New Ways of Controlling Dangerous Torsional Vibration in Mechanical Systems

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Abstract — **In general terms the mechanical systems (MS) mean the systems of driving and driven machines arranged to perform the required work. We divide them into MS operating with constant speed and MS working within a range of speed. In terms of MS dynamics we understand the system of masses connected with flexible links between them, i.e. systems that are able to oscillate. Especially piston machines bring heavy torsional excitation to the system, which causes oscillation, vibration, and hence their noise. Based on the results of our research, the torsional vibration control as a source of given systems excitation, can be achieved by application of pneumatic couplings, or pneumatic tuners of torsional vibration. Existence of pneumatic couplings and pneumatic tuners of torsional vibration developed by us, creates the possibility of implementing new ways of torsional oscillating mechanical system tuning. Based on the above, the aim of the article is to present new ways of dangerous torsional vibration control of mechanical systems by application of pneumatic couplings and pneumatic tuners of torsional vibration developed by us.**

Keywords — torsion, oscillating mechanical system, pneumatic coupling, pneumatic tuner of torsional vibration, ways of torsional vibration control

I. Introduction

Any MS, in terms of dynamics, we understand the system of masses connected with flexible links between them, i.e. systems that are able to oscillate. Piston machines, either drivers or driven units, bring to the system significant torsional vibration. This means that MS with internal combustion engines, compressors and pumps can be characterized as torsional oscillating mechanical systems (TOMS). In the range of operating speed there can be a very intense resonance between the driver frequencies (reciprocating machines) and the natural frequencies of the system. Consequently, there comes to an excessive vibration and related excessive stress of the whole MS. The excessive dynamic stress often causes malfunction of various parts of the system, such as:

- fatigue fractures of shafts,
- gear failures,
- deformation failures of flexible couplings and the like.

Therefore, it is necessary to control their dangerous torsional vibration.

Currently, the torsional vibration is reduced to a permissible degree by appropriate adjustment, respectively tuning the system by application of an appropriately selected flexible coupling, based on a dynamic calculation. Thus the principle of tuning is an appropriate adaptation of the basic dynamic properties, particularly the dynamic torsional stiffness of the flexible coupling to the system.

The general characteristics of flexible couplings include their dynamic torsional stiffness and damping coefficient. It should be noted that, they are affected by material (metal, rubber, plastics), shape, number and dimensions of the flexible elements. It follows that they depend on various factors, which are divided into stable and unstable factors [1]. The shape, number, size and various structural modifications of flexible elements can be added to the group of stable factors, while the material of flexible members to the group of unstable factors, as a result of fatigue and aging which are changing their original characteristics. By changing original properties there is a change of coupling characteristics $M_k = f(\varphi)$ (with respect to initial characteristics), and thus a change of its basic characteristics, which has a largely positive impact on the magnitude of the dangerous torsional vibration of the mechanical system [1], [2], [3].

It should be emphasized that any linear or nonlinear flexible coupling is only one characteristic, tightly coupled to the used flexible element. In the case of a linear coupling it is only one characteristic of a constant dynamic torsional stiffness. Dynamic torsional stiffness of the nonlinear coupling varies in some extent of its characteristics, obviously dependent on the working mode of the system. Changing the characteristics of coupling due to appropriate dynamic tuning of TOMS means the use of an other coupling flexible element or other flexible shaft coupling.

Influences such as: temperature of flexible coupling elements and the number of cycles causes that, by effect of external forces any flexible member of the coupling is exposed to fatigue and aging. Consequently, there is a change of coupling characteristics, and thus a change of its basic characteristic properties. This leads to the fact that a suitably tuned TOMS becomes untuned. A flexible coupling in this case does not act as a tuner, but rather as a driver of torsional oscillations.

It should be noted that this method of tuning will be suitable only in cases where no previously unforeseen (random) effects occur during the operating mode, particularly in the turbo-machinery and reciprocating machinery [2], [3]. In case of random failure in an operating mode of MS a very intense resonance of lower harmonic excitation occurs, which is usually unexpected. Due to this fact, an intense torsional excitation causes increased torsional vibration, mechanical vibration and hence a noisy mechanism.

The torsional vibration control, based on the results of our research, is achieved by use of a pneumatic flexible coupling as well as by application of pneumatic flexible coupling with auto-regulation – pneumatic tuner of torsional systems.

The aim of this scientific paper will be presentation of the possibility to control dangerous torsional vibration of mechanical systems by ways suggested by us, by tuning and continuous tuning. Tuning of the given system is provided by application of the pneumatic flexible coupling while continuous tuning (tuning during operation) of the system will be ensured by application of the pneumatic flexible coupling with auto-regulation – pneumatic tuner of torsional vibration.

II. Proposed ways to torsional vibration control of mechanical systems

A change of the pneumatic couplings torsional stiffness can be realized by changing the pressure of gaseous media, out of operation (Fig. 3) or during operation (Fig. 5) of mechanical systems. This leads to two proposed ways to the torsional vibration control of mechanical systems [4]:

- the torsional vibration control of mechanical systems out of operation, ensuring the so called tuning of the system [5], [6], [7], [8],
- the torsional vibration control of mechanical systems during operation, ensuring the so called continuous tuning of the system [5], [9], [11].

Under the tuning of torsionally oscillating mechanical systems with pneumatic coupling we understand the inflation space of the coupling compression suitable to pressure value of the gaseous medium out of the operation system. The appropriate pressure value of the gaseous medium, and hence the appropriate value of the dynamic torsional stiffness of the coupling is based on the previously realized dynamic systems in terms of calculation of the torsional dynamics. The mechanical systems run during their entire operation with such inflated pneumatic coupling.

The principle of the torsional vibration control of the mechanical system during its operation at steady state by application of torsional oscillations pneumatic tuner [2], [3] shows the adaptation of the basic dynamic properties, particularly the dynamic torsional stiffness of the tuner to the system dynamic . The basic principle of the pneumatic tuner is the ability to auto-regulate the twist angle due to a current change of the load torque on a predetermined constant angular value φ_K . This will ensure the auto-regulation of gas pressure in the compression space of the tuner, thus adapting it to the current value of the load torque.

III. Characteristics of pneumatic flexible shaft couplings

The differential pneumatic coupling (Fig.1) consists of the driving part (1), driven part (2), between them there is located the compression space filled with gaseous medium (air in our case). The compression space consists of three circumferentially spaced and interconnected differential elements. Each differential element consists of a compressed (3) and expanded pneumatic-flexible element (4).

Interconnection of differential elements is provided by the interconnecting hose (5). The filling of compression space of coupling through the valve (6) changes the pressure p of the gas media in it. Varying pressure will ensure the fact that the coupling always works with different characteristics (Fig. 2), which is defined by the formula (1).

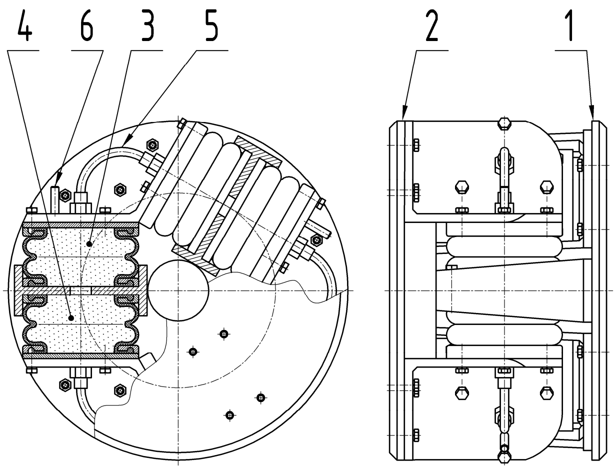


Fig. 1. Differential pneumatic flexible shaft coupling.

To another characteristic other characteristic properties always belong, thus still different torsional stiffness and damping coefficient. Therefore, each pneumatic coupling depending on the pressure is always defined by another course of the torsional stiffness in Fig. 3, as described by the formula (2).

$$M = a_0.\varphi + a_3.\varphi^3 , \qquad (1)$$

$$k = a_0 + \frac{3}{4} a_3.\varphi^3 , \qquad (2)$$

where: φ – twist angle of the coupling,

a_0, a_3 – constants of the coupling characteristics.

The orsional stiffness, as the main component in the field of the mechanical system tuning has a decisive influence on the natural frequency of the system

$$\Omega_0 = \sqrt{k / I_{red}} , \qquad (3)$$

where: I_{red} – reduced mass moment of inertia of the mechanical system.

It therefore follows the basic principle of the mechanical system tuning by pneumatic couplings. Its basic principle is to customize the natural frequency of the system Ω_0 to the angular excitation frequency ω, so that in the range of the system working mode there is no resonance condition $\omega = \Omega_0$ and hence dangerous torsional vibration.

The pneumatic tuner of the torsional vibrations (Fig. 4), which basic principle results from the patent claims [9], [11] is compared with the differential pneumatic coupling on a common structural base. The main difference is that it does not have the valve, but the controller (6) to ensure a coupling constant twist angle φ_k. The basic principle of the tuner is the ability to auto-regulate the twist angle due to the torque current load change on a predetermined constant angular value φ_k. This will ensure auto-regulation of the gaseous media pressure in the compression space of the tuner, thus adapting it to the current value of load torque.

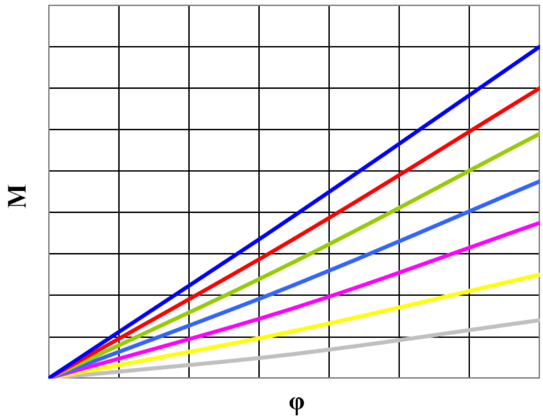


Fig. 2. Courses of static characteristics of the differential pneumatic coupling a, b, c, d, e, f, g shown in the general version belong to pressure of the gaseous medium at p = 100 ÷ 700 kPa with 100 kPa.

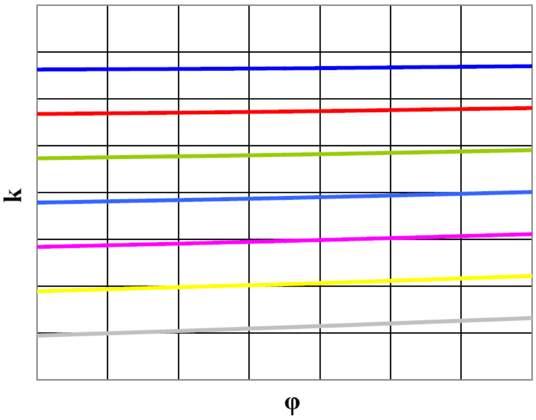


Fig. 3. Courses of the torsional stiffness k of the differential pneumatic coupling a, b, c, d, e, f, g shown in the general version belong to pressure of the gaseous medium at p = 100 ÷ 700 kPa with 100 kPa.

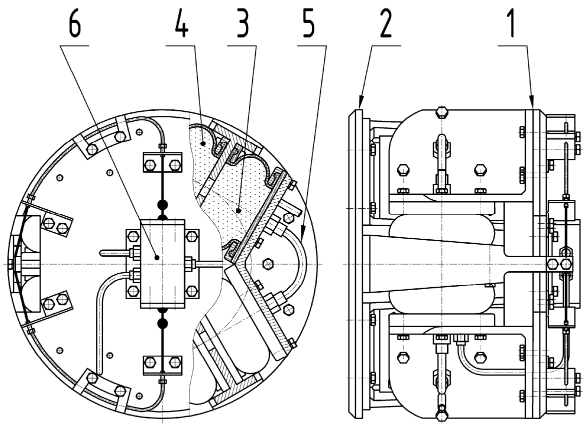
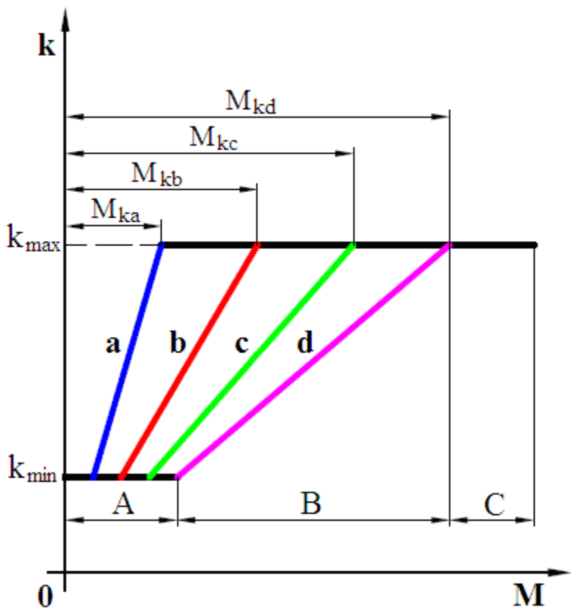


Fig. 4. Pneumatic tuner of the torsional vibration.

Auto regulation of he pressure of the gaseous media has a direct effect on the characteristics change of the pneumatic tuner (Fig. 2) Of course, for change of the torsional stiffness value (Fig. 5), as a result, we can tune the natural frequency of the system.

In Fig. 5 there are in general terms shown the traces of the pneumatic tuner of the torsional vibrations and torsional stiffness depending on the load torque. To each constant twist angle φ_{k1}, φ_{k2}, φ_{k3} and φ_{k4} based on a calculation one course of torsional stiffness labelled a, b, c, d is given.



Obr. 5. Courses of the pneumatic tuner of the torsional vibration and torsional stiffness depending on the load torque M.

IV. The results of the investigation of the proper tuning and continuous tuning of the torsionally oscillating mechanical system

When investigating an appropriate tuning, or any continuous tuning of the torsionally oscillating mechanical system we mostly start from the Campbell diagram showing the position of the critical speed n_K (or the position of the critical angular frequency ω_K) depending on the rotational speed frequency N (or natural angular frequency Ω_0).

Magnitude of the torsional vibration for the tuning and continuous tuning of the system is mostly presented by:

- courses of the dynamic torque amplitude excited by the torsional vibration in the mechanical system and hence to the pneumatic coupling, depending on the speed.

A. Characteristics of the realized torsionally oscillating mechanical system

The examination of an appropriate tuning and continuous tuning was performed on a realized torsionally oscillating mechanical system (Fig. 6). The realized system is composed of the driving part (1), pneumatic flexible shaft coupling (3) and driven part (2). The driving part, formed by a DC electric motor type SM 160 L with a power of 16 kW and an auxiliary thyristor controller of the rotational frequency (4) type IRO with the possibility of continuous control from n = 0 ÷ 2000 min^{-1}, using a pneumatic coupling that drives the exciter of the torsional vibrations represented by the three-cylinder compressor type 3–JSK–S. To increase the impact of the compressor torsional excitation to the mechanical system we used a compressor without a flywheel.

The load of the torsional oscillating mechanical system by the compressor will be adjusted (regulated) with throttle valve (6) integrated into the output pipe of the compressor.

The analyzis the pneumatic coupling load at work of the mechanical system in the steady state will be investigated by a schematic model of the realized torsional oscillating mechanical system (Fig. 7).

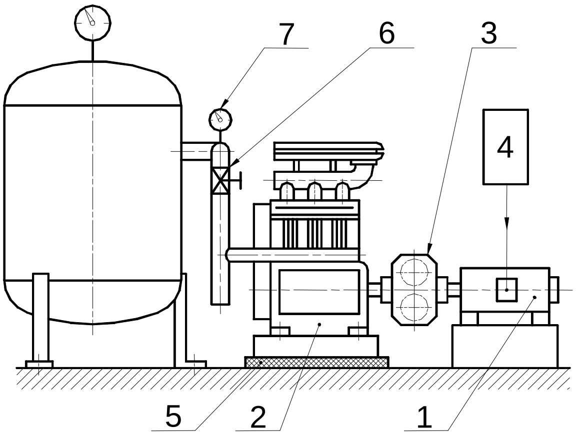


Fig. 6. Realized torsional oscillating mechanical system.

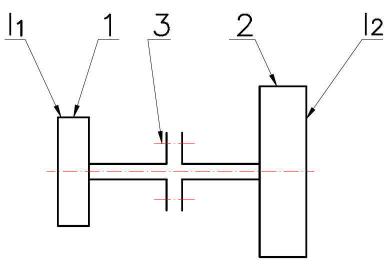


Fig. 7. Schematic model of the realized torsional oscillating mechanical system.

When calculating the loads for the steady-state mechanical system within its operational mode, we suppose that the mechanical system is rotating at an angular speed, which varies in a wide range. On mass (1) with the mass moment of inertia I_1 a load torque $M_N + \sum M_i.\sin(i.\omega.t + \gamma_i)$ acts. From the above it is clear that pneumatic coupling and thus the whole torsional oscillating mechanical system is loaded by both unvariable with time medium torque M_N in steady state and excitation of harmonic components M_i. On this basis an additional component of dynamic torque M_d is introduced in the pneumatic coupling. Thus pneumatic coupling will be in this case loaded with load torque M_S that causes the maximum twist angle φ_S:

$$M_S = M_N + M_d, \tag{4}$$

The magnitude of the additional dynamic torque calculated from the equations of motion (5) can be described by equation (6).

$$\begin{aligned} &I_1.\ddot{\varphi}_1 + b.(\dot{\varphi}_1 - \dot{\varphi}_2) + k.(\varphi_1 - \varphi_2) = M_i.\sin.(i.\omega.t + \gamma_i), \\ &I_2.\ddot{\varphi}_2 - b.(\dot{\varphi}_1 - \dot{\varphi}_2) - k.(\varphi_1 - \varphi_2) = 0. \end{aligned} \tag{5}$$

$$M_d = \sum_{i=1}^{n} M_i.\frac{I_2}{I_1 + I_2}.\theta.\sin[(i.\omega.t + \gamma_i) + \beta_i + \vartheta_i] \tag{6}$$

while dynamic coefficients θ, ξ and phase angles β_i, υ_i are described by formulas

$$\theta = \frac{\sqrt{1 + \left(\frac{i.\omega}{\Omega_0}\right)^2 . \left(\frac{2\chi}{\Omega_0}\right)^2}}{\sqrt{\left[1 - \left(\frac{i.\omega}{\Omega_0}\right)^2\right]^2 + \left(\frac{i.\omega}{\Omega_0}\right)^2 . \left(\frac{2\chi}{\Omega_0}\right)^2}}, \tag{7}$$

$$\xi = \frac{1}{\sqrt{\left[1 - \left(\frac{i.\omega}{\Omega_0}\right)^2\right]^2 + \left(\frac{i.\omega}{\Omega_0}\right)^2 . \left(\frac{2\chi}{\Omega_0}\right)^2}}, \tag{8}$$

Where for the damping coefficient $2.\chi$, natural angular frequency of the system Ω_0 and separation margin η it is applied

$$2.\chi = b.\left(\frac{1}{I_1} + \frac{1}{I_2}\right), \qquad \Omega_0 = \sqrt{k.\left(\frac{1}{I_1} + \frac{1}{I_2}\right)}, \qquad \eta = \frac{i.\omega}{\Omega_0} = \frac{i.n}{N}. \tag{9}$$

B. Results of a proper tuning of the torsionally oscillating mechanical system

The Campbell diagram according Fig. 8 describes the tuning of the realized mechanical system by the tangential pneumatic coupling type name 4–1/70–T–C in the speed range n = 0 ÷ 2000 min^{-1}. Operating mode of the system is defined in the speed range n = 750 ÷ 1500 min^{-1}.

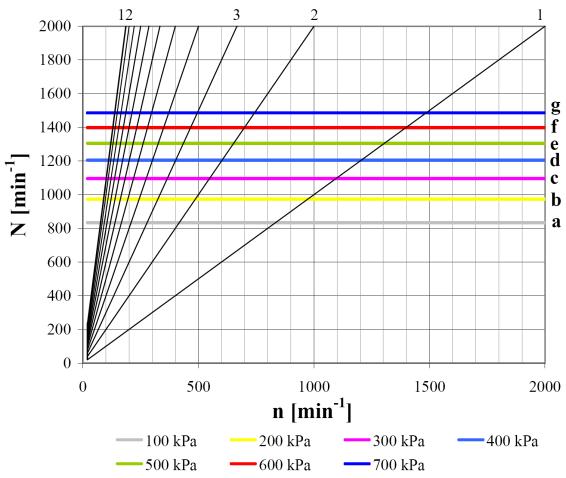


Fig.8. The Campbell diagram of the mechanical system with the applied tangential pneumatic coupling at the constant pressure p = 100 ÷ 700kPa.

The diagram shows the position of the critical speed, depending on the natural speed frequencies. Those given pneumatic couplings are nearly linear, natural speed frequencies are shown by the horizontal straight lines a, b, c, d, e, f, g for the entire range of the gaseous medium pressure p = 100 ÷ 700 kPa. Based on the diagram it is possible to say that the pneumatic coupling is capable to operate at all pressures of the gaseous medium (p = 100 ÷ 700 kPa).

On the other hand, in terms of the dynamic tuning we conclude that the pneumatic coupling is suitable for the given system in the pressure range p = 200 to 600 kPa. This is due to the fact that at the beginning of the operation mode at p = 100 kPa there is a resonance with the first harmonic component of the load torque speed n = 820 min^{-1}, while at p = 700 kPa a resonance occurs also with the first harmonic component but at the speed n = 1480 min^{-1}.

Based on the above we shall in following focus on the dynamic tuning characteristics of the realized system by the tangential pneumatic coupling at the pressure range p = 200 to 600 kPa.

Based on the figure of the Campbell diagram it is possible to say that a given coupling extends from the range of operating speed harmonic series i = 2 ÷ 12 The critical speed due to the main harmonic component (i = 3) at pressures p = 200, 300, 400, 500 and 600 kPa appears at the speed n_K = 330, 360, 405, 440 and 460 min^{-1}. These values indicate that the realized mechanical system is well-tuned with regard to the operating mode beginning. This is confirmed by the separation margin η = i. n / N, which for the investigated pressures has relatively high values of η = 2,3 to 1,6 for the investigated pressures. At the same time we can see in the figure that the harmonic series i = 1 extends in the range of the gaseous medium pressure p = 200 to 600 kPa into operating speed range (OSR). It follows that when using the pneumatic coupling there is a resonance with a harmonic component at these pressures. Specifically, for the pressure p = 200, 300, 400, 500 and 600 kPa resonances occur at speeds n_K = 980, 1090, 1220, 1330 and 1430 min^{-1}.

The tuning system realized by the pneumatic coupling for one disabled cylinder with regard to the main (i = 3) and secondary (i = 2, 1) harmonics within the operating speed (n = 750 ÷ 1500min^{-1}) is characterized in Fig. 9.

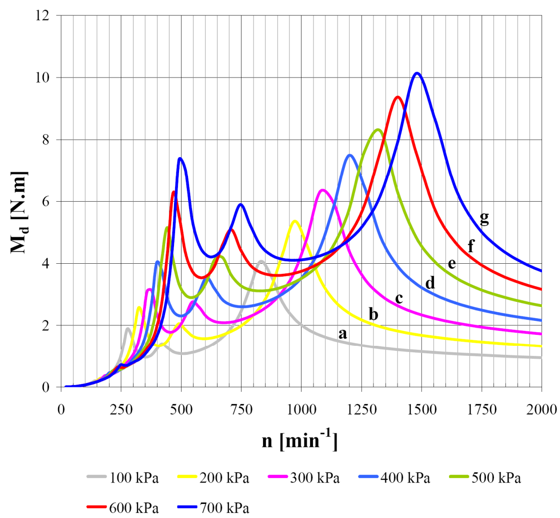


Fig. 9. The dependence of the dynamic component of the torque M_d at speeds in the range n = 0 ÷ 2000min^{-1} of the mechanical system on the tangential pneumatic coupling application with constant pressure at p = 100 ÷ 700 kPa.

The overall analysis shows that differential pneumatic coupling can be applied in the torsional oscillating mechanical system with a range of speed only in a fault-free case of work of the piston device. In case of faults caused mainly by the piston device (unbalanced excitation of engine cylinders, one disabled cylinder) it is to use the linear coupling in mechanical systems with a range of unsuitable speed . The reason of this is that, in this case, particularly lower harmonics cause increased amplitude of the mechanical load across the system (Fig. 9).

The results indicate that the linear differential pneumatic coupling would be particularly suitable for the mechanical system operating with constant operating speed.

C. The results of the continuous tuning of the torsionally oscillating mechanical system

The result of continuous tuning of the system realized by a pneumatic tuner of the torsional oscillation type 4–1/70–T–C is presented by the Campbell diagram in Fig. 10. In the figure there are represented eight waveforms of the natural speed frequencies marked a, b, c, d, e, f, g, h, corresponding to a constant twist angle of the pneumatic tuner φ_K = 0,5 °; 1 °; 1,5 °; 2 °; 2,5 °; 3 °; 3,5 °; 4 ° and are characterized by a broken line.

Based on the Campbell diagram it is possible to say that critical speeds by φ_K = 0,5 ° and 1 ° from the main harmonic i = 3 of the load torque are in a sufficient distance with regard to the start of the operating mode (n = 750 min^{-1}) by the lowest pressure p = 100 kPa and also by the highest pressure p = 700 kPa. This fact is confirmed by the separation margin, which for that case has values in the range η_{100} = 2,3 and η_{700} = 1,51. At the same time we can see that within the operating speed range of the system, particularly for the speed n = 1480 min^{-1}, a resonance is at the harmonic component series of i = 1 by φ_K = 0,5° and 1° of the pneumatic tuner with the maximum pressure value of the gaseous medium p = 700 kPa. Based on the above it can be concluded that the constant twist angles of the pneumatic tuner φ_K = 0,5 ° and 1 ° are not suitable for the realized system.

By the minimum pressure value of the gaseous medium of the pneumatic tuner p = 100 kPa with φ_K = 1,5 ° a resonance occurs at the harmonic component i = 1 at the operating speed n = 850 min^{-1} . With rising pressure up to the maximum value no resonance is at the harmonic component i = 1. For example at the maximum pressure the separation margin for i = 1 has a value η = 1,34. It indicates that the pneumatic coupling with φ_K = 1,5 ° is appropriate for the realized system except the beginning of the operating mode.

When using the pneumatic tuner with constant angles φ_K = 2 °; 2,5 °; 3 °; 3,5 ° and 4 ° no resonance is within the operating speed range of the realized system from any harmonic components of the load torque.

The results of the torsional vibration magnitude of the realized mechanical systems in the case of a disabled cylinder are shown in Fig.11. They are characterized by courses of the dynamic torque amplitudes M_d depending on the operating speed in the range n = 0 ÷ 2000min^{-1}.

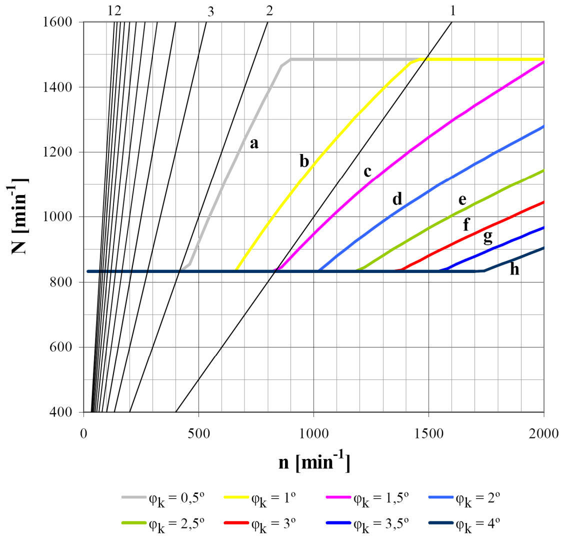


Fig. 10. The Campbell diagram of the realized mechanical system by use of the pneumatic tuner of the torsional vibration type 4–1/70–T–C.

It results from the overall analysis in Fig. 11 that the lowest dynamic loads of the mechanical system in the range of the operating speed $n = 750 \div 1500 min^{-1}$, are obtained at constant twist angles of the pneumatic tuner of torsional vibration φ_K = 2°; 2,5°; 3°; 3,5° and 4°. It is caused by the fact that the pneumatic tuner in that case acts as a highly flexible pneumatic coupling, thus coupling with a relatively low torsional stiffness.

The results indicate that the pneumatic tuner of torsional vibration will be especially suitable for mechanical systems working within a range of operating speed.

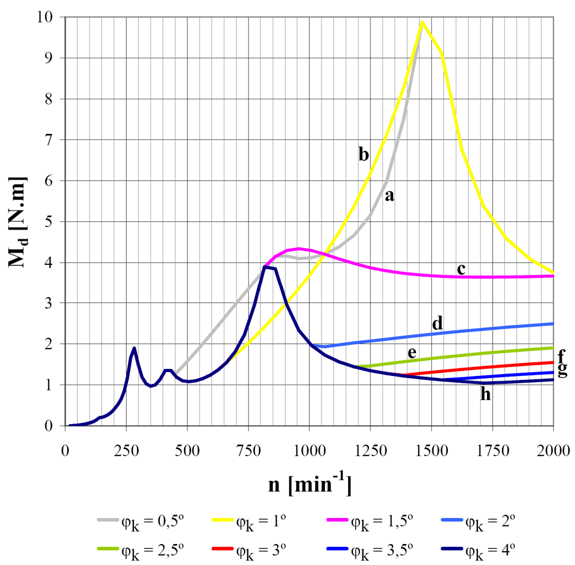


Fig.11. Courses of the dynamic torque amplitudes Md depending on the operating speed in the range $n = 0 \div 2000 min^{-1}$ for the realized mechanical system with application of the pneumatic tuner of torsional vibration

V. Conclusion

Based on presented results we can say that negative impact of the dangerous torsional vibration is possible to reduce by application of classical flexible couplings. On this occasion it is necessary to note, that each linear or nonlinear presently used flexible coupling has only one characteristic. The change of the flexible coupling characteristics, due to appropriate adaptation of its dynamic properties to the system dynamics means to use a different element of the flexible coupling or using a different flexible shaft coupling. In any case, it is not possible to forget the fatigue and aging of flexible materials, which finally have a major impact on the initial dynamic properties. Thus the unsteadiness of flexible coupling dynamic properties caused by aging and fatigue of their flexible elements and as well as the frequent failure rate of some other elements of the system causes the detuning of the tuned torsional oscillating mechanical system. In this case its tuning element, the flexible coupling, has no possibility to remove or reduce the increasing dangerous torsional vibration.

Taking into account the given facts we propose to use the pneumatic flexible shaft couplings developed by us in order to reduce dangerous torsional vibration by optimal tuning or rather optimal continuous tuning of torsional oscillating mechanical systems. Based on the presented results it is possible to say that presented differential pneumatic coupling, as well as the pneumatic tuner of the torsional vibration, fulfil all the requirements for their application in torsional oscillating mechanical systems. Based on the detailed analysis of the realized mechanical system we can say that linear pneumatic couplings are especially suitable for mechanical systems operating with constant operating speed. On the other hand, the pneumatic tuners of torsional vibrations will fulfil all the requirements of mechanical systems within a range of operating speeds.

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