Mathematical Model of Induction Motor with Ferromagnetic Rotor

prof. N. Zablodskiy [1], doc. V. Pliugin [2], p.g. V. Skryl [3], prof. J. Lettl [4]

[1] Donbas State Technical University/Automation of Electrical Engineering Systems, Alchevsk,Ukraine, *info@dmmi.edu.ua*

[2] Donbas State Technical University/Automation of Electrical Engineering Systems, Alchevsk, Ukraine, *vlad.plyugin@gmail.com*

[3] Donbas State Technical University/Automation of Electrical Engineering Systems, Alchevsk, Ukraine, *vvskryl@yandex.ru*

[4] Czech Technical University in Prague/Faculty of Electrical Engineering, Prague, Czech republic, *lettl@fel.cvut.cz*

Annotation — **Results of theoretical and experimental research as well as outcome of simulation by means of the established mathematical model of induction motor with ferromagnetic rotor in static and transient states are presented and analyzed.**

Keywords — induction motor, hollow ferromagnetic rotor, mathematical model, transient, simulation

I. Introduction

Induction motors with ferromagnetic rotors continue to be used among the traditional design machines. Their multifunctional complex use for transportation, fusion, and drying of friable materials and also for fusion and heating of fusible substances [1, 2] should be emphasised.

In the view of development of theoretical basis for Electro-Mechanical Transformers of Energy (EMTE), it arises the problem of acceptance of a mathematical model of induction motor with a ferromagnetic rotor putted together in accordance with such machine speciation features [3]. These features are especially:

a) absence of a wire winding;

b) presence of ferromagnetic massive rotor with magnetic permeability dependence on the electromagnetic field penetration depth to the rotor body (finally, also the dependence of rotor parameters on rotational speed).

In the article [4] the model of an induction motor with a ferromagnetic rotor was presented, however insertion of auxiliary coefficients to the model does not reflect physical sense of changes made in a rotor circuit. Thus the obtained model became too difficult and uncomfortable to be employed for simulation.

In this work establishment of mathematical model of EMTE with a ferromagnetic rotor in accordance with the theory of the generalized electric machine [5] is examined.

By solving the problem, the physically logical input elements of the model calculation matrix are most exposed, linked up with the evolution base kind [3].

II. Nomenclature

x'_{20} – reactance of ferromagnetic rotor
x''_{20} – reactance of rotor winding
α – constant respecting properties of ferromagnetic materials
U_1, U_2 – stator resp. rotor voltage
E_0 – EMF (Electro-Motive Force)
I_1, I'_2 – stator resp. reduced rotor current
I_0 – current of the magnetic circuit
r_1 – resistance of stator winding
r'_2 – reduced resistance of rotor winding
s – slip
$\Psi_{s\alpha}$, $\Psi_{s\beta}$ – flux of the stator winding in the axes α, β
$\Psi_{r\alpha}$, $\Psi_{r\beta}$ – flux of the rotor winding in the axes α, β
$u_{s\alpha}$, $u_{s\beta}$ – instant voltages of the stator winding in the axes α, β
$i_{s\alpha}$, $i_{s\beta}$ – instant currents of the stator winding in the axes α, β
$i_{r\alpha}$, $i_{r\beta}$ – instant currents of the rotor winding in the axes α, β
$L_{r\sigma}$ – leakage inductance of the rotor ferromagnetic body
ω_0 – electromagnetic field synchronous rotation speed
ω_e – mechanical speed
M_e, M_c – active electromagnetic resp. load torque
J_r – rotor moment of inertia
L_1 – full stator winding inductance
L_2 – full rotor winding inductance
M – mutual inductance
p – number of pole pairs

III. EMTE Mathematical Model

The starting point of the generalized model forming will be the mathematical description of the induction motor for the static mode, graphically presented as an equivalent T-circuit (Fig. 1):

$$\begin{cases} \dot{U}_1 = -\dot{E}_0 + \dot{I}_1(r_1 + jx_1); \\ \dot{U}_2 = \dot{E}_0 - \dot{I}'_2(r'_2 + jx'_{20})\dfrac{1}{s^{\alpha}} - \dot{I}'_2\, jx''_{20}; \\ \dot{I}_0 = \dot{I}_1 + \dot{I}'_2. \end{cases} \quad (1)$$

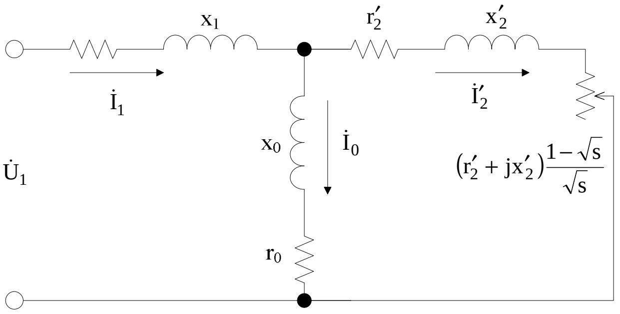

Fig. 1. Generalized induction machine equivalent T-circuit

Equations of a common induction motor (in further we shall thus call an induction motor with a squirrel cage rotor) get out from (1) if $\alpha = 1$ and $x'_{20} = 0$.

We get equations of the induction motor with a ferromagnetic rotor from (1) if α = var (usually $\alpha = 0.5$) and $x''_{20} = 0$.

We shall consider how to pass from the generalized model of the static mode (1) the system of differential equations of the transient mode.

Interest presents the second equation of the system (1) for the rotor circuit. All equation conversions for the stator circuit will be analogical to known procedure for the generalized EMTE [5]. Let write down the detailed expression for the rotor circuit (in the case of ferromagnetic or squirrel cage rotor $U_2 = 0$):

$$0 = \dot{E}_0 - \dot{I}'_2\frac{r'_2}{s^{\alpha}} - jI'_2\frac{x'_{20}}{s^{\alpha}} - jI'_2x''_{20}. \quad (2)$$

Multiplication of both equation (2) parts by s^{α} corresponds to the transition from a machine with the braked rotor to the machine with the revolving rotor:

$$0 = \dot{E}_0 s^{\alpha} - \dot{I}'_2 r'_2 - jI'_2x'_{20} - jI'_2x''_{20}s^{\alpha}. \quad (3)$$

Taking into account $\dot{E}_0 = -jx_m\dot{I}_m$ and $\dot{I}_m = \dot{I}_1 + \dot{I}'_2$

$$0 = -jx_m\dot{I}_1 s^{\alpha} - jx_m\dot{I}'_2 s^{\alpha} - r'_2\dot{I}'_2 - jx'_{20}\dot{I}'_2 - jx''_{20}\dot{I}'_2 s^{\alpha}. \quad (4)$$

By accepted notations $\dot{I}_s = \dot{I}_1$; $\dot{I}_r = \dot{I}'_2$; $r_r = r'_2$ and writing down the equation (4) for projection in the axis α of the two-phase coordinate system revolving with a rotor speed we get

$$0 = -jx_m\dot{I}_{s\alpha}s^{\alpha} - jx_m\dot{I}_{r\alpha}s^{\alpha} - r'_r\dot{I}_{r\alpha} - jx'_{20}\dot{I}_{r\alpha} - jx''_{20}\dot{I}_{r\alpha}s^{\alpha}. \quad (5)$$

We shall unite parameters with corresponding currents and multiply both parts of equation (5) by -1:

$$0 = \left(r'_r + jx_m s^{\alpha} + jx''_{20}s^{\alpha} + jx'_{20}\right)\dot{I}_{r\alpha} + jx_m\dot{I}_{s\alpha}s^{\alpha}. \quad (6)$$

The mathematical model of the generalized machine, being a base for all machines of Family “cylindrical” is written down in the general case as shown in (7).

We get an induction machine with a squirrel cage rotor from a base model (7) with the next specific mutations:

a) in the coordinate system α, β the rotation speed is $\omega_к = 0$;

b) sinusoidal voltages $u_{s\alpha}$, $u_{s\beta}$ shifted in time by 90^o with the frequency f_1 are connected on the stator windings;

c) for the squirrel cage type rotor, $u_{r\alpha} = u_{r\beta} = 0$;

d) because machine is cylindrical, then by virtue of symmetry the resistances and reactances of the stator and rotor of the same axes we accept equal: $r_{s\alpha} = r_{s\beta} = r_s$; $r_{r\alpha} = r_{r\beta} = r_r$.

Thus the system (7) simplifies to (8).

A model for an induction machine with a ferromagnetic rotor will be analogical to (8), but it should additionally take into account absence of wire winding and features of rotor realization:

1) a ferromagnetic rotor does not have the obviously expressed winding and reactance of the wire windings x''_{20}, expected for machines with squirrel cage or phase rotors are therefore absent;

2) resistances and reactances of the ferromagnetic rotor body are in general case functions of slip s with the power α.

Transform the expression (6) so that to bring it over in accordance with the type of the second line of the system (8).

In Tab. 1 (taking into account $I_{s\beta} = jI_{s\alpha}$) it is evidently shown how to make transformation of the equation written in relation to the axis α.

TABLE I.
TRANSFORMATION OF EXPRESSIONS IN THE MODEL MATRIX

Current	Own parameters, from (6)	Coercion parameters		Resulting parameter
$I_{r\alpha}$	$r_r + jx'_{20}$	$- x'_{20}\cdot s^{\alpha}$	$+jx_m + jx'_{20} + jx''_{20}$	$r_r + [jx_m + jx'_{20} + jx''_{20}] + (1 - s^{\alpha})jx'_{20}$
$I_{r\beta}$	$x_m\cdot s^{\alpha} + x''_{20}\cdot s^{\alpha}$	$+ x'_{20}\cdot s^{\alpha}$	$- x_m - x'_{20} - x''_{20}$	$(s^{\alpha} - 1)(x_m + x'_{20} + x''_{20})$
$I_{s\alpha}$		$+jx_m$		jx_m
$I_{s\beta}$	$x_m\cdot s^{\alpha}$	$- x_m$		$(s^{\alpha} - 1)x_m$

Note: in the column of own parameters in the axis β, at currents with rotation EMF in the system (8), passed parameters at the slip s^{α}.

$$\begin{bmatrix} u_{su} \\ u_{ru} \\ u_{rv} \\ u_{sv} \end{bmatrix} = \begin{bmatrix} r_{su} + \frac{d}{dt}L_{su} & \frac{d}{dt}M & M\omega_к & L_{sv}\omega_к \\ \frac{d}{dt}M & r_{ru} + \frac{d}{dt}L_{ru} & L_{rv}(\omega_к - \omega_r) & M(\omega_к - \omega_r) \\ -M(\omega_к - \omega_r) & -L_{ru}(\omega_к - \omega_r) & r_{ru} + \frac{d}{dt}L_{ru} & \frac{d}{dt}M \\ -L_{su}\omega_к & -M\omega_к & \frac{d}{dt}M & r_{su} + \frac{d}{dt}L_{su} \end{bmatrix} \times \begin{bmatrix} i_{su} \\ i_{ru} \\ i_{rv} \\ i_{sv} \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} u_{s\alpha} \\ 0 \\ 0 \\ u_{s\beta} \end{bmatrix} = \begin{bmatrix} r_s + \frac{d}{dt}L_s & \frac{d}{dt}M & 0 & 0 \\ \frac{d}{dt}M & r_r + \frac{d}{dt}L_r & -L_r\omega_r & -M\omega_r \\ M\omega_r & L_r\omega_r & r_r + \frac{d}{dt}L_r & \frac{d}{dt}M \\ 0 & 0 & \frac{d}{dt}M & r_s + \frac{d}{dt}L_s \end{bmatrix} \times \begin{bmatrix} i_{s\alpha} \\ i_{r\alpha} \\ i_{r\beta} \\ i_{s\beta} \end{bmatrix} \quad (8)$$

Thus, the second line of matrix for an induction motor with a solid ferromagnetic rotor (at the account of $x^{//}_{20} = 0$) will be as follows:

$$\begin{aligned} 0 = jx_m I_{s\alpha} + \left[r_r + jx_r + jx'_r\left(1 - s^{\alpha}\right)\right] I_{r\alpha} + \\ + x_r\left(s^{\alpha} - 1\right) I_{r\beta} + x_m\left(s^{\alpha} - 1\right) \end{aligned}, \quad (9)$$

where

$jx_r = jx_m + jx^{/}_{20} + jx^{//}_{20}$ is a full inductive rotor reactance;

$jx^{/}_r$ is own leakage inductance of rotor ferromagnetic body.

Write down equation (9) for instantaneous values taking into account $x_m = \omega M$, $x_r = \omega L_r$, $x^{/}_r = \omega L_{r\sigma}$ и $j\omega = d/dt$, $\omega_s = 1 - s^{\alpha}$:

$$\begin{aligned} 0 = \frac{d}{dt}Mi_{s\alpha} + r_r i_{r\alpha} + \frac{d}{dt}L_r i_{r\alpha} + \frac{d}{dt}L_{r\sigma} i_{r\alpha}\omega_s - \\ - L_r i_{r\beta}\omega_0\omega_s - Mi_{s\beta}\omega_0\omega_s \end{aligned}. \quad (10)$$

So, for an induction motor with a ferromagnetic rotor we have a system (11).

It is possible to make next conclusions for the system (11) comparing with (8):

1) the full rotor resistance differs in the size of the leakage inductance of the ferromagnetic body taking into account the speed approximation coefficient ω_s;
2) the machine nonlinear parameter change is taken into account by including of speed approximation coefficient of ω_s.

If we put $L_{r\sigma} = 0$ and $\alpha = 1$ for the system (11) (condition, given above for a squirrel cage motor) then

we shall get $\omega_s = s - 1 = \left(1 - \frac{\omega_r}{\omega_0}\right) - 1 = -\frac{\omega_r}{\omega_0}$ and thus

pass to the system (8), made for the induction motor with a squirrel cage rotor.

$$\begin{bmatrix} u_{s\alpha} \\ 0 \\ 0 \\ u_{s\beta} \end{bmatrix} = \begin{bmatrix} r_s + \frac{d}{dt}L_s & \frac{d}{dt}M & 0 & 0 \\ \frac{d}{dt}M & r_r + \frac{d}{dt}L_r + \frac{d}{dt}L_{r\sigma}\omega_s & -L_r\omega_0\omega_s & -M\omega_0\omega_s \\ M\omega_0\omega_s & L_r\omega_0\omega_s & r_r + \frac{d}{dt}L_r + \frac{d}{dt}L_{r\sigma}\omega_s & \frac{d}{dt}M \\ 0 & 0 & \frac{d}{dt}M & r_s + \frac{d}{dt}L_s \end{bmatrix} \times \begin{bmatrix} i_{s\alpha} \\ i_{r\alpha} \\ i_{r\beta} \\ i_{s\beta} \end{bmatrix} \quad (11)$$

This way leads to the mathematical model (11) adequately reflecting the features of the motor with the ferromagnetic rotor realization.

A next step is a verification of the model simulation in a transient mode in MatLab. Rewriting of equation (10), executing the grouping of elements:

$$0 = r_r i_{r\alpha} + \frac{d}{dt}\left(L_r i_{r\alpha} + M i_{s\alpha}\right) + \frac{d}{dt} L_{r\sigma} i_{r\alpha}\omega_s - \\ - \left(L_r i_{r\beta} + M i_{s\beta}\right)\omega_0\omega_s \quad . \qquad (12)$$

In (12) the expression $\frac{d}{dt}\left(L_r i_{r\alpha} + M i_{s\alpha}\right)$ is just a change of ampere-turns $\frac{d}{dt}\Psi_{r\alpha}$ and $\left(L_r i_{r\beta} + M i_{s\beta}\right)$ is ampere-turns $\Psi_{r\beta}$.

Then it is possible to describe the system (11) in a general view as follows:

$$\begin{cases} \dfrac{d\Psi_{s\alpha}}{dt} = u_{s\alpha} - r_1 i_{s\alpha} + \omega_e \Psi_{s\beta}; \\ \dfrac{d\Psi_{s\beta}}{dt} = u_{s\beta} - r_1 i_{s\beta} - \omega_e \Psi_{s\alpha}; \\ \dfrac{d\Psi_{r\alpha}}{dt} = -r_r i_{r\alpha} - \dfrac{d}{dt} i_{r\alpha} L_{r\sigma}\omega_s - \Psi_{r\beta}\left(\omega_e - \omega_0\omega_s\right); \\ \dfrac{d\Psi_{r\beta}}{dt} = -r_r i_{r\beta} - \dfrac{d}{dt} i_{r\beta} L_{r\sigma}\omega_s + \Psi_{r\alpha}\left(\omega_e - \omega_0\omega_s\right), \end{cases} \qquad (13)$$

where $\omega_s = 1 - \sqrt{1 - \frac{\omega_r}{\omega_0}}$ (we accept α = 0.5 for the rotor material).

IV. Simulation and Experimental Results

In Simulink it is comfortable to establish a model for the case of coordinate system rotation speed equal to the rotor speed $\omega_e = \omega_r$. A simulation was performed for a four-pole induction motor with a solid ferromagnetic rotor of power 1.1 kW [6]. Simulation results are shown in Fig. 2 and in Fig. 3.

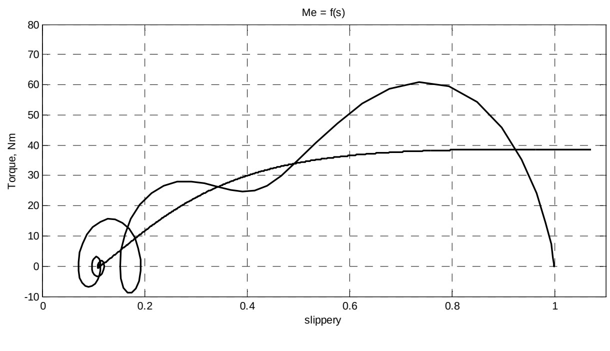

Fig. 2. Mechanical characteristic of induction motor with a ferromagnetic rotor

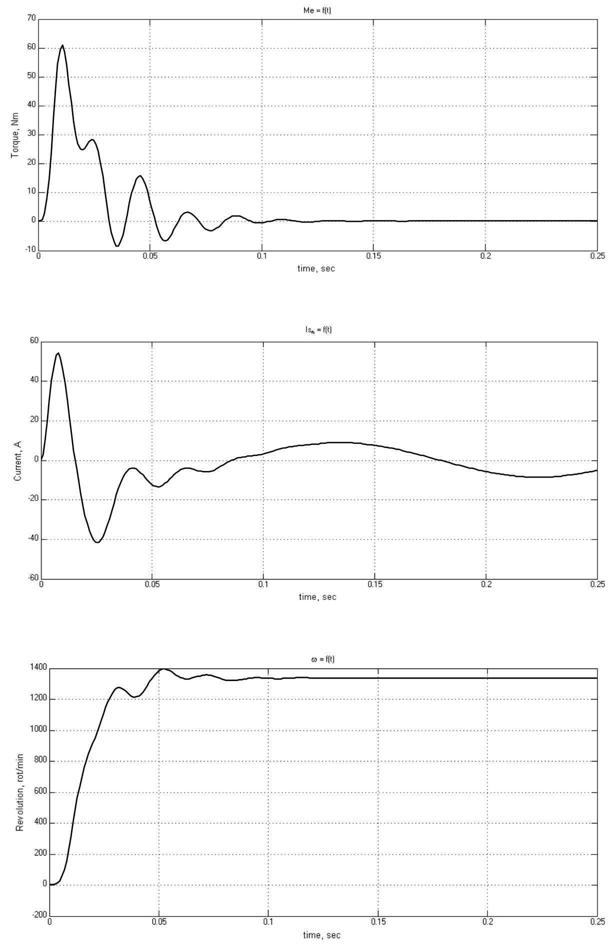

Fig. 3. Transient characteristics of induction motor with a ferromagnetic rotor

Simulation results presented in Fig. 2 and in Fig. 3 match satisfactorily the data obtained experimentally for an analogical motor [6]. Thus, it is possible to make a conclusion about adequacy of the considered model and to use it in future for research of the transient and static modes of machines with a ferromagnetic rotor.

Question of creation of a model in Simulink of the machine with a ferromagnetic rotor and simulation results are in detail considered in [7].

V. Conclusions

1. Clarification of creation of mathematical model of the induction motor with a ferromagnetic rotor by means of the incremental change of model of the generalized electromechanical transformer of energy is presented fo the first time. Such method allows us to use the worked out model in the development of electric machines [3].

2. Unlike work written earlier [4], corrections in model in the part going near modification of variable equations of the generalized machine are introduced. In addition, the new model is deprived of such defects, as the auxiliary coefficients are difficult to perceive and logically explainable.

3. When comparing the design results with the experimental research results, it is possible to state satisfactory convergence.

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