Experimental Verification of Model Simulation Results of Heating Cycles within the Electric Muffle Furnaces

Jiří Lucák

Department of Electrical Power Engineering and Environmental Engineering, University of West Bohemia in Pilsen, Czech Republic, e-mail: *jlucak@kee.zcu.cz*

Abstract — **The paper [1] described a mathematical model of the electric muffle furnace. A specific solution of real models has been already solved and presented within this paper. The first one is the muffle heating furnace equipped with a fireproof clay muffle (MODEL I) and thermal insulation of expanded fireproof clay. As the second variant the furnace equipped with a silicon carbide muffle (MODEL II) and with a thermal insulation of fibre ceramic boards of Sibral type is presented. Further the paper presents the results of simulations of the same heating cycles performed using the mathematical model presented in the paper [1] and comparing these results with the progression of the values measured in real furnaces.**

Experiment and simulation assessment will be carried out as the final step hereof. Equipment parameters are sources from technical documentations of the manufacturers.

Keywords — *muffle furnace, thermal process, heating element, furnace lining and mathematical model*

I. MODEL I

Manufacturer

Elektrim – Poland, year of equipment manufacture: 1963

Fig. 1. MODEL I – Fireproof clay muffle furnace.

A. Technical Description

The muffle furnace is designed for the performance of heating processes; see [1].

Furnace steel construction is made of 2.5 mm sheet metal. Inside the furnace body is a fireproof clay muffle with a coiled heating element. The muffle inside the muffle is covered with a ceramic material. The heating element is made of the resistance wire KANTHAL A1.

The heating insulation consists of extended fireproof clay furnace blocks. The furnace body is coated with protecting paint.

B. Technical Parameters

Furnace type	PPEM-2/156
Rated output	3.6 kW
Rated voltage	1×220 V
Rated temperature	1100 °C
Heating element resistance at 20 °C	13.2 Ω
Heating element material	Kanthal A1
Diameter of wire	2.0 mm
Specific surface load	2.0 W/cm^2

C. Furnace Workspace Dimensions

Width	180 mm
Height	120 mm
Length	400 mm

D. Furnace Outer Dimensions

Width	520 mm
Height	722 mm
Length	830 mm
Mass	150 kg

II. MODEL II

A. Technical Description

The muffle furnace was designed for the purposes of experimental measurement of heating processes using the present-day knowledge within the field, see [1].

Inside the furnace body the SiC muffle with a coiled heating element of the resistance wire KANTHAL A1 is placed.

Furnace heat insulation is made of fibre materials of high thermal insulation capacity and low energy accumulation.
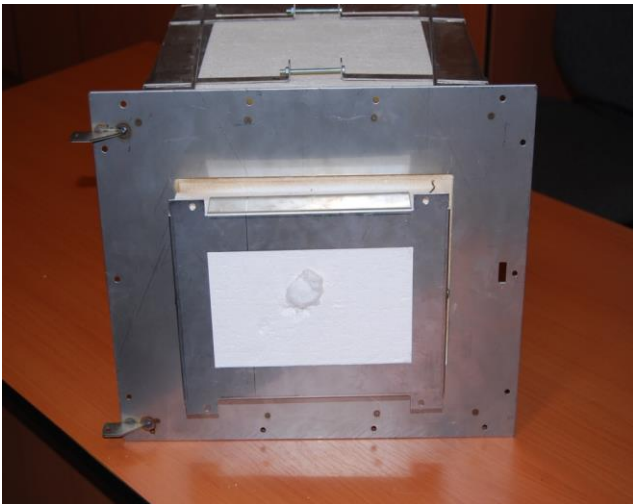
Fig. 2. MODEL II – Fibre materials furnace.

B. Technical Parameters

Rated output	2.2 kW
Rated voltage	1×230 V
Rated temperature	1100 °C
Heating element resistance at 20 °C	24.6 Ω
Heating element material	Kanthal A1
Diameter of wire	1.4 mm
Specific surface load	1.925 W/cm^2

C. Furnace Workspace Dimensions

Width	170.mm
Height	95 mm
Length	320 mm

D. Furnace Outer Dimensions

Width	400 mm
Height	600 mm
Length	530 mm
Mass	27 kg

III. TECHNICAL IMPLEMENTATION OF THE EXPERIMENTS IN THE FURNACES

The present case means: the furnaces were connected to variable voltage source which consists of a tap-changing transformer of 220 V, 180 V and 155 V switchable taps. The input power of the heating system is as follows: a step pulse of a constant value of supply voltage for the whole period of the experiment; see Fig. 3.

Resistance of the heating element is temperature dependent; it goes up slightly together with the material temperature, which means that together with the increasing temperature the performance output is falling down.

Manufacturers set the maximum permissible temperature in the furnaces to 1100 °C. Having been aimed not to exceed this temperature during the experiments, it was used the controller with two-position control (ON/OFF) that kept the temperature inside the furnace at this level. The steady-state would reach, at the value of constant output of the used temperature, a higher value than the maximum permissible temperature stated by the manufacturer. However, the maximum permissible temperature could not be exceeded when this kind of regulator was used.

When the experiment was finished, the furnace was switched out and spontaneous cooling was then initiated, starting at the temperature of 1100 °C.
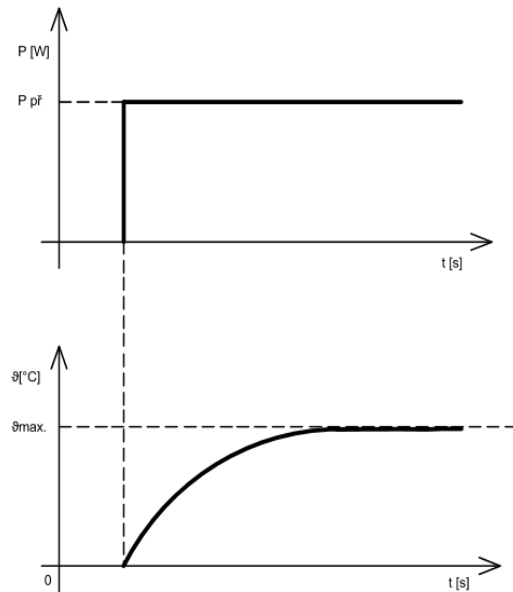

Fig. 3. The performance of the heating system in form of step unit. The estimated run of temperature.

During the experiments it were recorded:
- Thermocouple temperature inside the furnace.
- Temperature of a thermocouple attached to the outer furnace surface.
- Electric voltage at the heating element terminals.
- Electric current flowing through the heating element.

The following charts show the measurement records related to the Model I and Model II.

IV. DESCRIPTION OF THE CHANNELS OF EUROTHERM A 6100 RECORDER

|			
Channel 1	Red	0 – 1200 °C	Temperature progress in the furnace workspace
Channel 2	Light blue	0 – 1200 °C	Temperature progress on the furnace surface
Channel 3	Yellow	0 –250 V	Voltage on the heating element
Channel 4	Light green	0 – 50 A	Current flowing through the heating element
Channel 5	Dark green	0 – 1	A signal specifying the area of heating element connection to electric energy
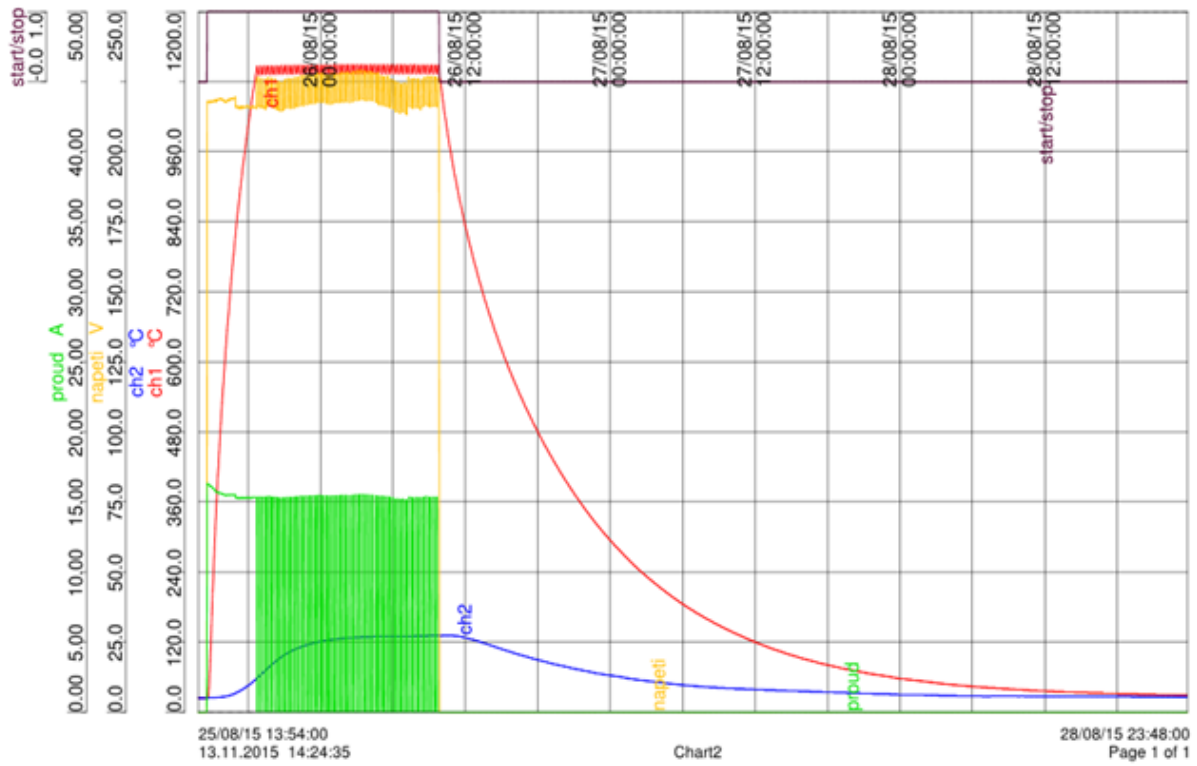
Fig. 4. Records sourced from the EUROTHERM A 6100 recorder – a temperature cycle progression – MODEL I – 220 V voltage tap.
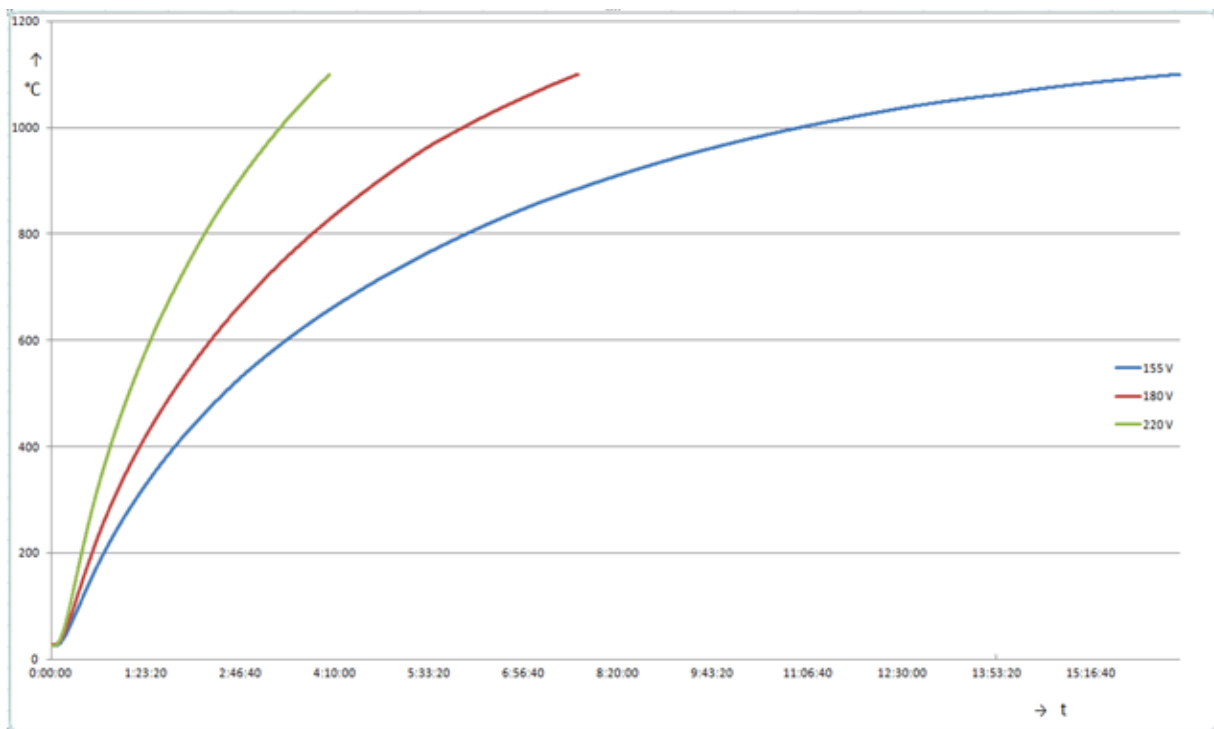

Fig. 5. Temperature progression in the furnace area in MODEL I for the 220 V, 180 V and 155 V voltage taps.
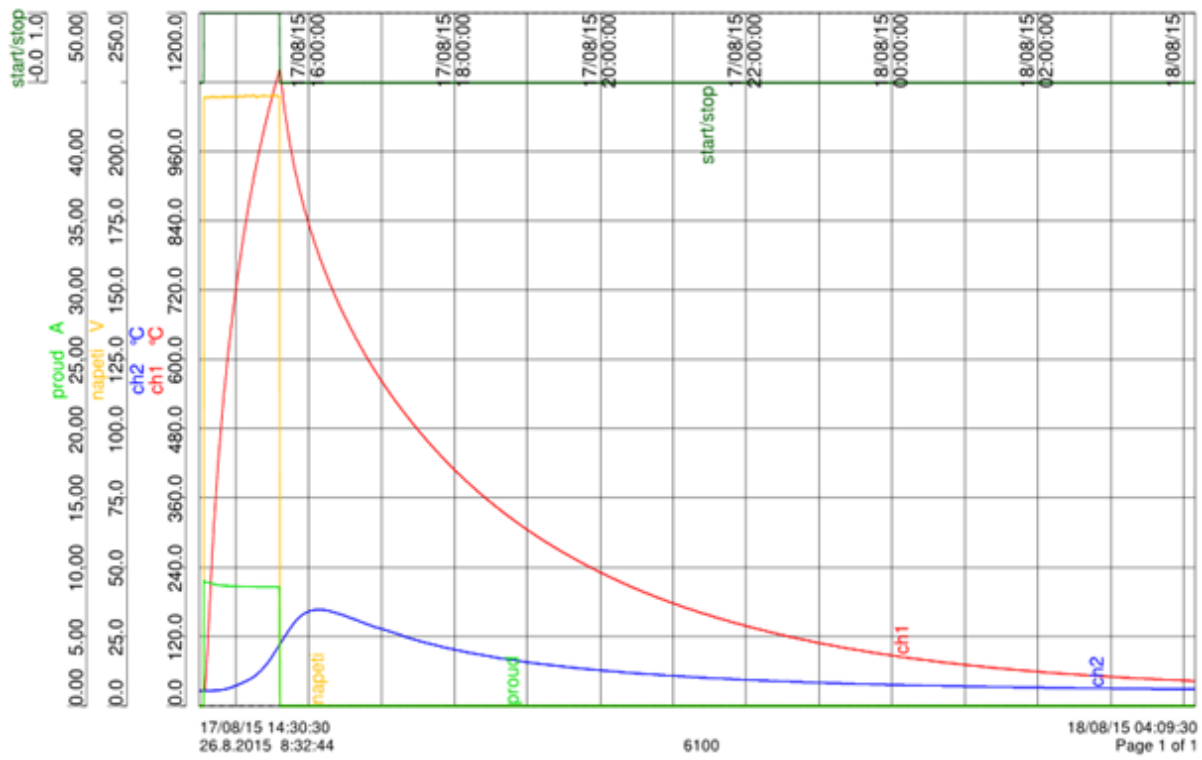
Fig. 6. Records sourced from the EUROTHERM A 6100 recorder – a temperature cycle progression – MODEL II – 220 V voltage tap.
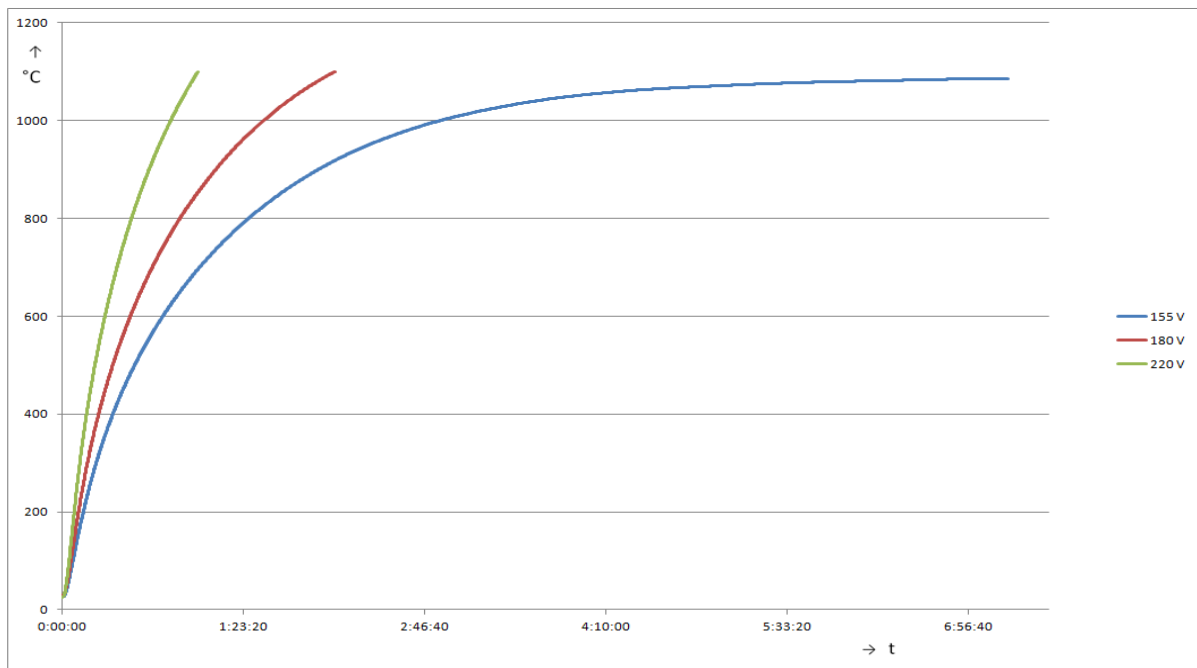

Fig. 7. Temperature progression in the furnace area in MODEL II for the 220 V, 180 V and 155 V voltage taps.

V. Heating Simulation Using a Mathematical Model of the Paper [1]

Due to different placing of the heating element in both models, modifications were performed in the differential equation system, as well as in the heat flow relations.

VI. A System of Temperature Differential Equations of State

Generally: The change of the thermal capacity content is given by the sum of the input heat which flows into the thermal capacity and the heat which flows out the thermal capacity.

a) At Model I

Fig. 8. MODEL I – Fireproof clay muffle.

A furnace made of fireproof clay materials consists of a heating coil which is poured with and embedded in the clay material which covers the muffle. In this case, the mass of the coil and the clay material is included in the $M1$ and a transfer carried out from it into the muffle and the surrounding insulation is understood as conductive transfer.

b) At Model II

Fig. 9. MODEL II – The SiC muffle.

The Sibral furnace + SiC, where the heating coil is placed unfixed in the space between the muffle and the Sibral boards the heat transfer is understood as the thermal radiation from a coil towards the muffle $q_{1,2}$, from a coil towards the Sibral $q_{1,4}$ and from the muffle towards the Sibral $q_{2,4}$. The convection is not considered.

For the reasons of clarity it is to show the modified relations within the Model I and II.

Heating coil:

$$M_1 c_1 \frac{\mathrm{d}t_1}{\mathrm{d}\tau} = q_{\mathrm{el}} - q_{1,2} - q_{1,4} \cdot \qquad (1)$$

Model I – the fireproof clay muffle:

$$M_2 c_2 \frac{\mathrm{d}t_2}{\mathrm{d}\tau} = q_{1,2} - q_{2,3} \, , \qquad (2.\mathrm{a})$$

Model II – the SiC muffle:

$$M_2 c_2 \frac{\mathrm{d}t_2}{\mathrm{d}\tau} = q_{1,2} - q_{2,3} - q_{2,4} \, , \qquad (2.\mathrm{b})$$

$$M_3 c_3 \frac{\mathrm{d}t_3}{\mathrm{d}\tau} = q_{2,3} - q_{3,8S} - q_{3,13K} - q_{3,14S} \, , \qquad (3)$$

Side walls + ceiling + hearth lining of the furnace:
Model I:

$$M_4 c_4 \frac{\mathrm{d}t_4}{\mathrm{d}\tau} = q_{1,4} - q_{4,5} \, , \qquad (4.\mathrm{a})$$

Model II:

$$M_4 c_4 \frac{\mathrm{d}t_4}{\mathrm{d}\tau} = q_{1,4} + q_{2,4} - q_{4,5} \, , \qquad (4.\mathrm{b})$$

$$M_5 c_5 \frac{\mathrm{d}t_5}{\mathrm{d}\tau} = q_{4,5} - q_{5,6} \, , \qquad (5)$$

$$M_6 c_6 \frac{\mathrm{d}t_6}{\mathrm{d}\tau} = q_{5,6} - q_{6,7} \, , \qquad (6)$$

$$M_7 c_7 \frac{\mathrm{d}t_7}{\mathrm{d}\tau} = q_{6,7} - q_{7,\mathrm{okk}} - q_{7,\mathrm{oks}} \cdot \qquad (7)$$

The door + back walls lining:

$$M_8 c_8 \frac{\mathrm{d}t_8}{\mathrm{d}\tau} = q_{3,8\mathrm{s}} - q_{8,9} - q_{8,13\mathrm{k}} - q_{8,14\mathrm{s}} \, , \qquad (8)$$

$$M_9 c_9 \frac{\mathrm{d}t_9}{\mathrm{d}\tau} = q_{8,9} - q_{9,10} \, , \qquad (9)$$

$$M_{10} c_{10} \frac{\mathrm{d}t_{10}}{\mathrm{d}\tau} = q_{9,10} - q_{10,11} \, , \qquad (10)$$

$$M_{11} c_{11} \frac{\mathrm{d}t_{11}}{\mathrm{d}\tau} = q_{10,11} - q_{11,12} \, , \qquad (11)$$

$$M_{12} c_{12} \frac{\mathrm{d}t_{12}}{\mathrm{d}\tau} = q_{11,12} - q_{12,\mathrm{okk}} - q_{12,\mathrm{oks}} \cdot \qquad (12)$$

Gas in the furnace:

$$M_{13} c_{13} \frac{\mathrm{d}t_{13}}{\mathrm{d}\tau} = q_{3,13\mathrm{k}} + q_{8,13\mathrm{k}} - q_{13,14\mathrm{k}} \cdot \qquad (13)$$

Thermometer:

$$M_{14}c_{14}\frac{dt_{14}}{d\tau} = q_{13,14k} + q_{3,14s} + q_{8,14s} \ . \tag{14}$$

VII. RELATIONS FOR HEAT FLOWS

The input power flowing into the furnace is affected by the change of rated resistivity of the heating element according to the temperature t1 and may be regulated according to the thermocouple temperature t_{14}, eventually according to the time τ.

$$q_{el} = q_{el}(\tau, t_1, t_{14}) \tag{15.a}$$

Model I – the flow $q_{2,4}$ is not in the fireproof clay furnace version:

$$q_{1,2} = S_{1,2}(t_1 - t_2)/(s_1/2\lambda_1 + s_2/2\lambda_2) \tag{16.a}$$

$$q_{1,4} = S_{1,4}(t_1 - t_4)/(s_1/2\lambda_1 + s_4/2\lambda_4) \tag{16.1.a}$$

Model II – from the Sibral and SiC furnace version:

$$q_{1,2} = \varepsilon_{\check{c}}\varepsilon_{1,2}\left((t_1 + 273)^4 - (t_2 + 273)^4\right)S_{1,2}\varphi_{1,2} \tag{16.b}$$

$$q_{1,4} = \varepsilon_{\check{c}}\varepsilon_{1,4}\left((t_1 + 273)^4 - (t_4 + 273)^4\right)S_{1,4}\varphi_{1,4} \tag{16.1.b}$$

$$q_{2,4} = \varepsilon_{\check{c}}\varepsilon_{2,4}\left((t_2 + 273)^4 - (t_4 + 273)^4\right)S_{2,4}\varphi_{2,4} \tag{16.2}$$

Next, for both versions it is applied the following:

$$q_{2,3} = S_{2,3}(t_2 - t_3)/(s_2/2\lambda_2 + s_3/2\lambda_3) \tag{17}$$

$$q_{3,13k} = S_{3,13}(t_3 - t_{13})\alpha_{k3} \tag{18}$$

$$q_{3,8s} = \varepsilon_{\check{c}}\varepsilon_{3,8}\left((t_3 + 273)^4 - (t_8 + 273)^4\right)S_{3,8}\varphi_{3,8} \tag{19}$$

$$q_{3,14s} = \varepsilon_{\check{c}}\varepsilon_{3,14}\left((t_3 + 273)^4 - (t_{14} + 273)^4\right)S_{3,14}\varphi_{3,14} \tag{20}$$

$$q_{1,4} = S_{1,4}(t_1 - t_4)/(s_4/2\lambda_4) \tag{21}$$

$$q_{4,5} = S_{4,5}(t_4 - t_5)/(s_4/2\lambda_4 + s_5/2\lambda_5) \tag{22}$$

$$q_{5,6} = S_{5,6}(t_5 - t_6)/(s_5/2\lambda_5 + s_6/2\lambda_6) \tag{23}$$

$$q_{6,7} = S_{6,7}(t_6 - t_7)/(s_6/2\lambda_6 + s_7/2\lambda_7) \tag{24}$$

$$q_{7,okk} = S_{7,ok}(t_7 - t_{ok})\alpha_{k7} \tag{25}$$

$$q_{7,oks} = \varepsilon_{\check{c}}\varepsilon_{7,ok}\left((t_7 + 273)^4 - (t_{ok} + 273)^4\right)S_{7,ok}\varphi_{7,ok} \tag{26}$$

$$q_{8,9} = S_{8,9}(t_8 - t_9)/(s_8/2\lambda_8 + s_9/2\lambda_9) \tag{27}$$

$$q_{8,13k} = S_{8,13}(t_8 - t_{13})\alpha_{k8} \tag{28}$$

$$q_{8,14s} = \varepsilon_{\check{c}}\varepsilon_{8,14}\left((t_8 + 273)^4 - (t_{14} + 273)^4\right)S_{8,14}\varphi_{8,14} \tag{29}$$

$$q_{9,10} = S_{9,10}(t_9 - t_{10})/(s_9/2\lambda_9 + s_{10}/2\lambda_{10}) \tag{30}$$

$$q_{10,11} = S_{10,11}(t_{10} - t_{11})/(s_{10}/2\lambda_{10} + s_{11}/2\lambda_{11}) \tag{31}$$

$$q_{11,12} = S_{11,12}(t_{11} - t_{12})/(s_{11}/2\lambda_{11} + s_{12}/2\lambda_{12}) \tag{32}$$

$$q_{12,okk} = S_{12,ok}(t_{12} - t_{ok})\alpha_{k12} \tag{33}$$

$$q_{12,oks} = \varepsilon_{\check{c}}\varepsilon_{12,ok}\left((t_{12} + 273)^4 - (t_{ok} + 273)^4\right)S_{12,ok}\varphi_{12,ok} \tag{34}$$

$$q_{13,14k} = S_{13,14}(t_{13} - t_{14})\alpha_{k14} \tag{35}$$

$$q_{ok} = q_{7,okk} + q_{7,oks} + q_{12,okk} + q_{12,oks} \tag{36}$$

The time course of the thermal process in a real furnace is in the mathematical model simulated using the movement trajectory in a 15 dimension space-time coordinates of which are: $(\tau, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13}, t_{14})$.

VIII. PHYSICAL PARAMETERS OF MATERIALS

Heating coil – Kanthal A1

Model I – Kanthal + fireproof clay material
Specific mass ρ (kg/m³):   1071
Specific heat c (J/kgK): $934 + 0.671t$
Heat conductivity λ (W/mK): $0.417 + 8.69 \cdot 10^{-5}t$
Diameter of heating coil: 0.014 m, diameter of wire: 2.0 mm, length of wire: 30.3 m.

Model II
Mass M (kg):   0.281
Specific heat c (J/kgK):   460
Emissivity ε (1):   0.7
Diameter of heating coil: 0.010 m, diameter of wire: 1.4 mm, length of wire: 25.75 m.

Muffle

Model I – dense fireproof clay SII
Specific mass ρ (kg/m³):   1900
Specific heat c (J/kgK): $934 + 0.671t$
Heat conductivity λ (W/mK): $1.12 + 1.75 \cdot 10^{-4}t$
Emissivity ε (1): $0.85 - 1 \cdot 10^{-4}t$

Model II – Silicon carbide SiC
Specific mass ρ (kg/m³):   3100
Specific heat c (J/kgK):
     $642 + 1.648t - 0.0015t^2 + 0.0000005t^3$
Heat conductivity λ (W/mK):   60
Emissivity ε (1):   0.93

Furnace lining

Model I – Foamed clay
Specific mass ρ (kg/m³):   732
Specific heat c (J/kgK): $934 + 0.671t$
Heat conductivity λ (W/mK): $0.188 + 3.91 \cdot 10^{-5}t$
Emissivity ε (1): $0.85 - 1 \cdot 10^{-4}t$
Steel sheathing: 2.5 mm, emissivity of the Al paint: 0.5 .

Model II – Sibral boards (Ceraboard)
Specific mass ρ (kg/m³): $310 \cdot 1.5$
Specific heat c (J/kgK): $724 + 1.21t - 0.0005t^2$
Heat conductivity λ (W/mK):
     $0.078 - 1.74 \cdot 10^{-5}t + 1.65 \cdot 10^{-7}t^2$
Emissivity ε (1):   0.56

Gas inside the furnace – Nitrogen
Specific mass ρ (kg/m³): $1.25 \cdot 273/(t_{13}+273)$
Specific heat c (J/kgK):   1043

Thermometer – sheathed thermocouple NiCr-Ni
Specific mass ρ (kg/m³):   4000
Specific heat c (J/kgK):   700
Emissivity ε (1): $0.65 + 1.22 \cdot 10^{-4}t$

IX. GEOMETRY OF MODEL

Model I

Furnace outer dimensions

Width (m):	0.520
Height (m):	0.520
Depth (m):	0.600

Muffle outer dimensions

Width (m):	0.210
Height (m):	0.160
Depth (m):	0.415

Muffle inner dimensions

Width (m):	0.180
Height (m):	0.130
Depth (m):	0.400

Thickness of the insulation lining of the side walls (m): 0.135
Thickness of the insulation lining of the ceiling and furnace hearth (m): 0.160
Thickness of the insulation lining of the door and back wall (m): 0.140
Thickness of the furnace steel sheathing (m): 0.0025

Sheathed thermocouple

Diameter (m): 0.002
The length sticking out into the furnace (m): 0.050

Model II

Furnace outer dimensions

Width (m):	0.325
Height (m):	0.245
Depth (m):	0.320

Muffle outer dimensions

Width (m):	0.185
Height (m):	0.105
Depth (m):	0.290

Muffle inner dimensions

Width (m):	0,175
Height (m):	0.095
Depth (m):	0.280

Thickness of the insulation lining of the side walls, ceiling and furnace hearth (m): 0.060
Thickness of the insulation lining of the door and back wall (m): 0.040

Sheathed thermocouple

Diameter (m): 0.002
The length sticking out into the furnace (m): 0.050

X. DETERMINATION OF THE α, ε, φ VALUES

Coefficients of convective heat passage $\alpha_{k,i}$

Inside the furnace, as well as on its outer surface, there is no forced gas flow, so in both cases we can count with natural convection.

To determine the coefficients of convection passage the criterion equations of similarity analysis are used.

Considering the fact that together with the growing temperature the convection ratio against the radiation is rapidly falling down, any detailed calculation will not be presented for the convection case and similar cases values, verified by practice, will be used.

The natural convection on the furnace outer surface may be calculated as follows: $\alpha_k = 4$ (W/m^{2}K). On the furnace inner surface: $\alpha_k = 8$ (W/m^{2}K).

Relative emissivity $\varepsilon_{i,j}$

Within a closed set of two isothermal areas of the relative emissivity ε_1 and ε_2 is valid:

$$\varepsilon_{1,2} = 1/\left(1/\varepsilon_1 + 1/\varepsilon_2 - 1\right) \tag{37}$$

Angular coefficients of the exposure to radiation $\varphi_{i,j}$

For a closed set of two isothermal areas F_1 and F_2 it is valid:

$$\varphi_{1,2} = 1 \tag{38}$$

$$\varphi_{1,2} = \frac{F_1}{F_2} \tag{39}$$
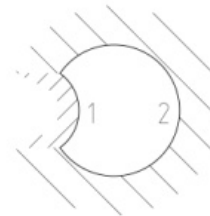
$$F_{1,2} = F_1 \tag{40}$$

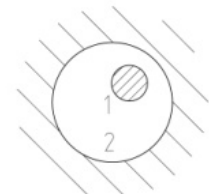Fig. 10.a A closed set of curved concave and non-concave surfaces.

Fig. 10.b A body of non-concave surface is inside a body of a concave surface.

These relations will be used to determine

$$\varphi_{3,8} = S_8/S_3 \ , \ S_{3,8} = S_8 \tag{41}$$

Temperature sensor is from all its sides surrounded with radiating surfaces S_3 and S_8.

For angular coefficients $\varphi_{3,14}$ and $\varphi_{8,14}$:

$$\varphi_{3,14} + \varphi_{8,14} = 1 \tag{42}$$

Using the mathematical models of fireproof clay and Sibral furnaces the simulations imitating the measured heats, heat persistence and cooling of real furnaces were performed.

XI. GRAPHICAL REPRESENTATION OF SIMULATION RESULTS

The dashed line is used for the temperature courses measured at the furnace, and the dotted line is used to mark the corresponding course which was calculated.
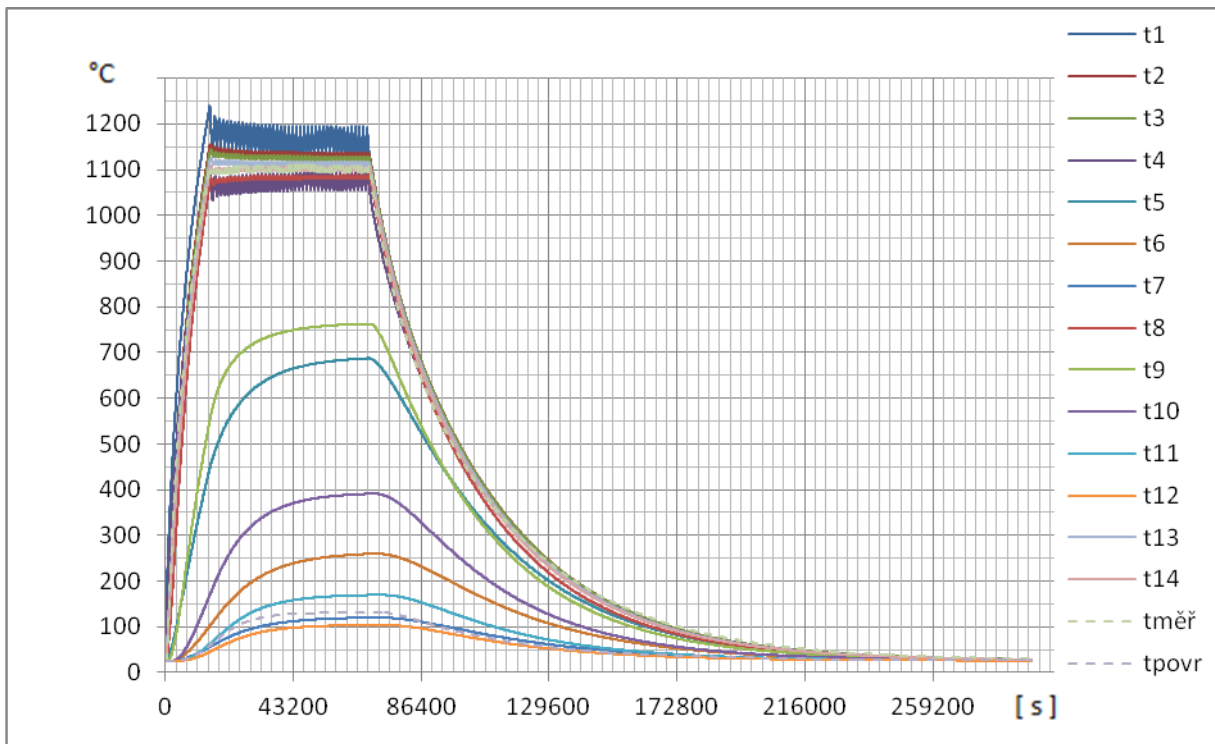
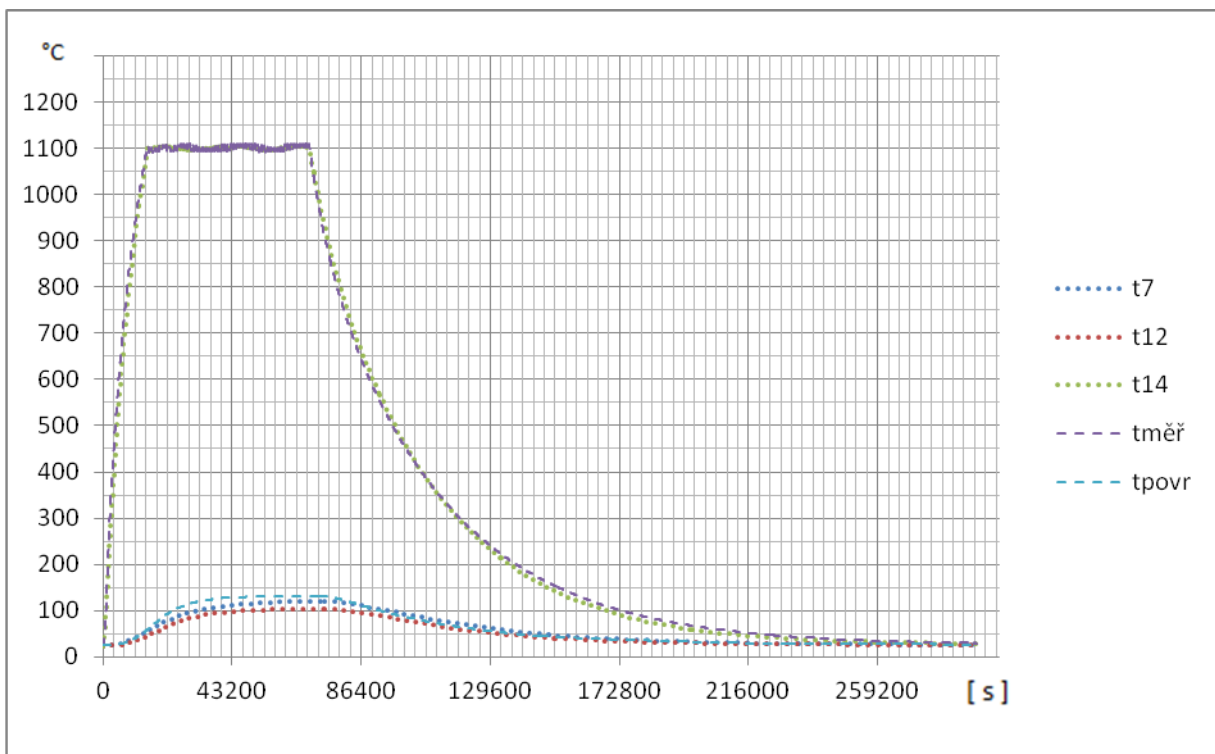Fig. 11. MODEL I – Temperatures – Tap voltage 220 V.

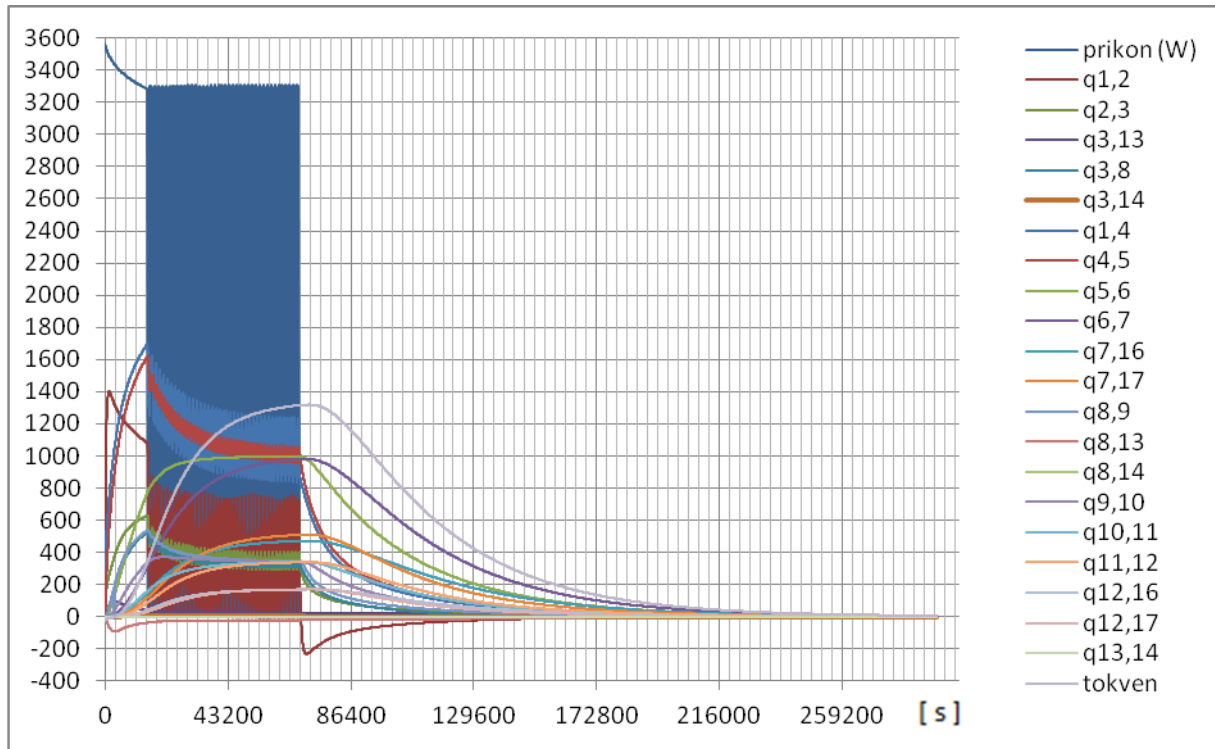Fig. 12. MODEL I – Selected temperatures – Tap voltage 220 V.

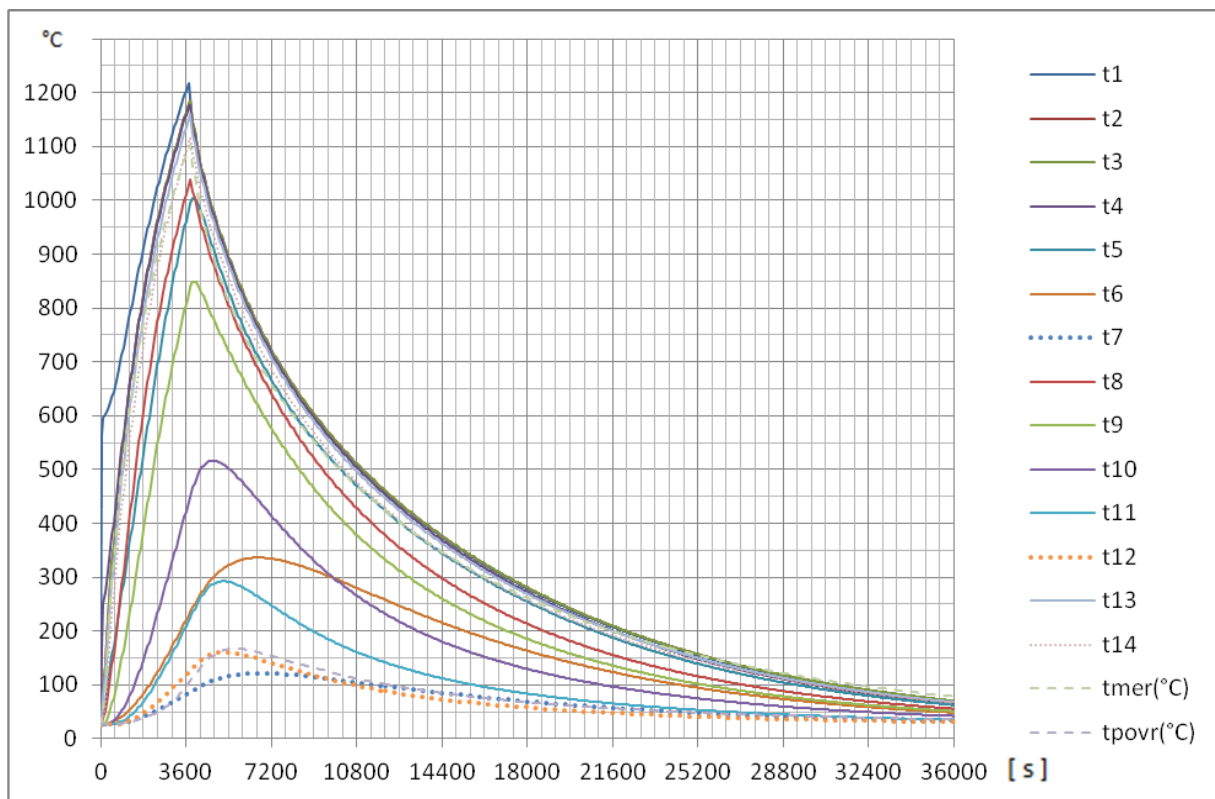Fig. 13. MODEL I – Heat flows – Tap voltage 220 V.

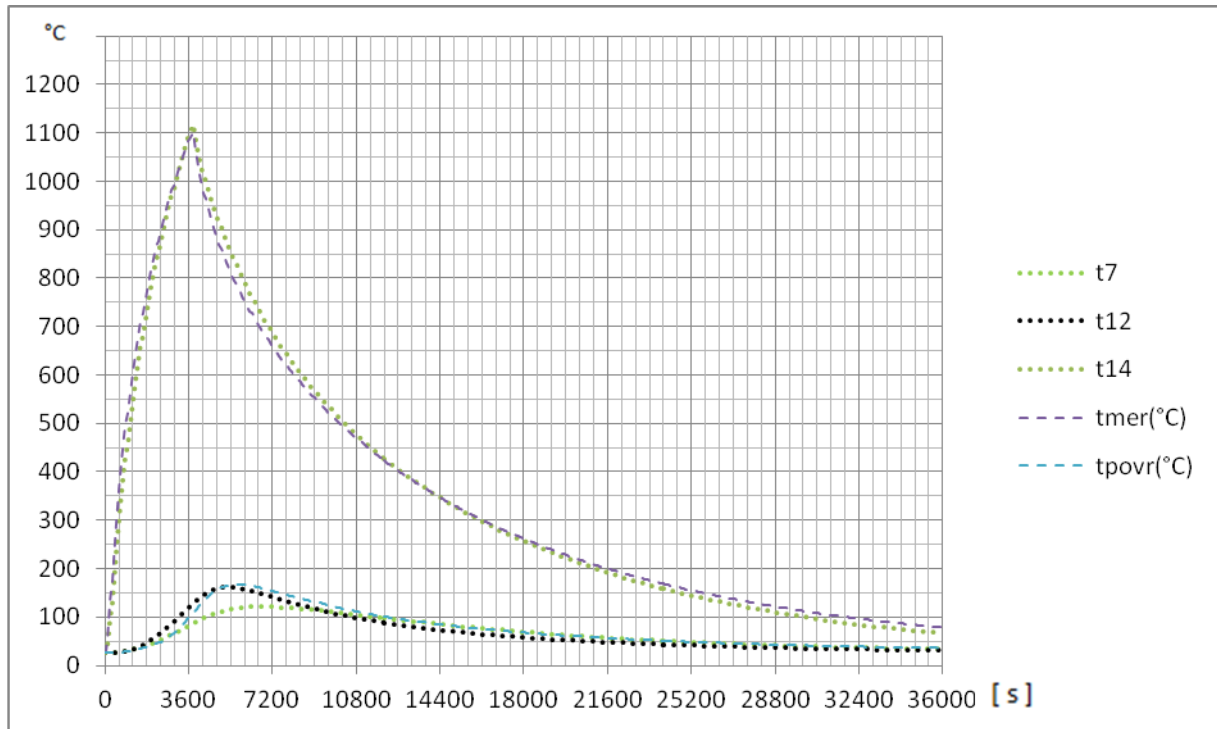Fig. 14. MODEL II – Temperatures – Tap voltage 220 V.

Fig. 15. MODEL II – Selected temperatures – Tap voltage 220 V.
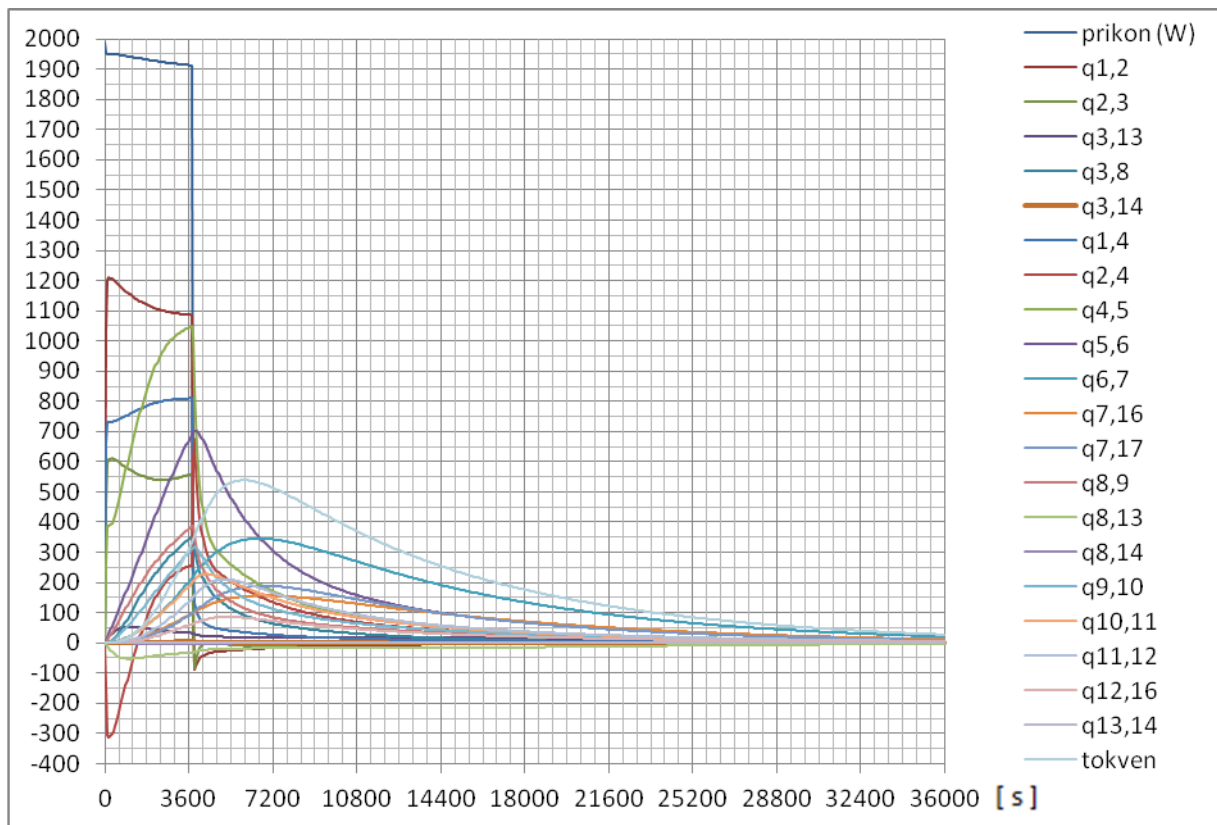

Fig. 16. MODEL II – Heat flows – Tap voltage 220 V.

XII. CONCLUSION

Comparing the courses of temperatures measured in real furnaces with the courses of the calculated simulations using the mathematical models shows the results of very good matching.

Thus, it can be concluded that the developed mathematical model of a heating and cooling within the electric muffle furnace imitates at a good level the processes which are being performed in real furnaces.

Generally, it is valid that simulations in computer models of dynamic systems are typical for a number of advantages against the experiments performed on real equipment and devices. Using them we are allowed to test the behaviour of such equipment that are being designed and have not been existed yet. Also, in this way, we have the possibility to simulate the states or processes that would be in case of real equipment understood as risky ones or high-priced.

However, the mathematical models bear also significant risks, their behaviour differs significantly from the real situation, if, in particular, in case of their development were used improper simplifications or if wrong ideas of their developers were applied.

REFERENCES

[1] J. Lucák, "A Mathematical Model of Thermal Processes in the Empty Electric Muffle Furnace," *Transactions on Electrical Engineering*, vol. 5, pp. 14-20, March 2016. http://dx.doi.org/10.14311/TEE.2016.1.014