3D Numerical Calculation of Electric Field Intensity under Overhead Power Line Using Catenary Shape of Conductors

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Abstract — **This article presents a superposition method in combination with the Coulomb's law and the Method of image charges for calculation of the electric field distribution generated by high voltage overhead power lines above a flat surface in every dimension. Such calculations are required to ensure the operational safety of people exposed to the action of external electric field as well as to reduce the cost of people protection. The method provides options for calculation of the field around the wire of a general shape. This substantial improvement of the method could be applied to eliminate the usual error in the calculation created using approximation of catenary shape conductors by infinite straight conductors. The method has been extensively tested on a set of shapes with known analytical solutions. It has been shown that the numerical solution converges uniformly to the analytical solution and the accuracy depends only on the number of finite elements.**

Keywords — electric field, FEM, power transmission line, charge

I. Introduction

The overhead transmission lines are source of magnetic as well as electric field. This electromagnetic field (EMF) is of low frequency and it is time-varying [1]. It is believed that the influence of the EMF field is harmful only in certain way to the human health so the effects of the field have to be taken in consideration in the power transmission line project design [2]. An increasing demand on operational safety in the vicinity of high voltage lines calls for a more precise project preparations. The project preparation mainly involves numerical calculations, the result of which can facilitate and cheapen particular project activities. An exposure to EMF field causes flow of induced currents in living organisms, and can have other unpleasant effects on human body [3]. In many cases this considerations has not been proven, but studies show a potential risk. According to this health risks non-governmental organization International Commission on Nonionizing Radiation Protection (ICNIRP) established for population reference levels for the exposure to time-varying electric and magnetic fields shown in Table 1 [2]. These reference levels can vary according to the international standards. The reference levels for the Slovak and Czech Republic are shown in Table 1. [4], [5]. Several national and international organizations have formulated guidelines establishing limits for the occupational and residential EMF exposure. The exposure limits for EMF fields developed by ICNIRP – formally recognized by World Health Organization (WHO), were developed following reviews of scientific literature, including thermal and non-thermal effects. The standards are based on evaluations of biological effects that have been established to have health consequences [3] [6].

TABLE I.
Legislative Valid Reference Levels in Slovak and Czech Republic for Exposure to Time-Varying Electric and Magnetic Fields (Unperturbed RMS Values)

Frequency	Electric field E_{rms} [kV.m^{-1}]	Magnetic flux density B_{rms} [μT]
50 Hz	5	100

The main conclusion from the WHO reviews is that the electromagnetic field exposures below the limits recommended in the ICNIRP international guidelines do not appear to have any known consequence on health. In the past years the European Commission gave a new recommendation, based on the ICNIRP study, to establish that all European Union (EU) states observe standard reference levels of the exposure to the field. Although reference levels vary in different countries they cannot be lower than the EU standards. The best way to deal with these standards is to have a truth worthy method for the calculation of the electric field. From Maxwell`s equations with a combination of the method of image charges dependence of electric field from the position of the conductor, the observer and its mirror image can be derived. This complex approach gives in combination with finite element method tool for calculation of the electric field intensity.

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TABLE II.
REFERENCE LEVELS FOR EXPOSURE TO TIME-VARYING ELECTRIC AND MAGNETIC FIELDS (UNPERTURBED RMS VALUES) [2]

Frequency range	Electric field E [kV.m^{-1}]	Magnetic field H [A.m^{-1}]	Magnetic flux density B [T]
1 Hz – 8 Hz	5	$3.2\times10^2/f^2$	$4\times10^{-2}/f^2$
8 Hz – 25 Hz	5	$4\times10^3/f$	$5\times10^{-3}/f$
25 Hz – 50 Hz	5	1.6×10^2	2×10^{-4}
50 Hz – 400 Hz	$2.5\times10^2/f$	1.6×10^2	2×10^{-4}
400 Hz – 3 kHz	$2.5\times10^2/f$	$6.4\times10^4/f$	$8\times10^{-2}/f$

A common simplification in the evaluation is to calculate the electric field intensity with the real conductor approximation by the infinite straight conductor placed in the lowest height of the sag. However this condition does not reflect most of real situations. This simplification can be applied only on symmetric spans.

II. TERMINOLOGY

A. Quasi-static Field

The time varying EMF is called quasi-static if we can neglect time changes of the EMF spread by finite speed. For a harmonic EMF field, which spread in the air and which variables vary with angular frequency ω, the quasi-static criterion can be expressed by Eq. 1, where σ is enviromental conductivity, f is frequency [7].

$$f \ll \frac{2\pi\varepsilon_0}{\sigma} \tag{1}$$

For air σ gets values from 3,10e-15 to 8,10e-15 [S/m]. The criterion is valid for both sides of the interval.

$$50 \ll 18544,16 \qquad 50 \ll 6954,06 \tag{2}$$

For the quasi-static field applies, that we can neglect time derivations in I. and II. Maxwell's equation [7].

$$\frac{\partial \vec{E}}{\partial t} \approx 0 \qquad \frac{\partial \vec{B}}{\partial t} \approx 0 \tag{3}$$

The equations simplify to the form:

$$\begin{aligned} div\,\vec{E} &= \frac{\rho}{\varepsilon_0} & rot\,\vec{E} &= 0 \\ div\,\vec{B} &= 0 & rot\,\vec{B} &= \mu_0\vec{J} \end{aligned} \tag{4}$$

B. Phasors and Vectors

The EMF field near transmission lines are described in this article using phasors and vectors. A vector is characterized by a magnitude and angle in space or by three spacial components, Eq. 5 [1].

$$\vec{E} = [E_x; E_y; E_z] \tag{5}$$

A phasor on the other hand is a quantity with a sinusoidal time variation described by a magnitude and a phase angle (Eq. 6). The angle φ describes a phase shift [1].

$$E_x(t) = E_x cos(\omega t + \phi_x) = Re\{E_x e^{j(j\omega t + \phi_x)}\} = E_x\angle\phi_x \tag{6}$$

Three orthogonal components of a vector may be phasors with different magnitude and phase angles. These components are called phasor-vector (Eq. 7). In this article, a vector is indicated with an arrow and phasor-vectors with a hat over the arrow [8].

$$\hat{\vec{E}} = [\hat{E}_x; \hat{E}_y; \hat{E}_z] = [E_x\angle\phi_x; E_y\angle\phi_y; E_z\angle\phi_z] \tag{7}$$

In many cases the single RMS value is necessary to evaluate the field. This value is calculated from magnitudes values of the phase-vector components as follows:

$$E_{RMS} = \sqrt{E_x{}^2 + E_y{}^2 + E_z{}^2}\,\frac{1}{\sqrt{2}} \tag{8}$$

C. Catenary Shape of Conductors

Conductor attached on two sides, in this case on transmission towers, will form curve in shape of a catenary.[1] The catenary can be considered as symmetric, if conductors are at their ends in the same height above a flat surface, or asymmetric if not. Figure1 shows the general asymmetric span, where A is the length of the span. V_1 and V_2 are the heights at each end of the catenary above the flat ground. A_1 is the distance from the beginning of the catenary to the middle of the span.

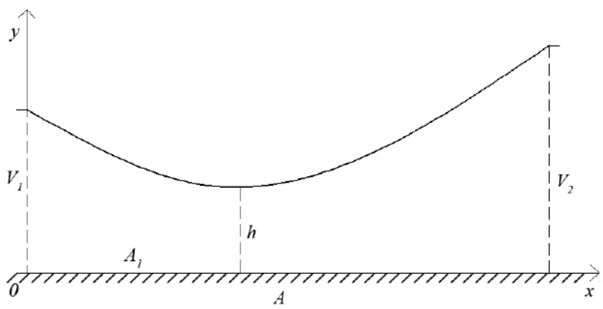

Fig. 1. Asymmetric span.

If A_1 is exactly half of the A then the catenary is symmetric. A_1 is in general calculated as follows:

$$A_1 = A - \left(asinh\left(\frac{V_2 - V_1}{2c.sinh\frac{A}{2c}} \right) + \frac{A}{2c} \right) c \tag{9}$$

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The constant c is the parameter defining the shape of the catenary. The parameter h is the height at the lowest point of the sag and it is calculated as follows:

$$h = V_1 - c\left(\cosh\frac{A_1}{c}\right) + c \qquad (10)$$

The height of the catenary y at the place x is calculated as follows:

$$y = c\left(\cosh\frac{x - A_1}{c}\right) - c + h \qquad (11)$$

III. Methodology

From the III. Maxwell's equation we can derive the Coulomb's law, in the integral form Eq. 12.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_l \frac{\tau\, dl}{r^3}\vec{r} \qquad (12)$$

Let the point of an observer P be the point where we want to calculate the electric field intensity. The vector $\vec{r}$ starts at the conductor element dl and points to the observer P. The vector $\vec{r}$ can be written also as:

$$\vec{r} = (\vec{r}_p - \vec{r}_0) \qquad (13)$$

where $\vec{r_p}$ vector points from the coordinate system origin to the observer P and $\vec{r_0}$ points from the coordinate system origin to the conductor element dl, Fig.2. The Coulomb's law equation in the analytical form is after a substitution as follows:

$$\vec{E} = \int_l d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_l \frac{\tau\, dl}{|(\vec{r}_p - \vec{r}_0)|^3}(\vec{r}_p - \vec{r}_0) \qquad (14)$$

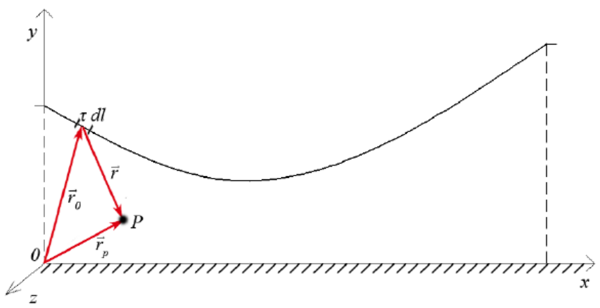

Fig. 2. Positions of vectors $\vec{r}$, $\vec{r_0}$, and $\vec{r_p}$ in relation to the observer P and catenary.

A. Method of Mirror Images

It is not possible to find out $\vec{E}$ in a dielectric environment, where conductors hang above a conductive plate, just by Eq. 14. The electric field in this model is created not only by charges in conductors but also by charges created by electrostatic induction in the conductive plate (terrain) with potential φ = 0. The charges distribution and density in this plate is uneven. Solving this problem is done by the method of image charges. The method creates a new mirror conductor axially symmetrical according the boundary plane to each real one. The mirror conductor has the opposite charge as the real one, Fig. 3.

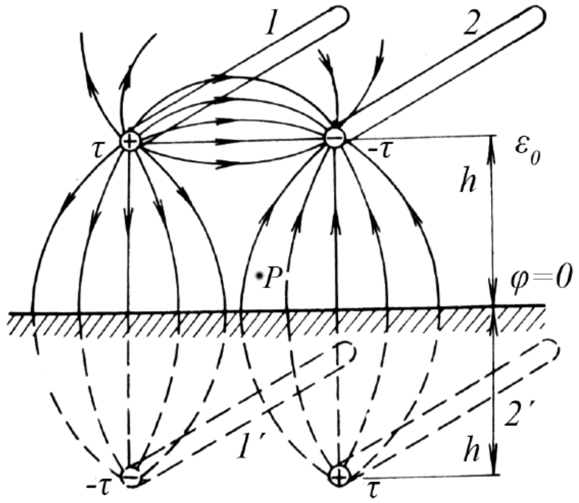

Fig. 3. Example of mirror conductors above the conductive plate with the potential φ = 0 [7].

In this new model the charge distribution in the conductors remained unchanged and the potential on the boundary plate is also zero as in the basic model. The solution of the new model will have the same solution as the initial boundary value problem. The electric field intensity $\vec{E}$ in point of the observer P, Fig. 3 equals the superposition of electric intensities created by each conductor, real and mirror one, Eq. 15.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_{1'} + \vec{E}_{2'} \qquad (15)$$

B. Change from Derivations to Differences

The catenary shape of the conductor makes the analytical calculation of Eqs. 14 and 15 too complicated. To overcome this issue a numerical method based on the superposition of finite elements has to be used. The core of this method is to change the derivations to the differences to determine the length of the conductor element Δl.

$$d\vec{E} \approx \Delta\vec{E} \qquad dl \approx \Delta l \qquad (16)$$

It is important to realize that the linear charge density changes with every one element due to a different distance from the ground. The vectors $\overrightarrow{r_{0n}}$ and $\overrightarrow{r_{0'n}}$ will determine the conductor element position and its image according to the coordinate system beginning. We can determine values of the vectors in X and Y directions by Eq. 11, value in the Z direction equals to the overhang of the conductor on a transmission tower. The electric field intensity $\vec{E}$ can be now determined for one conductor over a flat surface as follows:

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$$\vec{E} = \sum_{n=1}^{l} [\Delta \vec{E_1}_n + \Delta \vec{E_1}`_n] = \frac{1}{4\pi\varepsilon_0} \sum_{n=1}^{l} \left[ \frac{\tau_n \Delta l}{|(\vec{r}_p - \vec{r}_{0_n})|^3} (\vec{r}_p - \vec{r}_{0_n}) + \frac{-\tau_n \Delta l}{|(\vec{r}_p - \vec{r}_{0`_n})|^3} (\vec{r}_p - \vec{r}_{0`_n}) \right] \quad (17)$$

A conductor of any given shape can be by this method "chopped" to finite elements, Fig. 4.

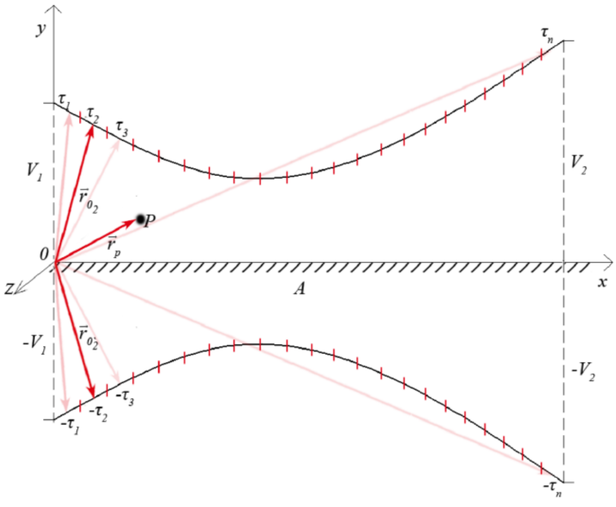

Fig. 4. Visualization of calculation of electric field intensity by the Finite element method and method of mirror images.

C. Calculation of Linear Charge Density

It is necessary to evaluate the linear charge density τ_n for each conductor element Δl. This is possible from known voltages on each conductor and from geometry of each catenary element Δl. In general for an infinite straight conductor linear charge density can by calculated by Eq. 18, where $[\tau_n]$ is one 1D matrix of the linear charges densities at k conductors, $[P_{kk}]$ is 2D matrix of Maxwell's potential coefficients, by the unit [F/m], finally $[U_k]$ is 1D matrix of voltages at k conductors.

$$[\tau_k] = [P_{kk}]^{-1} [U_k] \quad (18)$$

When calculating the field created by the conductor catenary shape, the matrix $[P]$ will be different for every conductor element Δl. For the n[th] element from the system of k conductors we can write:

$$[\tau_{kn}] = \left[[P_{kk}]_n^{-1} \right] [U_k] \quad (19)$$

The result is 2D matrix $[\tau_{kn}]$ consisting linear charge densities on conductor k for every nth element of the catenary. The components of matrix $[P_{kj}]$ for every element can be determined by following Eqs. 20 and 21. The matrix $[P_{kj}]$ is symmetric and for components not in diagonal equals $P_{kj} = P_{jk}$.

Distances between conductors are shown in simple example in Fig. 5 where P_{kk} is the self-potential coefficient of the conductor k, P_{kj} is the mutual potential coefficient of the conductors k and j, h_k is height of the conductor k element above ground, R_k is radius of the conductor k, D_{kj} is distance between the elements of the conductors k and j in the same distance X from of the coordinate system beginning, D'_{kj} is the distance between the element of the conductor k and mirror element of the conductor j in the same distance X from the coordinate system beginning.

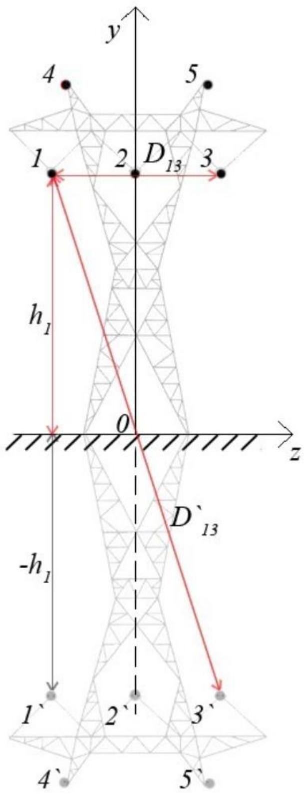

Fig. 5. Conductor positions and their mutual distances for calculation of coefficients P_{kj}.

$$P_{kk} = \frac{1}{2\pi\varepsilon_0} \ln\left(\frac{2h_k}{R_k}\right) \quad (20)$$

$$P_{kj} = \frac{1}{2\pi\varepsilon_0} \ln\left(\frac{D`_{kj}}{D_{kj}}\right) \quad (21)$$

D. Generalization

So far we have analyzed the calculation of $\vec{E}$ only as a stationary field formed by a constant voltage U. In the calculation of harmonically oscillating field where the intensity $\hat{\vec{E}}$ is the phasor-vector unit we shall use as input phasor of effective phase voltage of each conductor $\widehat{\overrightarrow{U_k}}$. The linear charge density will also harmoniously change so Eq. 22 can be rewritten as follows:

$$[\hat{\tau_{kn}}] = \left[[P_{kk}]_n^{-1} \right] [\hat{U}_k] \quad (22)$$

The final equation for the electric field intensity under a power transmission line consisting of k conductors in shape of catenary at point of the observer P with use of FEM method is as follows:

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$$\hat{\vec{E}} = \sum_{k=1}^{k}\left[\sum_{n=1}^{l}[\Delta\hat{\vec{E}}_{k_n} + \Delta\hat{\vec{E}}_{k`_n}]\right] = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{k}\left[\sum_{n=1}^{l}\left[\frac{\hat{\tau_{k_n}}\,\Delta l}{|(\vec{r}_p - \vec{r}_{0_{k_n}})|^3}(\vec{r}_p - \vec{r}_{0_{k_n}}) + \frac{-\hat{\tau_{k_n}}\,\Delta l}{|(\vec{r}_p - \vec{r}_{0`_{k_n}})|^3}(\vec{r}_p - \vec{r}_{0`_{k_n}})\right]\right] \tag{23}$$

IV. Analytical Verification

The numerical method has been tested on a set of conductor shapes which analytical solutions are known [9]. It was shown that the numerical solution converges uniformly to the analytical solution and the accuracy depends only on the number of finite elements, Fig. 4. It is important to note that the conductor segment vector Δl is in each case computed differently according to the conductor shape. In both cases the τ = 1.10e-4 C/m and the analytical results are RMS values [10].

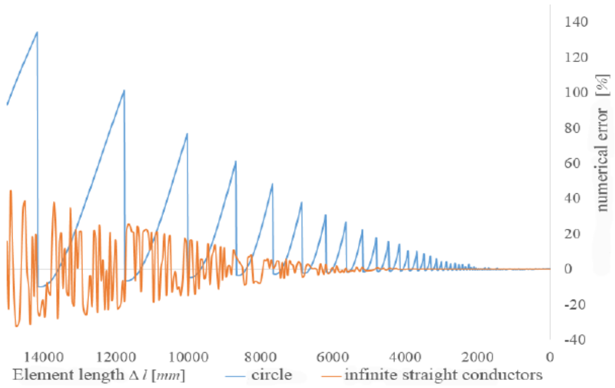

Fig. 6. Dependence of the numerical error from the length of the element for given conductor shapes.

Circle loop conductor: Conductor in shape of a circle loop with the radius R = 100 m, Eq. 24. Point of the observer is in distance 10 m from the circle at the central axis, Fig. 7.

$$E_x = 0,\quad E_y = 0$$
$$E_z = \frac{\tau R}{2\varepsilon_0}\frac{r_{p_z}}{\sqrt{(R^2 + r_{p_z}^2)^3}} \tag{24}$$

Two infinite straight conductors: The distance between conductors is a = 10 m, Eq. 25, and the point of the observer is placed at r_{px} = 5 m, r_{pz} = 0 m from the coordinate system beginning, Fig. 8.

$$E_x = \frac{\tau}{\pi\varepsilon_0}\frac{a({r_{p_x}}^2 - a^2) - {r_{p_y}}^2}{|\vec{r}_1|^2|\vec{r}_2|^2}$$
$$E_y = \frac{\tau}{\pi\varepsilon_0}\frac{2ar_{p_x}r_{p_y}}{|\vec{r}_1|^2|\vec{r}_2|^2} \tag{25}$$
$$E_z = 0$$

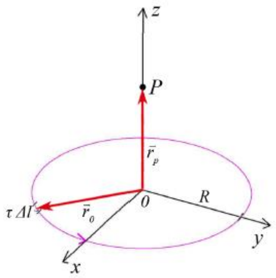

Fig. 7. Circle loop conductor.

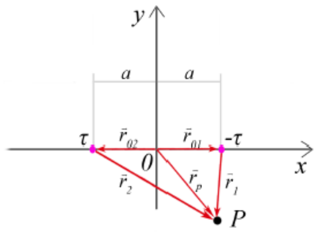

Fig. 8. Two infinite straight conductors.

V. Field under Power Line

In this farther example of results it is shown the calculation of the electric field intensity under the power line of the type 2×400 kV DONAU with two ground wires, Fig. 9. The distance between towers is 350 m. The parameters of towers for calculation are in Tab. 3. The lowest conductor is set to be 11.5 m above the ground. This minimum distance is no longer determined by the standard, which is 8 m above the ground, but according to the value of the field. It can vary approximately from 10 m to 12 m according to the mutual phase location, terrain curvature and many more factors. The current in each phase bundle is 2400 A and the phase voltage is set on 420 kV.

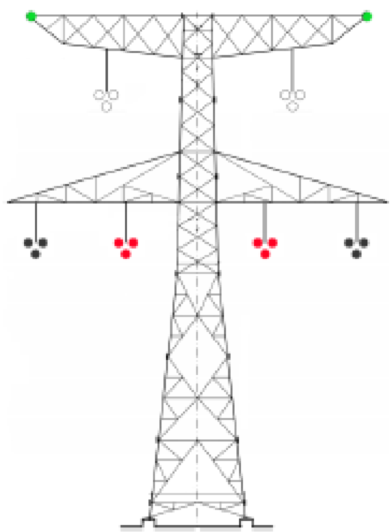

Fig. 9. Transmission tower type 2×400$ kV DONAU with two ground wires (green). Figure shows positions of phases L1 (white), L2 (black) and L3 (red).

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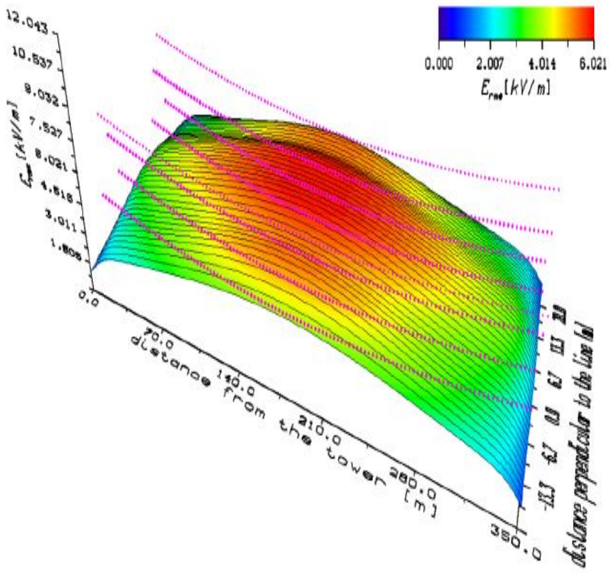

Fig. 10. E_{RMS} shown in 3D graph in horizontal plane in a constant height above ground, 1.8 m . The axis Z and colours display the field value.

TABLE III.
HANGING POINTS OF THE CONDUCTORS ON TOWER

Configuration		Position of poles	
		Height [m]	Center displacement [m]
System 1	Phase 1	29.8	-9.6
	Phase 2	18.6	-13.6
	Phase 3	18.6	-7.3
System 2	Phase 1	29.8	9.6
	Phase 2	18.6	13.6
	Phase 3	18.6	7.3
Ground wire 1		37.1	-12.1
Ground wire 2		37.1	12.1

Figures 11 and 10 show results that are calculated using the EMFTsim ultimate software (all graphical results are made by Dislin graphical library) which was built for this specific type of calculations by the authors of this article.

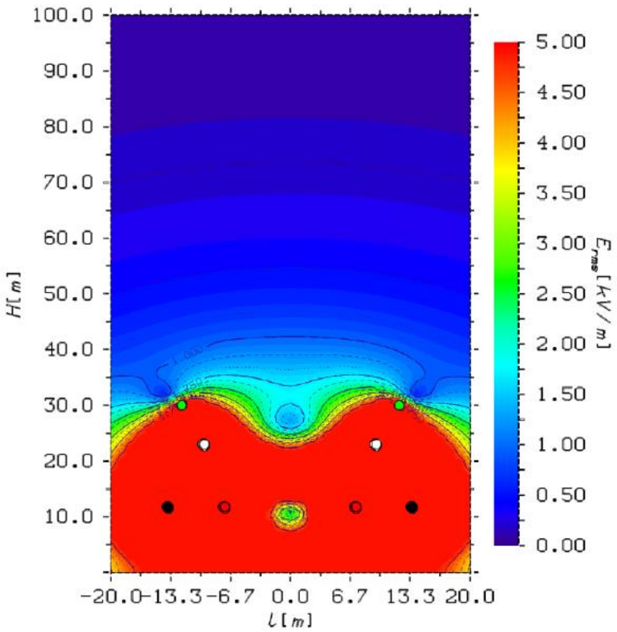

Fig. 11. E_{RMS} in a vertical plane crossing the transmission line. Calculation for the lowest distance of the conductors to the ground E_{RMS} is indicated by colour from dark to red.

VI. SHIELDING EFFECT OF THE GROUND WIRES

In past years it have been discussions in the power line community about the posibility of shielding the electromagnetic field by ground wires placed beneath the phase conductors. As it is shown in Figs. 12. and 13., the transmission tower Portál 400 kV with minimum distance of the phase conductors above the ground 12 m , such effect can be achieved, Fig. 13.

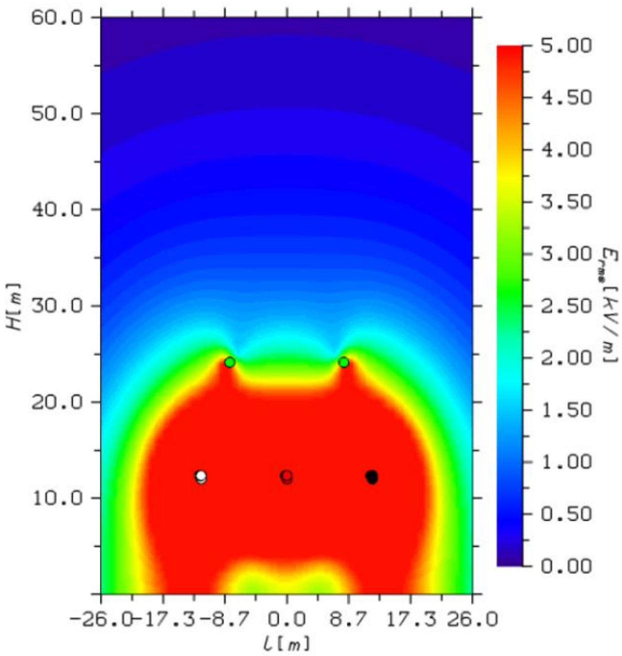

Fig. 12. E_{RMS} in a vertical plane crossing the transmission line of the type Portál 400 kV. Calculation for the lowest distance of the conductors to the ground. E_{RMS} is indicated by colour from dark to red.

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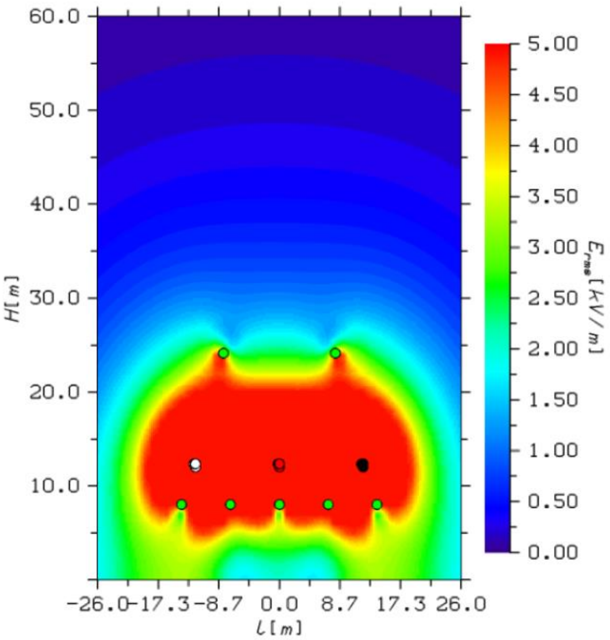

Fig. 13. E_{RMS} in a vertical plane crossing the transmission line of the type Portál 400 kV . Calculation for the lowest distance of the conductors to the ground. E_{RMS} is indicated by colour from dark to red.

As it is shown, the electric field can be dramaticly reduced, however shielding in reality will be highly unpractical, due to additional costs for development of new transmission towers. Also to maintain addition seperation distances between shielding wires and phase conductors and ground will in reality result into a transmission tower raising.

VII. Conclusion

Enumeration of the electromagnetic field generated by a power transmission line is one of crucial conditions for completing total project of the line. This article explained a numerical method for calculation of the electric field intensity in a complex way. The described method also tries to eliminate most of common approximations. As the paper shows, this method can be fully applied for any conductor shape. The results shown in this paper were verified not only analytically, but also using calculations of similar computation softwares. The value of electric field intensity is effected also by ground and all subjects including human itself. This means more study is needed to be done for ensuring operational safety of the transmission line but also for reducing the project costs. For a complex computation of the electromagnetic filed a simulation software EMFTsim Ultimate was developed. All graphical result in this paper are made by this software.

Acknowledgement

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References

[1] EPRI, “AC transmission line reference book – 200 kV and above.”

[2] WHO, “Environmental Health Criteria 238 – Extremely Low Frequency Fields.”

[3] D. M. Repacholi and E. Vandeventer, “WHO Framework for Developing EMF Standards,” pp. 1–13, October 2003.

[4] “Vyhláška Ministerstva zdravotníctva Slovenskej republiky o podrobnostiach o požiadavkách na zdroje elektromagnetického žiarenia a na limity expozície obyvateľov elektromagnetickému žiareniu v životnom prostredí,” Zbierka zákonov č. 534/2007, pp. 3812–3816, 2007.

[5] Sbírka zákonů České republiky, ročník 2008. “Nařízení vlády o ochraně zdraví před neionizujícím zářením,” Částka 1, pp. 2–29, 2008.

[6] ICNIRP, “Guidelines for limiting exposure to time-varying electric and magnetic fields (1 Hz to 100 kHz).” *Health physics*, vol. 99, no.6, pp. 818–36, Dec. 2010.

[7] D. Mayer and J. Polák, *Metody řešení elektrických a magnetických polí*, 1983.

[8] EPRI, “AC Transmission Line Reference Book – 345 kV and above,” Chapter 7, pp. 329–417, 1982.

[9] D. Mayer, *Aplikovaný elektromagnetizmus: úvod do makro-skopické teorie elektromagnetického pole pro elektrotechnické inženýry*, 2012.

[10] J. Bendík, M. Cenký, and Ž. Eleshová, “Complex calculation of intensity of electric field under power transmission line using catenary shape of conductors and flat surface”, in *Power engineering 2016: Energy-Ecology-Economy 2016: 13th International scientific conference. Tatranske Matliare, Slovakia*. May 31 – June 2, 2016. 1. vyd. Bratislava: Slovak University of Technology, 2016, s. 139–144. ISBN 978-80-89402-85-4

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