relation of the electrostatic dust modes using fluid model is presented. The dispersion relation of the low frequency electrostatic DLH mode is given in Section [3.](#page-3-0) Numerical results and discussions of the DLH mode and instability of this mode are analyzed in Section [4.](#page-3-1) Finally, conclusions of this research work are given in Section [5.](#page-4-0)

2. Dispersion relation of electrostatic dust modes

Let us consider homogeneous and uniform magnetized dusty plasma consists of different charged and neutral particles of masses m_{+}, m_{e}, m_{d} and m_{N} , where the subscripts $(+)$, (e) , (d) and (N) denotes respectively, the ions, electrons, dust grains and neutral particles. The dust charge is considered as negative, i.e., $Q_d = -Z_d e$, where Z_d and *e* being the number of electrons residing on a dust grain and elementary charge, respectively. An external magnetic field (**B**) and electric field (**E**) have been applied along the *z*-direction. We have considered the streaming of ions (electrons) relative to massive dust grains in the *z*-direction with constant velocity, v_{+o} (v_{e0}). It is considered that the low frequency electrostatic waves are propagating slantingly to **B** $(K_x^2 \gg K_z^2)$ with propagation vector **K** lying in the *xz*-plane.

Continuity and momentum equations for three charged elements of the dusty plasma are taken as follows, respectively:

$$
\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v_j}) = \mathbf{0},\tag{1}
$$

where j represents $+$, e and d, respectively.

$$
n_{+}m_{+}\frac{\partial \mathbf{v}_{+}}{\partial t} + n_{+}m_{+}\mathbf{v}_{+}\nabla \cdot \mathbf{v}_{+} + k_{B}T_{+}\nabla n_{+}
$$

$$
-en_{+}\mathbf{E} - en_{+}\mathbf{v}_{+} \times \mathbf{B} = -\nu_{+}n_{+}m_{+}\mathbf{v}_{+}, \quad (2)
$$

$$
k_B T_e \nabla n_e + en_e \mathbf{E} + en_e \mathbf{v}_e \times \mathbf{B} = -\nu_e n_e m_e \mathbf{v}_e, \quad (3)
$$

$$
n_{\rm d}m_{\rm d}\frac{\partial \mathbf{v}_{d}}{\partial t} + n_{\rm d}m_{\rm d}\mathbf{v}_{\rm d}\nabla \cdot \mathbf{v}_{\rm d} + k_{B}T_{\rm d}\nabla n_{\rm d}
$$

$$
+ eZ_{\rm d}n_{\rm d}\mathbf{E} + eZ_{\rm d}n_{\rm d}\mathbf{v}_{\rm d} \times \mathbf{B} = -\nu_{\rm d}n_{\rm d}m_{\rm d}\mathbf{v}_{\rm d}.
$$
 (4)

Here, n_j , and T_j are the densities, and temperature of the dusty plasma species, respectively and *k^B* is the Boltzmann constant.

The continuity and momentum equations are coupled to the following set of equations:

Poisson's equation:

$$
\varepsilon_0 \nabla \cdot \mathbf{E} = e(n_+ - n_e - Z_d n_d),\tag{5}
$$

quasi-neutrality equation:

$$
n_{+} = n_{\rm e} + Z_{\rm d} n_{\rm d} \tag{6}
$$

and the basic dust grain charging equation:

$$
\frac{\mathrm{d}Q_{\mathrm{d}}}{\mathrm{d}t} = I_{+} + I_{\mathrm{e}} + I_{\mathrm{pe}},\tag{7}
$$

where I_+ , I_e and I_{pe} represent ion current, electron current and photoelectron current, respectively which are collected by the dust grain and are given by [\[3,](#page-5-2) [5,](#page-5-6) [13\]](#page-5-8).

$$
I_{+} = \pi a_{\rm d}^{2} e n_{+} v_{+0} \left[1 - \frac{2e\Phi_{\rm d}}{m_{+} v_{+0}^{2}} \right],
$$
 (8)

$$
I_{\rm e} = -\pi a_{\rm d}^2 e_{\rm eq} v_{\rm e0} \left[1 - \frac{2e\Phi_{\rm d}}{m_{+} v_{\rm e0}^2} \right],\tag{9}
$$

$$
I_{\rm pe} = \pi a_{\rm d}^2 e n_{\rm e} \sqrt{\frac{2}{m_{\rm e}} (h\nu_p + e \Phi_{\rm d})} \exp\left\{\frac{e \Phi_{\rm d}}{h\nu_p + e \Phi_{\rm d}}\right\}.
$$
\n(10)

At zero-order state, the plasma is steady and uniform i.e., $\frac{\partial}{\partial t} = 0 = \frac{\partial}{\partial x} = \frac{\partial}{\partial y}$, the electric field **E**₀ is constant and the velocities of the three charged elements of the dusty plasma are then obtained from Eqs. $(2) - (4)$ $(2) - (4)$ $(2) - (4)$, respectively, as follows:

$$
e\mathbf{E_0} = \nu_+ m_+ \mathbf{v}_{+0},\tag{11}
$$

$$
e\mathbf{E_0} = -\nu_{\rm e}m_{\rm e}\mathbf{v}_{\rm e0},\tag{12}
$$

$$
eZ_{\rm d0}\mathbf{E_0} = -\nu_{\rm d}m_{\rm d}\mathbf{v_{\rm d0}}.\tag{13}
$$

The quasi-neutral condition at zero-order state becomes:

$$
n_{+0} = n_{e0} + Z_{d0} n_{d0}.
$$
 (14)

From Eqs. $(11) - (13)$ $(11) - (13)$ $(11) - (13)$, it is seen that streaming velocity of the plasma species depends on the charge and mass of the species, collisional frequency of the species and the zero order electric field.

Let us define the Doppler shifted frequencies as $\Omega_{+} = \omega - K_z v_{+0}, \Omega_{\rm e} = \omega - K_z v_{\rm e0}$ and $\Omega_{\rm d} = \omega - K_z v_{\rm d0}$ due to streaming of ions, electrons and dust grains, respectively, where $v_{e0} \gg v_{+0} \gg v_{d0}$. The quanti t ies $C_{+}^{2} = \frac{k_{B}T_{+}}{m}$ $\frac{k_B T_+}{m_+}, C_e^2 = \frac{k_B T_e}{m_e}$ $\frac{k_B T_{\rm e}}{m_{\rm e}}$, $C_{\rm d}^2 = \frac{k_B T_{\rm d}}{m_{\rm d}}$ $\frac{m_{\rm d}^2}{m_{\rm d}}$, $\omega_{\rm c+}$ = *eB^z* $\frac{eB_z}{m_+}, \omega_{\text{ce}} = -\frac{eB_z}{m_{\text{e}}}$ $\frac{eB_z}{m_e}$ and $\omega_{cd} = -\frac{eZ_{d0}B_z}{m_d}$ $\frac{a_0^2}{m_d}$ are the ion thermal velocity, electron thermal velocity, dust thermal velocity, ion cyclotron frequency, electron cyclotron frequency and dust cyclotron frequency, respectively.

For linearizing Eqs. $(1) - (7)$ $(1) - (7)$ $(1) - (7)$, all first-order quantities are assumed to have the space and time dependence as $e^{i(K_x x + K_z z - \omega t)}$. From the continuity equa-tion [Eq. [\(1\)](#page-1-5)] for ion and considering $\epsilon_{+} = \frac{n_{+1}}{n_{-1}}$ $\frac{n+1}{n+0}$, we get

$$
\epsilon_+ \ \Omega_+ = K_x v_{+1x} + K_z v_{+1z}.\tag{15}
$$

From the momentum equation for ion $[Eq. (2)]$ $[Eq. (2)]$ $[Eq. (2)]$, the first-order quantities can be written as

$$
v_{+1x} = \frac{\left(C_+^2 \in + \frac{e}{m_+} \Phi_1\right) K_x (\Omega_+ + i\nu_+)}{(\Omega_+ + i\nu_+)^2 - \omega_{c+}^2}, \quad (16)
$$

$$
v_{+1y} = -\frac{i\omega_{c+}v_{+1x}}{(\Omega_+ + i\nu_+)},\tag{17}
$$

$$
v_{+1z} = \frac{\left(C_+^2 \in + +\frac{e}{m_+} \Phi_1\right) K_z}{\left(\Omega_+ + i\nu_+\right)},\tag{18}
$$

where Φ_1 is the perturbed electric potential. Putting Eqs. $(16) - (18)$ $(16) - (18)$ $(16) - (18)$ in Eq. (15) we get

$$
n_{+1} = \frac{n_{+0}e\Phi_1R}{m_+(\Omega_+ - C_+^2R)},\tag{19}
$$

where

$$
R = \frac{(K_x^2 + K_z^2)(\Omega_+ + i\nu_+)^2 - \omega_{c+}^2 K_z^2}{[(\Omega_+ + i\nu_+)^2 - \omega_{c+}^2](\Omega_+ + i\nu_+)}.
$$
 (20)

Following similar method, from the linearized continuity and momentum Eqs. (1) , (3) and (4) for the electron and dust grain we obtain

$$
n_{\rm e1} = \frac{in_{\rm e0}e\Phi_1F}{m_{\rm e}(\Omega_{\rm e} + iC_{\rm e}^2F)}\tag{21}
$$

and

$$
n_{\rm d1} = -\frac{eZ_{\rm d0}n_{\rm d0}\Phi_1 L}{m_{\rm d}(\Omega_{\rm d} - C_{\rm d}^2 L)},\tag{22}
$$

where

$$
F = \frac{(K_x^2 + K_z^2)\nu_e^2 + \omega_{ce}^2 K_z^2}{(\nu_e^2 + \omega_{ce}^2)\nu_e}
$$
(23)

and

$$
L = \frac{(K_x^2 + K_z^2)(\Omega_{\rm d} + i\nu_{\rm d})^2 - \omega_{\rm cd}^2 K_z^2}{[(\Omega_{\rm d} + i\nu_{\rm d})^2 - \omega_{\rm cd}^2](\Omega_{\rm d} + i\nu_{\rm d})}.
$$
 (24)

The first order perturbed photoelectron, electron and ion currents are obtained, respectively, from the Eqs. $(8) - (10)$ $(8) - (10)$ $(8) - (10)$ as

$$
I_{+1} = | I_{+0} | \frac{n_{+1}}{n_{+0}}, \tag{25}
$$

$$
I_{e1} = | I_{e0} | \frac{n_{e1}}{n_{e0}}, \tag{26}
$$

$$
I_{\text{pel}} = | I_{\text{pe0}} | \frac{n_{\text{el}}}{n_{\text{e0}}}.
$$
 (27)

Due to the presence of an electrostatic wave the dust will gain a perturbed charge Q_{d1} , which can be obtained from Eq. [\(7\)](#page-1-6) and using Eqs. $(25) - (27)$ $(25) - (27)$ $(25) - (27)$ we get

$$
Q_{d1} = \frac{i}{\omega} \left[(| I_{e0} | - | I_{pe0} |) \left(\frac{n_{+1}}{n_{+0}} - \frac{n_{e1}}{n_{e0}} \right) \right].
$$
 (28)

Since the ion streaming velocity is lower than that of the electrons and hence, $I_{+0} < I_{e0}$. Moreover, the irradiation frequency is much higher than that of the dust modes, Therefore, the contribution of I_{+0} is negligible and hence, I_{+0} can be ignored in dust grain charge fluctuation [cf. Eq. [\(28\)](#page-2-2)]. In this case, the piled up electrons on the dust grain surface is swept out by the photoelectric effects and hence, neutralized the fluctuated dust charge. From the linearized Eq. [\(5\)](#page-1-13), we get

$$
\varepsilon_0 K^2 \Phi_1 = e(n_{+1} - n_{\text{el}} + \frac{Q_{\text{d1}}}{e} n_{\text{d0}} - Z_{\text{d0}} n_{\text{d1}}), \tag{29}
$$

where $K^2 = K_x^2 + K_z^2$.

The required general dispersion relation for the electrostatic dust-modes in collisional, streaming and magnetized dusty plasma including DCF with photoelectric effect is then obtained from Eq. [\(29\)](#page-2-3) using the value of n_{+1}, n_{e1}, n_{d1} and Q_{d1} from Eqs. [\(19\)](#page-1-14), (21) , (22) and (28) , respectively. The obtained general dispersion relation is then given by

$$
1 = \frac{\omega_{\rm p+}^2 R}{K^2 (\Omega_{+} - C_{+}^2 R)} \left[1 + \frac{i}{\omega} (\beta_{\rm e} - \beta_{\rm p}) \frac{n_{\rm e0}}{n_{\rm +0}} \right] - \frac{i \omega_{\rm p\rm e}^2 F}{K^2 (\Omega_{\rm e} + i C_{\rm e}^2 F)} \left[1 + \frac{i}{\omega} (\beta_{\rm e} - \beta_{\rm p}) \right] + \frac{\omega_{\rm pd}^2 L}{K^2 (\Omega_{\rm d} - C_{\rm d}^2 L)}, \quad (30)
$$

where the quantities $\omega_{p+} = \left(\frac{n_{+0}e^2}{2m}\right)^2$ $\epsilon_0 m_+$ $\Big)^{\frac{1}{2}}$, $\omega_{\rm pe}$ = $(n_{\rm e0}e^2)$ *ϵ*0*m*^e $\int_{0}^{\frac{1}{2}}$ and $\omega_{\text{pd}} = \left(\frac{Z_{\text{d}0}^2 n_{\text{d}0} e^2}{\omega_{\text{rad}}}\right)$ *ϵ*0*m*^d $\int_{0}^{\frac{1}{2}}$ are the ion

plasma frequency, the electron plasma frequency and dust plasma frequency, respectively.

The DCF parameter due to electron current β_e = $\frac{|I_{e0}|}{|I_{e0}|} \frac{n_{d0}}{n_{d0}}$ and that of the photoelectron current *e n*e0 $\beta_{\rm p} = \frac{\mid I_{\rm pe0} \mid}{\sqrt{\cdot}}$ *e* $n_{\rm d0}$ $\frac{n_{\text{d0}}}{n_{\text{e0}}}$ in Eq. [\(30\)](#page-2-6), are like the effective collision frequency of the streaming electrons with the dust grains and the effective detachment frequency of photoelectrons from the dust grains, respectively [\[5,](#page-5-6) [13,](#page-5-8) [17\]](#page-5-12).

For low frequency electrostatic dust mode, we consider dust particles are cold $(C_d = 0)$ and unmagnetized $(\omega_{\rm cd} = 0)$ and ions are also cold $(C_{+} = 0)$, but strongly magnetized, electrons as well as photoelectrons form a hot Boltzmann gas at temperature *T*^e [\[5,](#page-5-6) [7,](#page-5-7) [17\]](#page-5-12), i. e., the electrostatic dust mode satisfy the following set of condition: $\omega_{\text{cd}} \ll \omega \ll \omega_{\text{c}+} \ll \omega_{\text{ce}}$, $KC_{d}, K_{z}C_{+} \ll \omega \ll K_{z}C_{e}$. Since $m_{e} \ll m_{+} \ll m_{d}$, collision of dust with neutrals can be neglected $(\nu_d =$ 0).

Under the above conditions, substituting the values of *R, F* and *L* from Eqs. [\(20\)](#page-2-7), [\(23\)](#page-2-8) and [\(24\)](#page-2-9), respectively, in equation (30) we get

$$
\omega^2 = \omega_{DM}^2 - i\beta - i\nu'_+ - i\nu'_e,\tag{31}
$$

where

$$
\omega_{DM}^2 = \frac{\left(\frac{K_z^2 \omega_{\rm p+}^2}{K^2 \Omega_+^2} + \frac{\omega_{\rm pd}^2}{\Omega_d^2}\right) \omega^2}{1 + \frac{1}{\lambda_{\rm de}^2 K^2} + \frac{K_x^2 \omega_{\rm p+}^2}{K^2 \omega_{\rm c+}^2}},\tag{32}
$$

$$
\beta = \frac{(\beta_{\rm e} - \beta_{\rm p}) \left[\frac{n_{\rm e0}\omega_{\rm p+}^2 K_x^2}{n_{\rm +0}K^2\omega_{\rm c+}^2} + \frac{1}{K^2\lambda_{\rm de}^2} \right] \omega}{1 + \frac{1}{\lambda_{\rm de}^2 K^2} + \frac{K_x^2 \omega_{\rm p+}^2}{K^2\omega_{\rm c+}^2}},\tag{33}
$$

$$
\nu'_{+} = \frac{\left(\nu_{+} \frac{\omega_{p+}^{2} K_{x}^{2}}{K^{2} \Omega_{+} \omega_{c+}^{2}}\right) \omega^{2}}{1 + \frac{1}{\lambda_{\text{de}}^{2} K^{2}} + \frac{K_{x}^{2} \omega_{p+}^{2}}{K^{2} \omega_{c+}^{2}}},\tag{34}
$$

$$
\nu'_{\rm e} = \frac{\left(\frac{\nu_{\rm e}\Omega_{\rm e}}{K^2\lambda_{\rm de}^2C_{\rm e}^2K_z^2}\right)\omega^2}{1 + \frac{1}{\lambda_{\rm de}^2K^2} + \frac{K_x^2\omega_{\rm p+}^2}{K^2\omega_{\rm e+}^2}}.\tag{35}
$$

Eq. (31) is the dispersion relation of low frequency electrostatic dust modes. There may have several modes but our interest is to observe DLH mode in magnetized dusty plasma which are derived in the next section from this equation using relevant conditions.

3. Dust lower hybrid (DLH) mode

For simplicity of the analysis of low-frequency, lowphase-velocity electrostatic DLH mode in the magnetized dusty plasma, we now consider $\frac{\omega_{\rm p+}^2}{2}$ $\omega_{\mathrm{c+}}^2$ $> \frac{1}{2}$ $\frac{1}{\lambda_{\text{de}}^2 K^2}$ $K_x^2 \gg K_z^2$ and $K_z v_{d0} \ll \omega$. In these conditions, we obtain the DLH mode from Eq. (31) as

$$
\omega^{2} = \omega_{\text{DLH}}^{2} \left(1 + \frac{K_{z}^{2}}{K_{x}^{2}} \frac{\omega_{\text{p}+}^{2}}{\omega_{\text{p}d}^{2} \Omega_{+}^{2}} \omega^{2} \right)
$$

$$
-i \left[(\beta_{\text{e}} - \beta_{\text{p}}) \frac{n_{\text{e}0}}{n_{+0}} \omega + \nu_{+} \frac{\omega^{2}}{\Omega_{+}} + \nu_{\text{e}} \frac{1}{\lambda_{\text{de}}^{2} \omega_{\text{p}+}^{2}} \frac{\Omega_{\text{e}} \omega_{\text{c}+}^{2} \omega^{2}}{C_{\text{e}}^{2} K_{z}^{2} K^{2}} \right].
$$
(36)

It is noted that without photoelectric effects, i.e. $\beta_{\rm p} =$ 0, Eq. [\(36\)](#page-3-2) reduces to the dispersion relation given by M. K. Islam et al. (2003) [\[5\]](#page-5-6).

From Eq. [\(36\)](#page-3-2), we can easily separate the real part (ω_R) and imaginary part (growth rate, γ) of the DLH mode, those are given by, respectively,

$$
\omega^2 = \omega_{\text{DLH}}^2 \left(1 + \frac{K_z^2}{K_x^2} \frac{\omega_{\text{P}+}^2}{\omega_{\text{pd}}^2} \right),\tag{37}
$$

and

$$
\gamma = \frac{1}{2} \left[(\beta_{\rm p} - \beta_{\rm e}) \frac{n_{\rm e0}}{n_{\rm +0}} - \nu_{\rm +} \frac{\omega^2}{\Omega_{\rm +}} - \nu_{\rm e} \frac{1}{\lambda_{\rm de}^2 \omega_{\rm p+}^2} \frac{\Omega_{\rm e} \omega_{\rm c+}^2 \omega}{C_{\rm e}^2 K_z^2 K^2} \right]
$$
(38)

where ω_{DLH} is the DLH frequency which is given by

$$
\omega_{\text{DLH}}^2 = \frac{\omega_{\text{pd}}^2 \omega_{\text{c}+}^2}{\omega_{\text{p}+}^2}.
$$
 (39)

The DLH mode is due to the presence of strongly magnetized ions and un-magnetized dust grains and propagate nearly perpendicular to the z-axis in the Cartesian co-ordinate system.

The first and second terms of the Eq. [\(38\)](#page-3-3) represent the DCF effects due to photoelectron and electron currents, respectively. The third and fourth terms represent the collisional effects on the DLH mode in the magnetized dusty plasmas. It is clear from Eq. (38) , at $v_{e0}(v_{+0}) < \frac{\omega}{\nu}$ $\frac{\infty}{K_z}$, without DCF effect collisional effects give damping of DLH mode. At $v_{e0}(v_{+0}) > \frac{\omega}{K}$ $\frac{\infty}{K_z}$ collisional effect between electrons (ions) and neutral atoms can unstable the DLH mode.

4. Numerical results and discussions

For the purpose of numerical study of the DLH mode and its instability, we implement a set of appropriate dusty plasma parameters that are given in Table [1](#page-4-1) [\[1,](#page-5-0) [8,](#page-5-15) [9,](#page-5-3) [18\]](#page-5-16).

Figure 1. The angular frequency vs. z component of the wave vector of DLH mode for $Z_{do} = 10^4$ and $K_x = 1000 \, m^{-1}$.

Figure 2. The growth rate vs. electron's streaming velocity for $Z_{do} = 10^4$, $K_x = 1000$ m^{-1} and $K_z = 5$ m^{-1} .

Fig. [1](#page-3-4) shows the linear relationship between the angular frequency and *z* component of the wave number (K_z) of the DLH mode. It is clear from this figure that the K_z has a significant impact to sustain DLH mode in magnetized dusty plasma. It is because wave is influenced by the external magnetic field which is applied along the *z*-direction.

In Fig. [2](#page-3-5) growth rate vs. electron's streaming velocity of DLH mode is plotted in the case of $\beta_{\rm p} = \beta_{\rm e} = 0$, i.e., without DCF effects. In this figure we find that at $v_{e0} = 3.8 \times 10^2 \text{ ms}^{-1}$ growth rate becomes zero. In this case the wave's parallel phase velocity is equal to the streaming velocity of electrons. i.e. in the absence of dust charge fluctuation effect critical streaming velocity is $3.8 \times 10^2 \text{ ms}^{-1}$. Below this critical streaming velocity, DLH mode becomes damped, i. e., in the case of $K_z v_{e0} < \omega$, wave losses energy to the streaming effect. On the other hand above the critical streaming velocity DLH mode start to grow i. e., in the case of $K_z v_{\text{e}0} > \omega$, wave gain energy from the streaming effect. In other words, without DCF effects, the collisional effects can make the DLH mode unstable in streaming dusty plasma if the streaming velocity of electron exceeds the wave's parallel phase velocity [cf. Eq. [\(38\)](#page-3-3)].

In Fig. [3,](#page-4-2) growth rate vs. photon energy of DLH

 \equiv

Parameter	Value
Ion mass (m_+)	4.7×10^{-26} (kg)
Dust grain mass (m_d)	4.2×10^{-15} (kg)
Electron temperature (kT_e)	$0.1~\mathrm{(eV)}$
Ion temperature (kT_+)	0.01~(eV)
Dust radius (a_d)	$10 \; (\mu m)$
Electron density (n_e)	3×10^{15} (m ⁻³)
Ion density (n_+)	3×10^{15} (m ⁻³)
Dust density (n_d)	1×10^{11} (m ⁻³)
Magnetic field (B)	0.1 T
Electron neutral collision frequency (ν_e)	$2.2 \times 10^{7}(s^{-1})$
Ion neutral collision frequency (ν_+)	$8.8 \times 10^{2}(s^{-1})$

Table 1. Set of plasma parameters.

Figure 3. The growth rate vs. photon energy for $Z_{do} =$ 10^4 , $K_x = 1000 \, \text{m}^{-1}$, $K_z = 5 \, \text{m}^{-1}$ and $v_{e0} = 1 \times 10^3$ ms^{-1} .

mode is plotted including the DCF effects. In this figure the dashed curve represents DCF effects in the presence of collisional and streaming effects whereas the solid curve represents only the DCF effects. These both curves (Solid and dashed) show that the growth rate increases with respect to photon energy, i. e., wave get more energy from photoelectric and streaming effects. It is observed from this figure that in the absence of streaming effects, growth rate of the DLH mode due to DCF effect becomes less than that of the combined effects of DCF and streaming of charged particles of plasma, it is because, in this case we include $v_{\text{e}0} = 1 \times 10^3$ and the condition $K_z v_{\text{e}0} > \omega$ is satisfied [cf. Eq. [\(38\)](#page-3-3)]. As a result, due to combined effects of DCF and streaming velocity of electron make the DLH mode more unstable as shown in Fig. [3.](#page-4-2) It is observed from Figs. [2](#page-3-5) and [3,](#page-4-2) growth rate of DLH mode due to DCF effect is five order more than that of without DCF effect.

5. Conclusions

Using the fluid model of plasma, a rigorous investigation of the photoelectric effect through DCF on the low frequency electrostatic DLH mode in a magnetized dusty plasma has been studied theoretically, considering lighter particles streaming to the direction

of external magnetic field in the presence of zero-order electric field and collisional effects of charged particles with neutral atoms. The dust grains are considered to be negatively charged. To analyze the low frequency and low phase velocity electrostatic dust modes, necessary general dispersion relation [cf. Eq. (31)]-in which many modes may present-has been derived using fundamental equations of fluid model of plasma. 2

In the limit,
$$
\frac{\omega_{\text{P}+}^2}{\omega_{\text{c}+}^2} > \frac{1}{\lambda_{\text{de}}^2 K^2}
$$
, the electrostatic ultra

low frequency DLH mode is obtained which propagate nearly perpendicular to the external magnetic field involving strongly magnetized ions and un-magnetized dust grains [cf. Eq. [\(36\)](#page-3-2)]. It has been found that the *z* component of the wave vector (K_z) has a significant impact to generate natural DLH mode in magnetized dusty plasma.

It is found that at $v_{e0} = 3.8 \times 10^2$ ms⁻¹ growth rate of the DLH mode become zero in the absence of DCF effects. Below this critical streaming velocity, DLH mode become damped, whereas above the critical streaming velocity DLH mode start to grow. This is because the Doppler shifted frequency in the collisional terms can make the DLH mode unstable in streaming dusty plasma if the streaming velocity exceeds the wave's parallel phase velocity [cf. Eq. [\(38\)](#page-3-3)].

The DCF effects can unstable the DLH mode [cf. Eq. [\(38\)](#page-3-3)] and it is seen that the growth rate increases with respect to photon energy. It is observed that due to the DCF effect, in the presence of streaming and collisional effects, the DLH mode become more unstable than that of in the absence of that effects. As seen from Figs. [2](#page-3-5) and [3,](#page-4-2) instability of DLH mode due to DCF effect is five order more than that of without DCF effect.

Therefore, it is understood that the instability of DLH mode occur significantly due to the photoelectric effect compared to that of the streaming and collisional effects.

Finally, it is concluded that the present study should be applied in understanding the photoelectric effects on electrostatic dust modes in the space science, where dust grains and radiation exist, such as planetary rings, the lower ionosphere/magnetosphere of the Earth, interstellar space cloud, etc. as well as irradiated laboratory dusty plasma, such as the edge plasma of the fusion devises, etc.

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