Mechanical Stress Distribution within a Cylinder Shape Winding

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Abstract — **The article treats assessment of the mechanical stress distribution within a cylinder shape coil using analytical method.**

Keywords — Windings, tangential stress, radial stress, magnetic field, flux density, thick-wall vessel, rotating disk, boundary conditions, superconductive coil.

I. INTRODUCTION

The theme of the mechanical stress distribution within cylinder shape winding was elaborated in the article “The calculation of mechanical stress in superconductive magnet with cylinder coil” that was published in EO [4]. The problem of mechanical stresses resolution leads to a differential equation. The equation solution that was published in the article was not correct because of regarding the presumption of the zero induction on the outer radius which is not generally valid (there is an opposite direction of induction than on the inner surface within real winding). This presumption leads to incorrect relations for the searched stresses. The aim of this article is to set this matter of fact right and to set the correct form of solving even for more general courses of forces.

II. THE ASSESMENT OF CORRECT RELATIONS FOR TANGENTIAL AND RADIAL STRESS

At cylinder windings that are constructed of several layers of conductor comes to unbalancing distribution of mechanical stresses. We can imagine this construction as a thick-walled vessel or as a rotating disk problem. In contrast to both a thick-walled vessel and a rotating disk the acting forces have a different trend. The general solution as the function of generally distributed electrodynamic forces is published in [1].
For practical purposes we can manage with simplified solution. The simplification consists in implementation hypothesis of linear magnetic induction decrease in independence on the radius. This is valid approximately for long windings. In the course of an inference we can emerge from the Fig. 1. There is illustrated the section of current-carrying cylinder coil. The mean current density is *J*. Simultaneously there is illustrated the supposed course of magnetic induction on the figure.

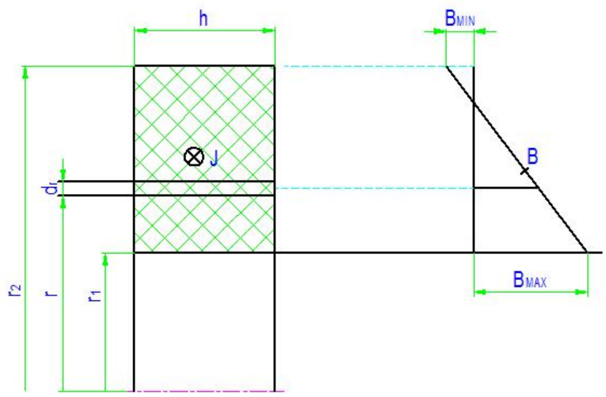

Fig.1. The section of cylinder coil and supposed magnetic field course.

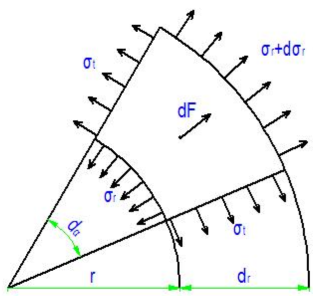

Fig. 2. The element of the winding.

Pointing out the element of winding its dimensions are dr, $d\alpha$ – see Fig. 2 – we can write the formula:

$$k_p(\sigma_r + d\sigma_r)(r + dr)\,.\mathrm{h}d\alpha - k_p\sigma_r r\mathrm{h}\, d\alpha + \mathrm{dF} = k_p\sigma_t \mathrm{h} dr d\alpha\,. \quad (1)$$

Where

k_p is the space factor, which is given as the rate of net cross section to whole cross section of the winding including spacers and cooling ducts ($k_p < 1$),

h axial length of winding,
dF is the force given by means of mutual action of field and current.

There acts the force dependent on magnetic induction (in given place) in an elemental volume dV according Fig.2. This force is given as:

$$dF = \left[\frac{(B_{max} - B_{min})(r_2 - r)}{r_2 - r_1} + B_{min}\right] Jhrdrd\alpha. \quad (2)$$

Where

B_{max} is a maximal value of magnetic induction inside the winding,
B_{min} is a minimal value of magnetic induction on outer side of the winding,
J is current density outspread to the whole cross section of winding.

Using the substitution (2) to the equation (1) after an arrangement we can get following formula:

$$d(\sigma_r r) - \sigma_t dr = -\left[\frac{(B_{max} - B_{min})(r_2 - r)}{r_2 - r_1} + Bmin J kp\, rdr. \right. \quad (3)$$

Provided that this equation is completed via of relations for tangential and radial deformation u according the literature e.g. [3]

$$\sigma_t = \frac{E}{1-\mu^2}\left(\frac{u}{r} + \mu\frac{du}{dr}\right)$$
$$\sigma_r = \frac{E}{1-\mu^2}\left(\frac{du}{dr} + \mu\frac{u}{r}\right) \quad (4)$$

and after the introduction $E^x = \frac{E}{1-\mu^2}$

we can obtain the differential equation:

$$ru'' + u' - \frac{u}{r} = -\frac{Jr}{E^x k_p}\left[\frac{(B_{max} - B_{min})(r_2 - r)}{r_2 - r_1} + Bmin\right. . \quad (5)$$

Particular solution of this equation we can assume in the form

$$u_p = k_1 r^3 + k_2 r^2 . \quad (6)$$

If we substitute this solution and its derivation to the equation (5) we get for constants k_1, k_2 :

$$k_1 = \frac{-J\,(B_{min} - B_{max})}{8k_p E^x}\frac{1}{r_2 - r_1} \quad (7)$$

$$k_2 = \frac{-J\,(B_{max} r_2 - B_{min} r_1)}{3k_p E^x}\frac{1}{r_2 - r_1} \quad . \quad (8)$$

The solution of homogenous equation that has a fundamental system x^1 ; x^{-1} is

$$u_0 = C_1' r + \frac{C_2'}{r} . \quad (9)$$

Complete solving is then:

$$u = C_1' r + \frac{C_2'}{r} - \frac{-J\,(B_{min} - B_{max})}{8k_p E^x}\frac{r^3}{r_2 - r_1} - \frac{J\,(B_{max} r_2 - B_{min} r_1)}{3k_p E^x}\frac{r^2}{r_2 - r_1} \quad (10)$$

Using $C_1 = E^x C'_1$, $C_2 = E^x C'_2$ we can obtain:

$$\sigma_t = C_1(1+\mu) + \frac{C_2}{r^2}(1-\mu) - \frac{JBr^2}{8A}(1+3\mu) - \frac{JDr}{3A}(1+2\mu) \quad (11)$$

$$\sigma_r = C_1(1+\mu) + \frac{C_2}{r^2}(1-\mu) - \frac{JBr^2}{8A}(3+\mu) - \frac{JDr}{3A}(2+\mu) \quad (12)$$

where

$$A = k_p(r_2 - r_1) \quad (13)$$

$$B = B_{min} - B_{max} \quad (14)$$

$$D = B_{max} r_2 - B_{min} r_1 . \quad (15)$$

The constants C_1, C_2 are possible to determine from boundary conditions for radial stress on the radiuses r_1, r_2.

$$\sigma_r(r_1) = 0 ; \quad \sigma_r(r_2) = 0$$

Using the substitution for constants we obtain following relations:

$$C_1 = \frac{JB(3+\mu)}{8A(1+\mu)}(r_1^2 + r_2^2) + \frac{JD(2+\mu)}{3A(1+\mu)}\left(r_1 + \frac{r_2^2}{r_1 + r_2}\right) \quad (16)$$

$$C_2 = \frac{JB(3+\mu)}{8A(1-\mu)}(r_1^2 r_2^2) + \frac{JD(2+\mu)}{3A(1-\mu)}\left(\frac{r_1 r_2^2}{r_1 + r_2}\right) \quad (17)$$

For example for an arrangement

J [A/m^2]	E [Pa]	μ	B_{min} [T]	B_{max} [T]	k_p	r_1 [m]	r_2 [m]
1.0070E+08	1.2E+11	0.35	-0.8	5.8	0.883	0.075	0.135

we obtain:

A	B	D	C_1	C_2
0.05	-6.60	0.84	57612512.4	114146.2

and the course of tangential and radial stresses for variable radius dimension r along the coil radius it is possible to trace in the Tab.1 and in the graph on the Fig.3.

TABLE I.

r	σ_r [Mpa]	σ_t [Mpa]
0.075	0.000	40.951
0.085	-1.225	34.094
0.095	-2.273	28.752
0.105	-2.827	24.610
0.115	-2.702	21.483
0.125	-1.784	19.257
0.135	0.000	17.858

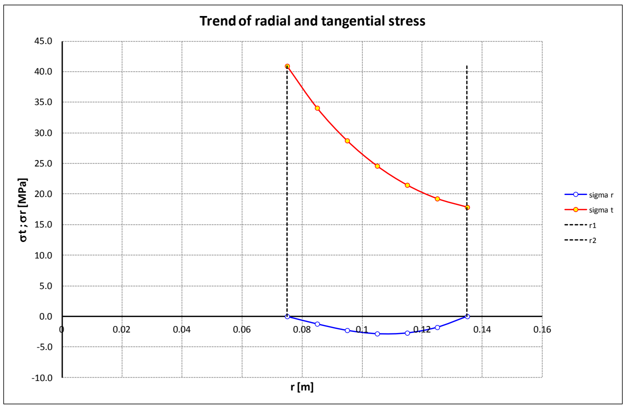

Fig. 3. Trend of radial and tangential stress.

III. Conclusions from Analytical Solution

The relations (11), (12), (16), (17) are valid providing that the properties of winding – Modulus of Elasticity E and Poisson's Ratio μ are the same in all directions. This hypothesis with regard to character of winding cannot be realized.

Modulus of Elasticity in radial direction will be lower than in tangential direction with regard to insulation spacers between conductors. This factor leads to decrease of radial stresses and to particular elimination of the tangential stress peak on the inner radius.

Nevertheless the mentioned solution gives a good conception about of a strain character within a cross section of windings that are loaded of great current density. Especially this solution enables to determine when using of inner reinforcement is effective.

It is concerned especially for solving of cylinder solenoid magnets that are intended for magnetic field limit values (these are realized like superconductive ones).

IV. The Comparison of Results Obtained by Means of Mentioned Analytical Calculation with Results that are Generated Using Finite Element Method

There was created a 3D model of coil in the FEM software ANSYS/MAXWELL. This model includes the quarter of coil geometry and it is parametric. The excitation of coil is set according to existing current density and stranded character of coil. Also Line for flux density distribution for graph is part of the model. Model was solved in solver for a magnetostatic field. The geometry of this one is illustrated on the Fig. 4.

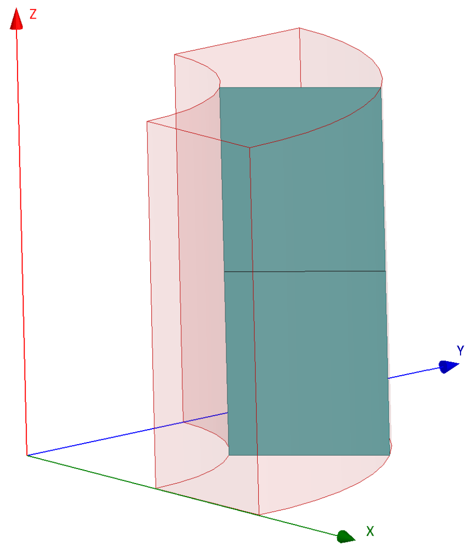

Fig. 4. Coil model geometry - 3D.

In the coil model after current loading we will obtain a distribution of a magnetic field. The module B distribution is illustrated on Fig. 5–7. There is well evident the reversing of the vectors B direction within coil body. Fig. 1 shows idealized line trend of radial flux density we supposed for analytical calculation and it´s compared with FEM calculation result in Fig. 8. Both lines are nearly agreeing.

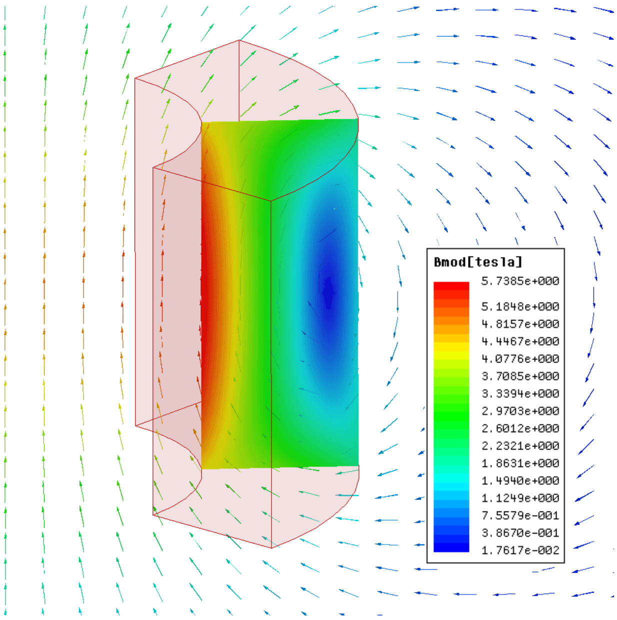

Fig. 5. Flux density vectors in space and flux density distribution in coil.

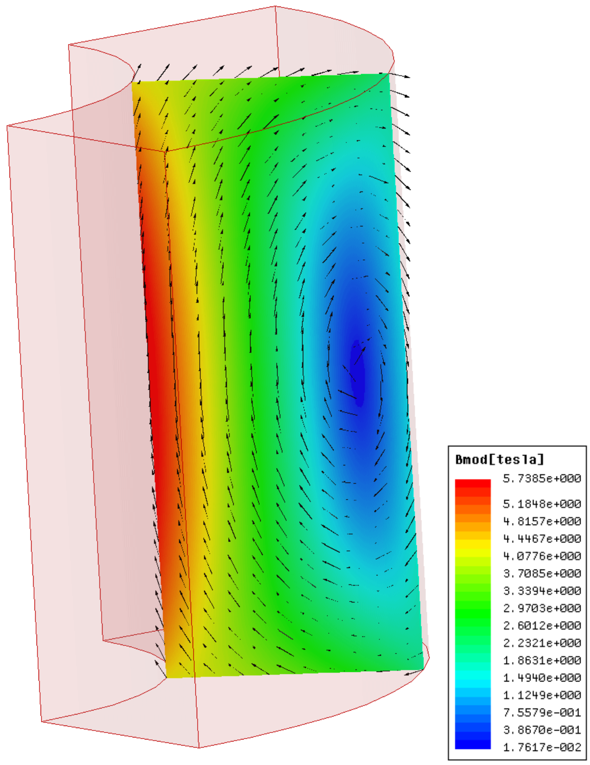

Fig. 6. Flux density distribution and vectors in coil.

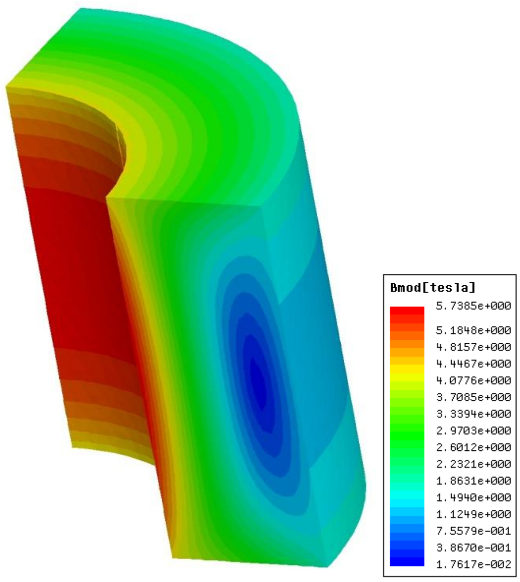

Fig. 7. Flux density distribution within coil body.

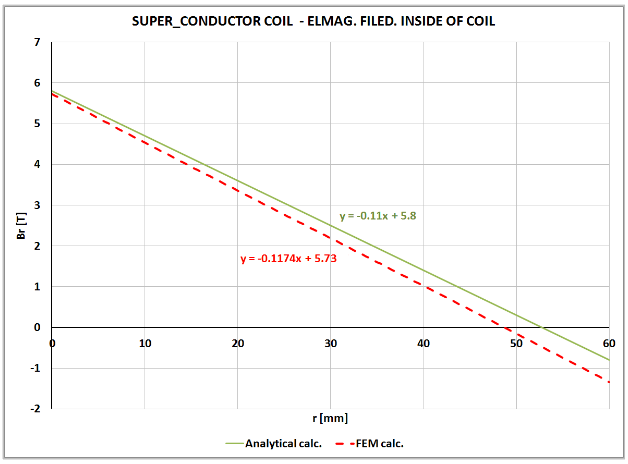

Fig. 8. The trend of flux density inside coil – idealized one & according FEM.

ACKNOWLEDGMENT

The article submits an analytical calculation of mechanic stresses or more precisely their trends along radial road midway of axial length within cylinder coil for great current load (i.e. for superconductive application mainly). Simultaneously there is carried out a confrontation of the solution of this problem using Finite Element Method by means of the evolutionary instrument ANSYS.

REFERENCES

[1] H. Brechna, “Superconducting Magnet Systems”, *Springer 1973*.

[2] F. Kuba, Teorie pružnost a vybrané aplikace, SNTL 1977.

[3] Composite authors, “Pružnost a pevnost,” in *Magnetism*, University mimeographed of ČVUT.

[4] J. Stauber, “The calculation of mechanical stresses in a superconducting magnet with a cylindrical coil,” *Elektrotechnický obzor vol. 75 No.2* pp. 94-98, February 1986.

[5] J. Stauber, “Superconductive Windings Construction” *Elektrotechnický časopis vol.19 No.4* pp. 284-297, April 1983.

[6] J. Stauber, “Výpočet mechanických napětí v solenoidu, vyvolaných elektrodynymickými silami” *EV1232P*, December 1989.