Acta Polytechnica **66**(1):109–116, 2026
`https://doi.org/10.14311/AP.2026.66.0109`

REGRESSION MODEL OF THE EXTRACTION FORCE OF AN AUTOMATIC RIFLE CARTRIDGE CASE

Dinh Dung Tran[a,*], Martin Macko[a], Duc Hoan Tran[b],
Thanh Loan To[c], František Racek[a]

[a] *University of Defence, Kounicova 65, 662 10 Brno, Czech Republic*
[b] *Le Quy Don Technical University, Hoang Quoc Viet 236, 10000 Hanoi, Vietnam*
[c] *Hanoi University of Science and Technology (HUST), School of Materials Science and Engineering, Dai Co Viet 1, 10000 Hanoi, Vietnam*
[*] corresponding author: `dinhdung.tran@unob.cz`

Abstract. Reliable extraction of the cartridge case after firing is essential to the proper functioning of a gas-operated gun. This study presents a predictive model for the extraction force of a 7.62×39 mm steel cartridge case, based on the maximum chamber pressure and the contact friction coefficient (between the cartridge case and the chamber). A Central Composite Design (CCD) with two factors was used to generate the simulation data from finite element models developed in ANSYS. A reduced quadratic regression model was constructed and statistically validated, showing a high predictive capability ($R^2 = 0.969$, adjusted $R^2 = 0.953$). The model reveals that friction has a stronger influence on the extraction force than the maximum pressure, and that their interaction is non-linear. Experimental validation at the design centre, using a custom-built extraction test rig, yielded an average measured force of 39.10 N, closely matching the predicted value of 38.90 N (error = 0.51 %). The proposed model is a fast and reliable tool for the design and optimisation of ammunition and extractor mechanisms in small arms.

Keywords: Cartridge case extraction force, ANSYS simulation, 7.62 × 39 mm ammunition, friction coefficient, maximum chamber pressure, central composite design, regression modelling, experimental validation.

1. Introduction

For gas-operated guns, reliable extraction of the cartridge case after firing is essential for maintaining uninterrupted cycling and optimal weapon performance. Failures in extraction, whether due to excessive chamber pressure, inadequate extractor design, or unfavourable frictional conditions, can lead to jamming, increased wear, and system failure, particularly in combat or during high-rate firing. As such, understanding and predicting the extraction force of a fired cartridge case has significant implications for both the ammunition design and the firearm reliability.

Previous studies have examined the aspects of the interaction between the cartridge case and the chamber through experimental observation, numerical simulation, and analytical modelling. For example, Zuo et al. [1] and Song et al. [2] demonstrated that increasing the chamber pressure leads to a radial expansion of the cartridge case, which increases contact pressure and extraction resistance. Similarly, Cai et al. [3] emphasised the role of friction in axial resistance, particularly in post-firing scenarios where thermal and plastic deformation effects are dominant. These works laid the foundation for understanding the mechanics of cartridge case behaviour within a chamber.

However, most prior studies focused primarily on local deformation, contact pressure distribution, or strain accumulation within the wall of the cartridge case. Despite providing valuable insights into failure modes and material behaviour, they did not directly quantify the overall extraction force as a function of key operating variables, such as internal pressure and friction coefficient. Moreover, even fewer studies have integrated high-fidelity numerical simulation with statistical modelling to establish predictive relationships that can be validated against actual measurements.

This gap is particularly relevant for small-calibre ammunition such as 7.62×39 mm rounds, where minor variations in chamber pressure or surface friction can cause disproportionate changes in extraction reliability. Access to a fast, physics-based model of extraction force would improve the design optimisation of cartridge cases and extractor mechanisms in such systems, reducing the need for costly trial-and-error testing.

This study aims to develop a regression-based [4] prediction model [5] for extraction force by combining finite element simulations (ANSYS) with a Central Composite Design (CCD) experimental matrix. The model quantifies the influence of maximum pressure and friction coefficient on the extraction force and is validated against physical measurements using a custom-built test rig. Therefore, this research provides a compact yet precise predictive framework to

Element	**Low level (−1)**	**Medium level (0)**	**High level (+1)**
Maximum Pressure $p_{\max}$ [MPa]	359.6	367.7	375.8
Friction coefficient μ	0.05	0.14	0.23

TABLE 1. Encoded levels of the variables.

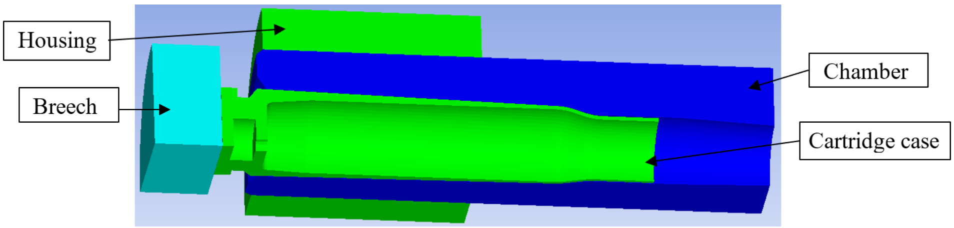


FIGURE 1. Cartridge case extraction force simulation model.

support future improvements in the design of small arms and ammunition.

2. MATERIALS AND METHODS

This study investigated the extraction force of a 7.62 × 39 mm steel cartridge case, widely used in military applications. All cartridge cases used were from the same manufacturing batch to ensure geometric and material consistency. Firing and extraction tests were conducted at room temperature (22 ± 2 °C) in a controlled environment, with dry, clean chamber surfaces to simulate typical operational conditions.

The two primary input variables studied were the maximum internal pressure ($p_{\max}$) and the friction coefficient (μ) at the cartridge case-chamber interface. The range for $p_{\max}$ (359.6 to 375.8 MPa) was selected based on the measured and simulated internal ballistics data of the 7.62 × 39 mm cartridge case reported by Cai et al. [3] and Zuo et al. [1]. This range captures the realistic variations that result from differences in propellant performance. The friction coefficient μ varied from 0.05 to 0.23, which encompasses the range typical of dry steel-steel contact as found in Cai et al. [3] and other tribological studies.

A Central Composite Design (CCD) was employed using two variables ($p_{\max}$ and μ), each at five levels, resulting in a total of thirteen simulation points, including five replicates at the centre point to evaluate curvature and repeatability. The encoded levels (x_1 for pressure and x_2 for friction) are summarised in Table 1, and the complete experimental matrix is presented in Section 3.1.

The replicated centre points were obtained through repeated numerical simulations conducted under identical forming conditions. To emulate small experimental variability, the friction coefficient at the centre point was varied slightly around $\mu = 0.14$ (ranging from 0.138 to 0.142). Since these replications were based on deterministic simulations, the random error was nearly zero.

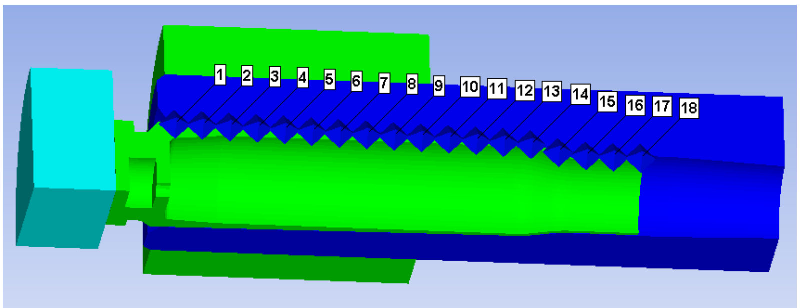
A finite element model [6, 7] and the appropriate initial and boundary conditions were built in the ANSYS Explicit Dynamics environment, and the data were then transferred to ANSYS AUTODYN. Appropriate material models and parameters were added to calculate the pressure value of the cartridge case that impacts the chamber:

- Material model: During firing, the cartridge case is affected by the Bauchinger effect. The Classical Bilinear Kinematic Hardening material model in ANSYS is built based on the Bauchinger effect. Therefore, the paper uses the shell material model as Bilinear Kinematic Hardening (BKIN). The breech, chamber, and housing only deform elastically during firing, so the material model is selected to deform isotropically.
- Boundary conditions: Fix the gun barrel, breech, and housing.
- Initial conditions: Apply a load equal to the pressure of the propellant gas ($p(t)$) acting on the inner surface of the cartridge case; the value of $p(t)$ is taken from the results of solving the internal ballistics of the 7.62 × 39 mm bullet [8–11]. The pressure is applied uniformly along the inner wall of the cartridge case. Frictional contact is modelled along the outer wall of the cartridge case using the μ values determined in the CCD. The geometry model used in the simulation is shown in Figure 1.

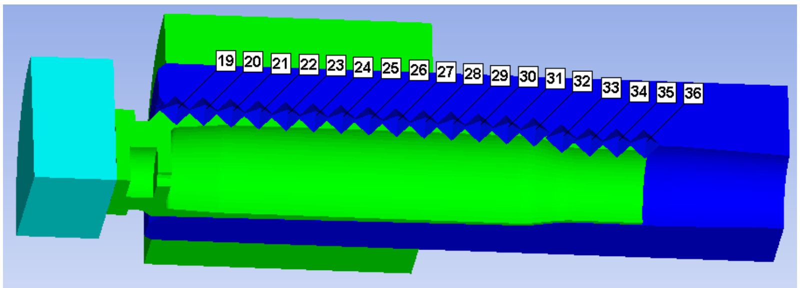
In the model, the author used the manual meshing method [7], then refined the mesh to obtain a uniform element size, increasing the accuracy of the results. To determine the pressure exerted by the cartridge case on the chamber, the survey points are placed 2 mm apart in length on the outer surface of the cartridge case and the inner surface of the chamber (Figure 2).

To validate the model, a custom-built test rig was developed to measure the extraction force experimentally (Figures 3, 4).

Ten rounds were fired under centre-point conditions ($p_{\max} = 367.7$ MPa, $\mu \approx 0.14$), and the peak extraction forces were recorded (Table 2). The resulting averages and standard deviations were then compared with the predicted regression model.



(A). Survey points on the outer surface of the cartridge case.



(B). Survey points on the inner surface of the chamber.

FIGURE 2. Setting the survey gauge points.

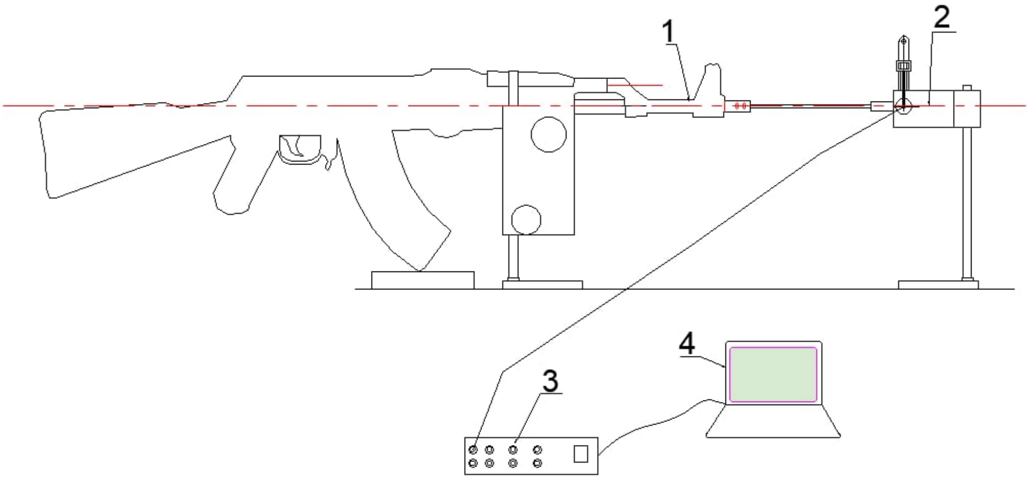


FIGURE 3. Layout of the extraction force testing equipment: 1 – Test weapon, 2 – Extraction force measuring device, 3 – Signal converter, 4 – Computer.

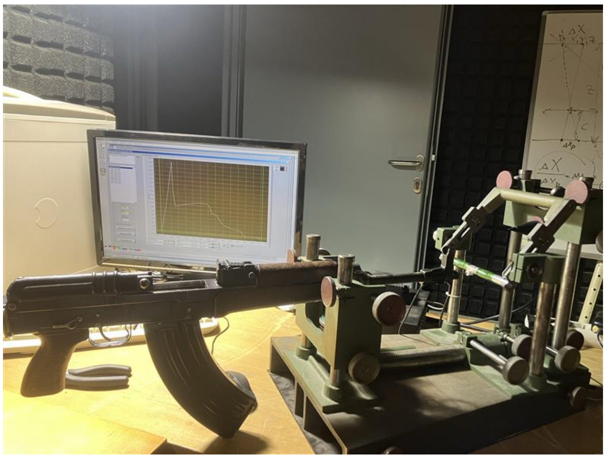


FIGURE 4. Test layout diagram.

Experiment number	**Maximum cartridge case extraction force [N]**
The 1st shot	34.81
The 2nd shot	40.20
The 3rd shot	40.20
The 4th shot	40.20
The 5th shot	40.20
The 6th shot	40.20
The 7th shot	40.20
The 8th shot	40.20
The 9th shot	39.19
The 10th shot	40.20

TABLE 2. Experimental results for cartridge case extraction force.

Simulation number	x_1	x_2	**The extraction force [N]**
1	−1	−1	16.23
2	−1	+1	26.67
3	+1	−1	42.55
4	+1	+1	146.30
5	−1.414	0	35.11
6	+1.414	0	92.35
7	0	−1.414	3.53
8	0	+1.414	74.12
9	0	0	38.82
10	0	0	39.10
11	0	0	38.55

TABLE 3. The simulated results.

3. RESULTS AND DISCUSSION

3.1. FINITE ELEMENT SIMULATION OF EXTRACTION FORCE BASED ON CCD DESIGN

A finite element model was developed in ANSYS to quantify the extraction force of a 7.62 × 39 mm steel cartridge case. Simulations were carried out following a central composite design (CCD) with two input factors: maximum internal pressure ($p_{\max}$) and chamber friction coefficient (μ), each at five levels. The matrix included 13 simulation runs, including five centre-point replicates to assess model curvature and reproducibility.

The simulated results are summarised in Table 3. The extraction force varied significantly depending on the input conditions, ranging from 3.53 N to 146.30 N:

- At low pressure and low friction ($p = 359.6$ MPa, $\mu = 0.05$), the extraction force was relatively small (16.23 N).
- At the highest tested levels ($p = 375.8$ MPa, $\mu = 0.23$), the extraction force increased steeply to 146.30 N.
- At a moderate pressure (367.7 MPa) and extremely low friction ($\mu \approx 0.013$), the force decreased to 3.53 N, illustrating the dominant effect of μ.

These results reflect the physical mechanism of cartridge case expansion and chamber contact. An increase in internal pressure results in greater plastic deformation of the cartridge case wall, reducing clearance and enhancing surface contact with the chamber. This contact substantially resists along-axis movement during extraction when combined with high friction [5].

While no regression analysis is performed at this stage, preliminary trends suggest that both $p_{\max}$ and μ positively influence the extraction force, with friction contributing more significantly. These trends justify the development of a regression model to quantify these effects more precisely.

3.2. REGRESSION MODELLING AND PARAMETRIC INFLUENCE ON EXTRACTION FORCE

Based on the simulation results in Table 3, a reduced second-order regression model was constructed to quantify the relationship between the extraction force (F) and the two main factors: maximum chamber pressure ($p_{\max}$) and friction coefficient (μ). The model equation in coded variables is:

$$\begin{aligned} F = 39.98 &+ 24.39x_1 \\ &+ 30.72x_2 + 23.33x_1x_2 + 13.9x_1^2. \end{aligned} \tag{1}$$

The regression model was evaluated for statistical adequacy using summary statistics and analysis of variance (ANOVA). The results are as follows:

- $R^2 = 0.969$, indicates that the model explains 96.9 % of the total variation in the extraction force.
- Adjusted $R^2 = 0.953$ and predicted $R^2 = 0.838$, which are in close agreement ($\Delta R^2 < 0.2$), indicate a good internal consistency and predictive capability.
- Adequate precision = 27.34, far exceeding the minimum threshold of 4.0, demonstrates a strong signal-to-noise ratio.

These metrics confirm that the regression model is statistically robust and can be reliably used to predict extraction forces within the design space.

Figure 5 shows a strong agreement between the values predicted by the model and the simulated data. All the points lie close to the 45° diagonal, further confirming the accuracy of the model.

The ANOVA results (Table 4) show that all model terms – linear (x_1, x_2), interaction ($x_1 \cdot x_2$), and quadratic (x_1^2) – are statistically significant with $p < 0.01$. Notably, the friction coefficient (x_2) has the largest linear coefficient (30.72), suggesting that it is the most influential factor in determining extraction force.

Although the lack-of-fit in Table 4 is statistically significant ($p < 0.05$), this effect is attributed to the

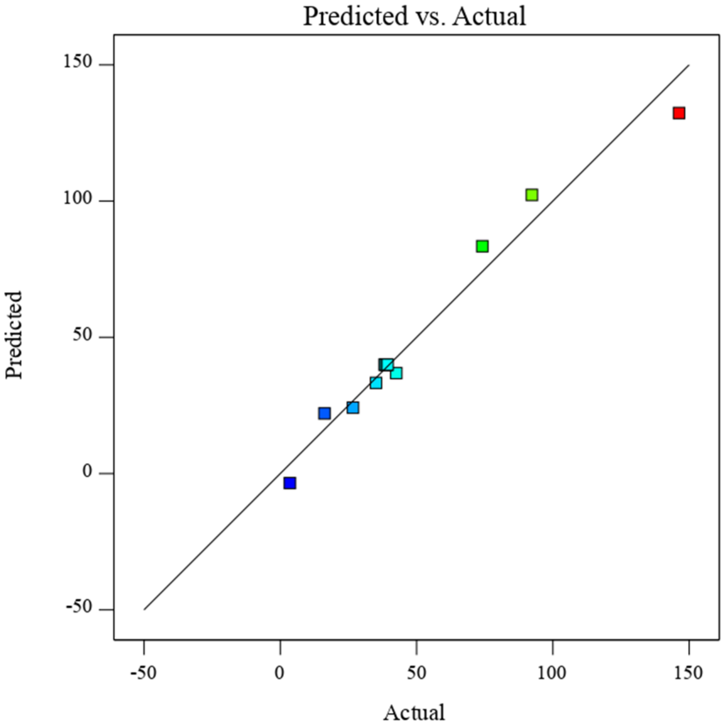


Figure 5. Predicted vs. simulated extraction force.

Source	Sum of squares	df	Mean square	F-value	p-value	
Model	15854.83	4	3963.71	61.82	< 0.0001	significant
A-p	4759.47	1	4759.47	74.23	< 0.0001	
B-m	7551.20	1	7551.20	117.77	< 0.0001	
AB	2176.62	1	2176.62	33.95	0.0004	
A^2	1367.54	1	1367.54	21.33	0.0017	
Residual	512.95	8	64.12			
Lack of Fit	512.18	4	128.04	666.01	< 0.0001	significant
Pure Error	0.7690	4	0.1923			
Cor Total	16367.78	12				

Table 4. ANOVA summary for the regression model.

almost zero random error in the replicated simulation data. The pure error is extremely small, which exaggerates the F-value of the lack-of-fit test, and hence the significance does not indicate model inadequacy.

The effect of $p_{\max}$ is positive and nonlinear (positive quadratic term), confirming that the extraction force increases more rapidly at high pressures.

The effect of the friction coefficient is dominant and linear, suggesting that even a small increase in μ greatly increases the required extraction force.

The positive interaction term ($x_1 \cdot x_2 = +23.33$) indicates a synergistic amplification: when pressure and friction increase together, the required extraction force increases disproportionately.

Figure 6 (3D surface plot) illustrates the combined effects of pressure and friction on extraction force. A sharp ridge in the response surface at high values of both variables indicates nonlinearity and strong interaction.

Figure 7 (contour plot) delineates zones of high (> 100 N) and low (< 30 N) extraction force, showing that it remains manageable only in the lower-left region (low $p_{\max}$ and μ).

Our findings are in good agreement with previous experimental and numerical studies. For instance, Zuo et al. [1] and Song et al. [2] reported that increasing chamber pressure causes greater radial plastic expansion of the cartridge case, which results in higher contact pressure along the cartridge case-chamber interface. This elevated contact pressure contributes directly to an increase in the extraction force. Similarly, Cai et al. [3] emphasised that the friction coefficient has a dominant influence on the movement of the cartridge case along the axis, particularly after the radial clearance has been reduced by pressure-induced expansion.

The regression model in the current study reveals a consistent pattern: both peak pressure and friction

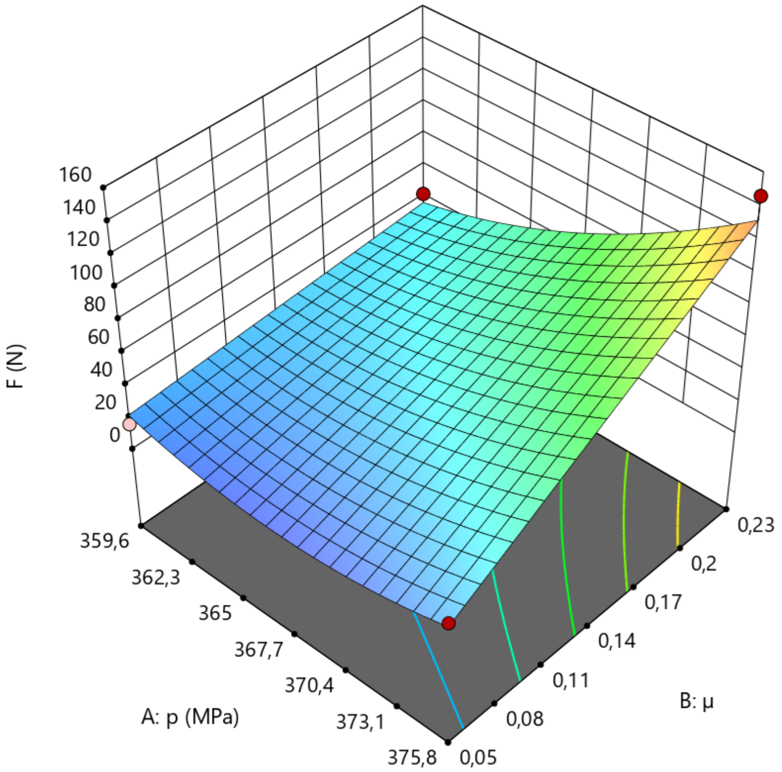


FIGURE 6. 3D surface plot of extraction force with $p_{\max}$ and μ.

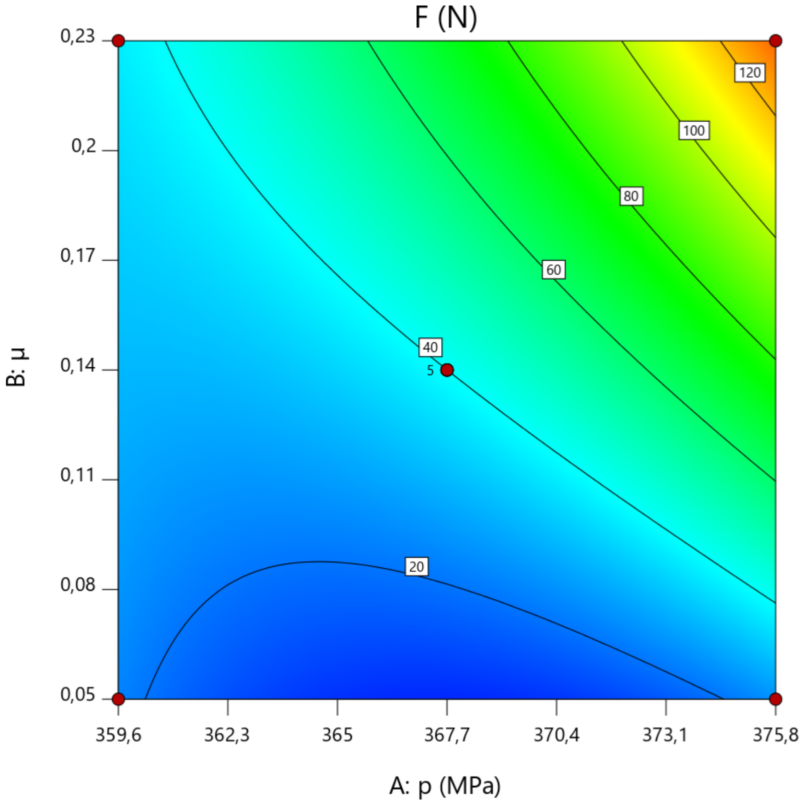


FIGURE 7. Contour plot of extraction force response.

coefficient have a strong positive effect on extraction force, with friction showing a more pronounced impact. The quadratic term for pressure confirms that the relationship is nonlinear – at lower pressure levels, force increases moderately, but beyond a certain threshold, the effect becomes steeper due to enhanced plastic expansion and surface contact. The interaction between pressure and friction is also significant, indicating that their combined effect is not merely additive but mutually reinforcing.

These insights not only corroborate existing mechanistic interpretations of extraction difficulty under high-pressure conditions but also quantify the trends within a predictive framework. The regression model captures these behaviours, providing a foundation for assessing the sensitivity of extraction force to variations in design or operating conditions, especially in scenarios involving elevated pressure or insufficient lubrication.

3.3. Experimental validation of the regression model

To evaluate the predictive accuracy of the regression model, an experiment was conducted using 7.62 × 39 mm steel-case ammunition. The test conditions were selected to match the centre point of the design matrix, corresponding to a maximum chamber pressure ($p_{\max}$) of 367.7 MPa and a friction coefficient (μ) of 0.14. The extraction force test rig described in Section 2 was used to measure the maximum axial force required to remove the fired cartridge case from the chamber.

A total of ten rounds were fired under identical conditions. The recorded extraction forces were consistent, with a mean value of 39.10 N and a standard deviation of 1.2 N. Under the same conditions, the regression model predicted a force of 38.90 N, yielding a relative error of only 0.51 %. This close agreement confirms the validity of the regression model in practical application and demonstrates that the simulation-based approach can provide physically meaningful predictions.

A key assumption in this validation was using a friction coefficient of $\mu = 0.14$ in the ANSYS simulations and in interpreting the experimental conditions. While a direct measurement of dynamic friction during firing is not feasible, this value was selected based on the relevant literature. Specifically, Cai et al. [3] used $\mu = 0.14$ in simulations of steel-steel contact under chamber pressure conditions.

Although the validation was conducted at a single design point, the low error and narrow experimental variation confirm that the model captures the dominant physical mechanisms governing the extraction force. In future work, further experiments will be conducted at multiple points across the design space to assess model generalisability more thoroughly. Moreover, a refinement of the assumed friction coefficient – either through inverse identification techniques or tribological testing under dynamic pressure – will enhance the physical accuracy of the model for a wider range of applications.

4. Conclusion

This study developed and validated a regression model to predict the extraction force of 7.62 × 39 mm steel cartridge cases as a function of maximum internal pressure and coefficient of contact friction between the chamber and the cartridge case. Using a combination of finite element simulations and Central Composite Design (CCD), a reduced quadratic model was constructed and statistically validated. The model demonstrated a high predictive capability, with $R^2 = 0.969$, Adjusted $R^2 = 0.953$, and Predicted $R^2 = 0.838$.

The results revealed that both input factors had significant effects on extraction force, with the friction coefficient exhibiting a stronger influence than maximum pressure. Notably, the interaction between the pressure and friction was also significant, indicating that the extraction force increases nonlinearly when both parameters rise simultaneously. These findings align well with studies on chamber pressure effects and cartridge case-chamber friction behaviour.

Experimental validation was conducted using a custom-built test rig under nominal firing conditions. The mean measured extraction force (39.10 N) closely matched the model-predicted value (38.90 N), with a relative error of just 0.51 %, confirming the accuracy and physical relevance of the proposed model.

The resulting model is a fast, flexible, and physically grounded tool for predicting extraction force during the design of ammunition and chamber optimisation. It can be valuable for future developments in small arms design, extraction mechanism tuning, and performance diagnostics. Future work will extend the validation to cover a range of various conditions and incorporate dynamic friction modelling for enhanced realism.

References

[1] H.-D. Zhao, J. qing Xie, Z. peng Li, H. suo Zhang. Experiment and simulation of launching process of a small-diameter steel cartridge case. *Defence Technology* **10**(4):349–353, 2014. `https://doi.org/10.1016/j.dt.2014.07.009`

[2] W.-H. Feng, J. Fang. Failure analysis of extractor slipping in the cartridge case extraction process under different working conditions. *Engineering Failure Analysis* **130**:105749, 2021. `https://doi.org/10.1016/j.engfailanal.2021.105749`

[3] S. Cai, C.-L. Huang, K. Liu, et al. Theoretical and numerical investigations on the headspace of cartridge cases considering axial deformation and movement. *Defence Technology* **16**(1):88–95, 2020. `https://doi.org/10.1016/j.dt.2019.05.023`

[4] J. Kačur, M. Durdán, M. Laciak, P. Flegner. A comparative study of data-driven modeling methods for soft-sensing in underground coal gasification. *Acta Polytechnica* **59**(4):322–351, 2019. `https://doi.org/10.14311/AP.2019.59.0322`

[5] M. Nedoma, M. Staf, J. Hrdlička. Experimental and simulation study of CO_2 breakthrough curves in a fixed-bed adsorption process. *Acta Polytechnica* **62**(3):370–385, 2022. `https://doi.org/10.14311/AP.2022.62.0370`

[6] R. Likhonina. Finite element modelling and simulation of vehicle impact on steel safety barriers. *Acta Polytechnica* **56**(6):455–461, 2016. `https://doi.org/10.14311/AP.2016.56.0455`

[7] T. Radnic, P. Šafařík, M. Luxa, D. Šimurda. Shock wave configuration in a transonic flow through a mid-section of a last stage turbine blade cascade equipped with a part-span connector. *Acta Polytechnica* **63**(4):268–272, 2023. `https://doi.org/10.14311/AP.2023.63.0268`

[8] F. Ongaro, C. Robbe, A. Papy, et al. Modelling of internal ballistics of gun systems: A review. *Defence Technology* **41**:35–58, 2024. `https://doi.org/10.1016/j.dt.2024.05.004`

[9] B. Vo Van, P. Nguyen Duy, P. Nguyen Minh. Study of the effect of some muzzle device types on the firing force and firing impulse. *Archives of Advanced Engineering Science* **2**(4):224–231, 2024. `https://doi.org/10.47852/bonviewAAES32021349`

[10] D. D. Tran, M. Phu Nguyen, V. B. Vo, et al. Analysis of gas flow losses in a gas-operated gun. In *2023 International Conference on Military Technologies (ICMT)*, pp. 1–7. 2023. `https://doi.org/10.1109/ICMT58149.2023.10171337`

[11] A. Catovic, E. Kljuno. A novel method for determination of lethal radius for high-explosive artillery projectiles. *Defence Technology* **17**(4):1217–1233, 2021. `https://doi.org/10.1016/j.dt.2020.06.015`