## Optimization of Fuzzy Logic Controller for Supervisory Power System Stabilizers

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#### Abstract

This paper presents a powerful supervisory power system stabilizer (PSS) using an adaptive fuzzy logic controller driven by an adaptive fuzzy set (AFS). The system under study consists of two synchronous generators, each fitted with a PSS, which are connected via double transmission lines. Different types of PSS-controller techniques are considered. The proposed genetic adaptive fuzzy logic controller (GAFLC)-PSS, using 25 rules, is compared with a static fuzzy logic controller (SFLC) driven by a fixed fuzzy set (FFS) which has 49 rules. Both fuzzy logic controller (FLC) algorithms utilize the speed error and its rate of change as an input vector. The adaptive FLC algorithm uses a genetic algorithm to tune the parameters of the fuzzy set of each PSS. The FLC's are simulated and tested when the system is subjected to different disturbances under a wide range of operating points. The proposed GAFLC using AFS reduced the computational time of the FLC, where the number of rules is reduced from 49 to 25 rules. In addition, the proposed adaptive FLC driven by a genetic algorithm also reduced the complexity of the fuzzy model, while achieving a good dynamic response of the system under study.

Keywords: fuzzy logic controller: adaptive fuzzy set (AFS), fixed fuzzy set (FFS) and genetic algorithm.

### 1 Introduction

Researchers usually employ the simple one-machine infinite-bus system to study a novel or modified control technique. This analysis of the simplified model is only indicative of generator behavior when connected to a rigid system. However, it cannot provide complete information about generator behavior when connected to an oscillating system of comparable size. This can be achieved by replacing the infinite bus by another synchronous generator. In this case, the mutual influence between the two machines depends not only on the relative sizes of the two machines, but also on their parameters and the initial working conditions [1].

The main stability-indicating factor in the twomachine system is the instantaneous variation of the angle between their rotors, which must be convergent for "synchronous" and "steady state" operation. If the system is subjected to various disturbances, e.g. a change in load, a sudden transient short circuit, or some other abnormal conditions, the machines will be able to remain synchronized if the angle between the rotors does not acquire an increasing manner or does not "theoretically" exceed the stability limit [2,3].

In general, a study of a two-machine system is acknowledged to represent a large power system concentrated in two distinct areas, and connected by a tie-line or by a short transmission line.

## 2 Power system structure and modeling

The system under study is shown in Figure 1. It consists of two synchronous generators, connected together by two short parallel transmission lines. The generators feed local loads at their terminal bus bars. Each generator is equipped with an automatic voltage regulator (AVR) as the main excitation control. The PSS also supports the excitation control of each generator. Each synchronous generator is represented by a third order model compromising three mathematical equations: two electromechanical, and one electromagnetic. A mathematical model of each generator may be written as follows [4,5]:

$$\dot{\omega} = \frac{1}{M} (T_m - T_e - T_D), \tag{1}$$

$$\dot{\delta} = \omega_b(\omega - 1), \tag{2}$$

$$\dot{e}'_{q} = \frac{1}{T'_{do}} (E_{FD} - e'_{q} - (x_d - x'_d)i_d), \qquad (3)$$

The electric output power is given by the following equation:

$$T_{e} \approx P_{e} = \left(\frac{e'_{q} \cdot V_{t}}{x'_{d} - x_{d}}\right) \sin \delta + \frac{V_{t}^{2}(x'_{d} - x_{q})}{2x'_{d}x_{q}} \sin 2\delta$$

$$e'_{q} = V_{t} + jx'_{d}i_{d} + jx_{q}(ji_{q}) \tag{4}$$

Where:  $\omega$  is the mechanical angular speed, M is the inertia constant, and  $T_m$ ,  $T_e$  and  $T_D$  are the mechanical, electrical, and damping torques, respectively. Symbol  $\delta$  defines the power angle, and  $\omega_b$  is the base angular speed.  $e'_q$  is the voltage behind the transient quadrature axis.  $T'_{do}$  is the field winding open circuit time constant (sec).  $E_{FD}$  defines the excitation internal voltage of the machine, while  $x_d$  and  $x'_d$  are the synchronous and transient direct-axis reactances, respectively, of the synchronous machine.  $V_t$  is the terminal voltage of the machine. The dot denotes the first time derivative of this variable.

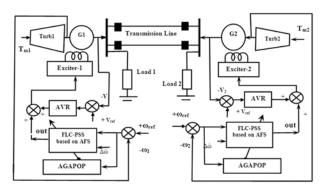


Fig. 1: The two-machine system under study

The mathematical model of the AVR and the exciter of each machine is given by [5]:

$$E_{FD} = \left(\frac{K_A}{T_A}\right) \left(V_{ref} - V_t + U_{PSS}\right) - \left(\frac{E_{FD}}{T_A}\right) \tag{5}$$

where

$$V_t = \sqrt{V_d^2 + V_q^2}, \ V_d = \frac{x_q E \sin(\delta)}{x_e + x_q}, \ V_q = \sqrt{V_t^2 - V_d^2}.$$

In the above equation,  $V_{ref}$  is the reference terminal voltage, Upss is the output of the power system stabilizer,  $T_A$  is the exciter time constant, and  $V_d$  and  $V_q$  are the direct and quadrature axis components of the terminal voltage.

For the conventional lead-lag PSS (CPSS), the following transfer function is considered during the simulation phase of the system under study:

$$U_{PSS} = -\frac{K_I}{K_A} \left( \left[ \frac{sT_Q}{1 + sT_Q} \right] \times \left[ \frac{1 + sT_1}{1 + sT_2} \right] \right) \times \dot{\delta} \quad (6)$$

Where,  $K_I$  and  $K_A$  are constants,  $T_Q$  is the time constant to be compensated, while  $T_1$  and  $T_2$  are the time constants of the lead-lag compensating network. More details of the CPSS can be found in references [5,6]. The values of these parameters and the controller gains are given Appendix A. The simulation study of the two machines is intended to determine their behavior in response to disturbances of the driving torque and terminal voltage of each generator.

## 3 Fuzzy logic controller

Fuzzy control systems are rule-based systems. A set of fuzzy rules represents the FLC mechanism for adjusting the effect of certain system stimuli. Thus, the aim of fuzzy control systems is to replace a skilled human operator with a fuzzy rules-based system. The FLC also provides an algorithm which can convert the linguistic control strategy, based on expert knowledge, to automatic control strategies. Figure 2 depicts the basic configuration of the FLC. It consists of a fuzzification interface, a knowledge base, decision making logic, and a defuzzification interface [7].

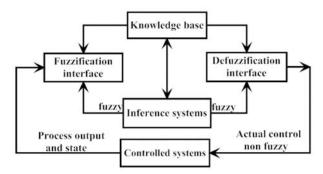


Fig. 2: Generic structure of the fuzzy logic controller

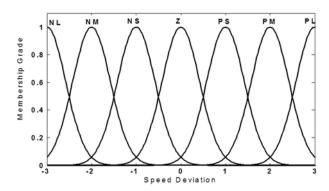


Fig. 3: Membership Functions (MFs) of Speed Deviation for SFLC

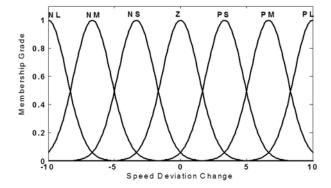


Fig. 4: Membership Functions of Speed Deviation Change for SFLC

### A. Global input variables

The fuzzy input vector of each SFLC-PSS of each generator consists of two variables: generator speed deviation,  $\Delta\omega$ , and speed deviation change,  $\Delta\omega'$ . Two static fuzzy controllers are then designed; one with seven linguistic variables, using fixed fuzzy sets, for each input variable, as shown in Figures 3, 4. The second SFLC has five linguistic variables, using fixed fuzzy sets for each input variable, as shown in Figures 6, 7 indicated by the solid lines. The fuzzy sets for the output variable, based on seven fuzzy sets, are shown in Figure 5, and the fuzzy sets for the output variable, based on five fuzzy sets, are shown in Figure 8 indicated by the solid lines. In these Figures, linguistic variables used are: PL (Positive Large), PM (Positive Medium), PS (Positive Small), Z (Zero), NS (Negative Small), NM (Negative Medium) and NL (Negative Large), as indicated in Tables 1–2.

Table 1: The look-up table relating input and output variables in a fuzzy set form for seven fuzzy sets of SFLC

$\begin{array}{c} \text{Speed} \\ \text{Deviation} \\ (\Delta\omega) \end{array}$	$\begin{array}{c} \text{Speed} \\ \text{Deviation Change} \\ (\Delta\omega') \end{array}$						
	NL	NM	NS	Z	PS	PM	PL
NL	NL	NL	NL	NL	NM	PS	Z
NM	NL	NM	NM	NM	NS	Z	PS
NS	NL	NM	NS	NS	Z	PS	PM
Z	NL	NM	NS	Z	PS	PM	PL
PS	NM	NS	Z	PS	PS	PM	PL
PM	NS	Z	PS	PM	PM	PL	PL
PL	Z	PS	PM	PL	PL	PL	PL

Table 2: A look-up table relating input and output variables in a fuzzy set form for five fuzzy sets for GAFLC

$\begin{array}{c} \text{Speed} \\ \text{Deviation} \\ (\Delta\omega) \end{array}$	Speed Deviation Change $(\Delta\omega')$				
	NL	NS	Z	PS	PL
NL	NL	NL	NL	NS	Z
NS	NL	NL	NS	Z	PS
Z	NL	NS	Z	PS	PL
PS	NS	Z	PS	PL	PL
PL	Z	PS	PL	PL	PL

The fuzzy input vector of each GAFLC-PSS of each generator consists of the previous variables used in SFLC with five linguistic variables using adaptive fuzzy sets. Only five linguistic variables (LV) are used for each of the input variables, as shown in Figures 6, 7, respectively. The output variable fuzzy set is shown in Figure 8. In these Figures, the fuzzy set of the related variables used with SFLC is indicated by the solid lines, while the dotted lines represent the simulation results of the fuzzy set when using GAFLC. Figure 9 shows the fuzzy surface for the rules. In these Figures, the LVs that we use are PL (Positive Large), PS (Positive Small), Z (Zero), NS (Negative Small) and NL (Negative Large), as indicated in Table 2.

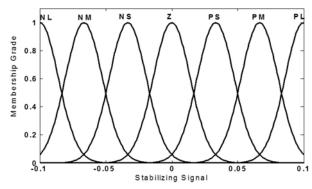


Fig. 5: Membership Functions of Stabilizing Signal for SFLC

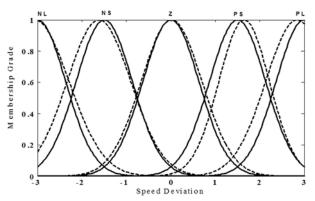


Fig. 6: Membership Functions (MFs) of Speed Deviation. SFLC is indicated by solid lines, while GAFLC is indicated by dotted lines

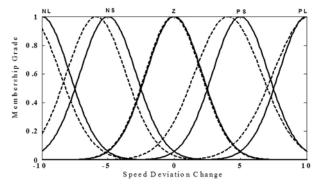


Fig. 7: Membership Functions of Speed Deviation Change. SFLC is indicated by solid lines, while GAFLC is indicated by dotted lines

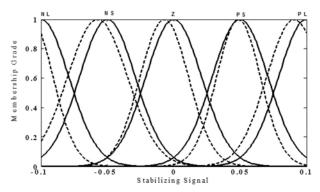


Fig. 8: Membership Functions of the Stabilizing Signal. SFLC is indicated by solid lines, while GAFLC is indicated by dotted lines

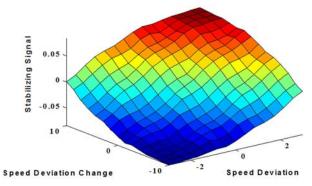


Fig. 9: Rules surface viewer for SFLC and GAFLC controllers

### B. Defuzzification method

The Minimum of Maximum value method was used to calculate the output from the fuzzy rules. This output is usually represented by a polyhedron map. The defuzzification stage is executed in two steps. First, minimum membership is selected from the minimum value of interest of the two input variables ( $\Delta\omega$  and  $\Delta\omega'$ ) with the related fuzzy set in that rule. This minimum membership is used to rescale the output rule, and then the maximum is taken to give the final polyhedron map. Finally, the centroid or center of area is used to compute the fuzzy output, which represents the defuzzification stage [7–9].

## 4 Genetic algorithm for optimizing fuzzy controllers

The adaptive fuzzy logic controller (GAFLC), using an adaptive fuzzy set based on a genetic algorithm, has the same inputs and output as the static fuzzy logic controller (SFLC) [10]. However, GAFLC uses five fuzzy sets for the inputs and output variable. Thus the full rule-base is (25 rules). SFLC is defined as an FLC using a fixed fuzzy set structure, as shown in Figures 3–5, in case of seven FS. For five FS, it is indicated by solid lines in Figures 6–8. The rules have

the general form given by the following statement:

If Vector  $(\Delta\omega)$ ,  $is\ NS$  and Change in Vector  $(\Delta\omega')$  is  $Z\ then$  Stabilizing Signal is NS.

where the membership functions  $(mf_i)$  are defined as follows:

 $mf_i \in \{NB, NS, Z, PS \text{ and } PB\}$  as in the static fuzzy case. However, the output space has 5 different fuzzy sets. To accommodate the change in operating conditions, the adaptation algorithm changes the parameters of the input and output fuzzy sets. The membership function parameters of the FLCs are optimized on the basis of the Adapted Genetic Algorithm with adjusting population size (AGAPOP) [12]. The simulation results using the GAFLC controller are denoted in dotted lines in Figures 6–8. This will be described later. AGAPOP is used to calculate the optimum value of the fuzzy set parameters based on the best dynamic performance and domain search of the parameters [11]. The objective function used in the AGAPOP technique is given by the following equation (F = 1/(1 + J)), where (J) is the minimum cost function. AGAPOP uses its operators and functions to find the values of the fuzzy set parameters of the FL controllers to achieve a better dynamic performance of the overall system. These parameter values lead to the optimum value for the control actions for which the system reaches the desired values, while improving the percentage of overshoot (P.O.S), the rising time and the oscillations.

The main aspect of the AGAPOP approach is to optimize the fuzzy set parameters of FL controllers. The flowchart procedure for the AGAPOP optimization process is shown in Figure 10 [12].

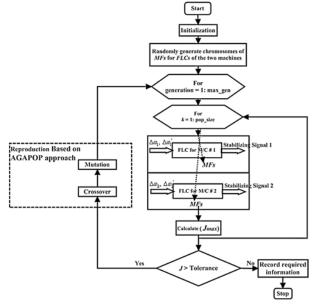


Fig. 10: Flowchart of the AGAPOP approach for optimizing MFs  $\,$ 

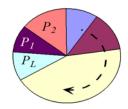


Fig. 11: Roulette Wheel Selection Scheme

### A. Representation of fuzzy set parameters in GA

The fuzzy set parameters of FL controllers are formulated using the AGAPOP approach [12], and are represented in a chromosome. The fuzzy set parameters of FL controllers are initially started using static fuzzy set parameter values. The intervals of acceptable values for each fuzzy set shape forming parameter ( $\Delta c = [c_{\min}, c_{\max}]$ , and  $\Delta \sigma = [\sigma_{\min}, \sigma_{\max}]$  for Gaussian) are determined based on  $2^{\rm nd}$  order fuzzy sets for all fuzzy sets, as explained in Appendix B. The Gaussian shape is chosen in order to show how the parameters of the fuzzy sets are formulated and coded in the chromosomes.

The minimum performance criteria J are [8]:

$$J = \int_0^T (\alpha_1 |e(t)| + \beta_1 |e'(t)| + \gamma_1 |e''(t)|) dt \qquad (7)$$

where e(t) is equal to the average error of  $\Delta\omega_1$  and  $\Delta\omega_2$ . Parameters  $(\alpha_1, \beta_1 \text{ and } \gamma_1)$  are weighting coefficients.

### B. Coding of fuzzy set parameters

The coded parameters are arranged on the basis of their constraints, to form a chromosome of the population. The binary representation is the coded form for parameters with chromosome length equal to the sum of the bits for all parameters. Tables 3, 4 show the coded parameters of FLCs for machines 1 and 2, respectively.

#### C. Selection function

The selection usually applies some selection pressure by favoring individuals with better fitness. After procreation, the suitable population consists, for example, of L chromosomes which are all initially randomized [12, 14] and [16]. Each chromosome has been evaluated and associated with fitness, and the current population undergoes the reproduction process to create the next population, as shown in Figure 11. The chance on the roulette-wheel is adaptive, and is given as  $P_l/\sum P_l$  as in equation (8) [8]:

$$P_l = \frac{1}{J_l}, \qquad l \in \{1, \dots, L\}$$
 (8)

where  $J_l$  is the model performance encoded in the chromosome measured in the terms used in equation (7).

### D. Crossover and mutation operators

The mating pool is formed, and crossover is applied. Then the mutation operation is applied followed by the AGAPOP approach [12]. Finally, the overall fitness of the population is improved. The procedure is repeated until the termination condition is reached. The termination condition is the maximum allowable number of generations. This procedure is shown in the flowchart given in Figure 10.

Chromosome	Sub-chro of in	Sub-chromosome of output	
	$\Delta\omega_1$	$\Delta\omega_1'$	Stabilizing Signal 1
Parameters	$c_1, \sigma_1, \ldots, c_5, \sigma_5$	$c_1, \sigma_1, \ldots, c_5, \sigma_5$	$c_1, \sigma_1, \ldots, c_5, \sigma_5$
30	$2 \times 5$	$2 \times 5$	$2 \times 5$

Table 3: Coded parameters of GAFLC for M/C # 1

Table 4: Coded parameters of GAFLC for M/C # 2

Chromosome	Sub-chro of in	Sub-chromosome of output	
	$\Delta \omega_2$	$\Delta \omega_2'$	Stabilizing Signal 2
Parameters 30	$c_1, \sigma_1, \dots, c_5, \sigma_5$ $2 \times 5$	$c_1, \sigma_1, \dots, c_5, \sigma_5$ $2 \times 5$	$c_1, \sigma_1, \dots, c_5, \sigma_5$ $2 \times 5$

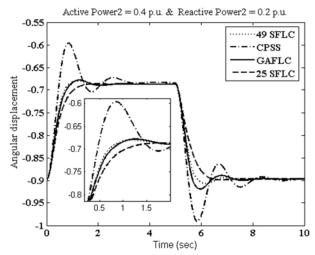
## 5 Simulation results and discussion

## 5.1 Dynamic performance due to sudden load variation

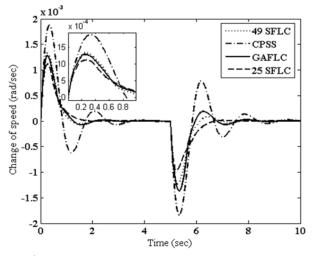
The system data given in Appendix A is used to test the proposed algorithm. Different simulation computations have been performed, and results were obtained for the two generators equipped with PSS driven by SFLC based on adaptive fuzzy sets. The simulation programs cover a wide range of operating conditions covering light, medium and heavy load conditions. Light load is represented by assuming both synchronous generators normally loaded and delivering 0.4 per unit (pu) active power  $(P_e)$  and 0.2 pu reactive power  $(Q_e)$ . In addition, the medium operating points are considered when both generators are normally delivering  $P_e$  and  $Q_e$  equal to 0.65 and 0.45 pu, respectively. For the case of heavy load,  $P_e$ and  $Q_e$  for the two generators, equal 0.9 and 0.4 pu, respectively.

# 5.2 Mechanical Torque DisturbanceA. Light load conditions

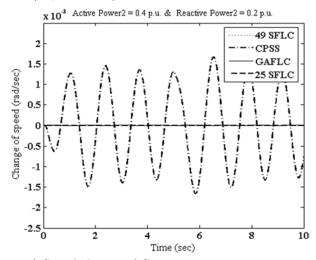
The first case was studied when both synchronous generators were loaded by  $P_e$ ,  $Q_e$  equal to 0.4, 0.2, respectively. Generator (Gen-1) was subjected to a 10 % step increase in the reference mechanical torque. The torque was then returned back to the initial condition. Figures 12a, b, c show the angular displacement between the rotors of the two machines  $(\delta)$ , in radians, and the speed deviation,  $\Delta\omega$  in rad/sec, for Gen-1 and Gen-2, respectively. These Figures include the simulation results for the system under study when equipped with various PSS controllers. These controllers are conventional PSS, PSS-SFLC using seven static fuzzy sets with overall rules equal to 49 rules, PSS-SFLC using five static FS with overall rules equal to 25 rules, and the PSS-genetic adaptive fuzzy logic controller (GAFLC) using five adaptive fuzzy sets with overall rules equal to 25 rules. It should be noted that the PSS-SFLC using seven fuzzy sets provides a better dynamic performance than the PSS-SFLC with five fuzzy sets. However, the main drawback of the PSS-SFLC using seven FS is the large computation time for 49 rules every sampling time when compared with the time required for 25 rules using PSS-FLC with five FS. Meanwhile, PSS-GAFLC almost coincides with PSS-FLC with seven FS. Table 1 and Table 2 show the rules for static and adaptive fuzzy controllers. The dynamic response, shown in Figure 12, depicts the superio-



a) The angular displacement between the two machine rotors under a light load

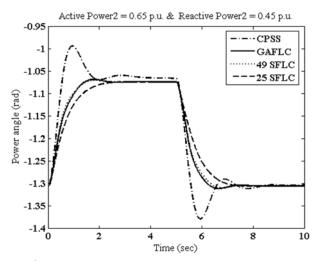


b) Speed change of Generator 1

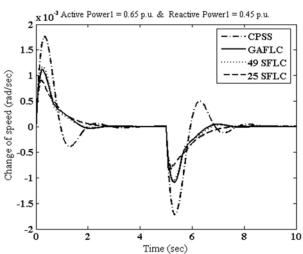


c) Speed change of Generator 2

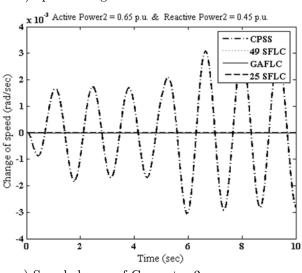
Fig. 12: Dynamic response of a synchronous generator equipped with SFLC-PSS, GAFLC-PSS and CPSS. Gen-1 is subjected to a step increase/decrease in  $T_m$ 



a) The angular displacement between the two machine rotors under a medium load

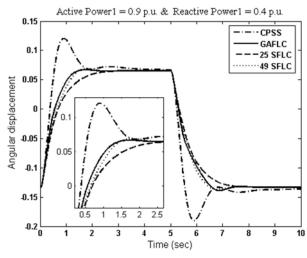


b) Speed change of Generator 1

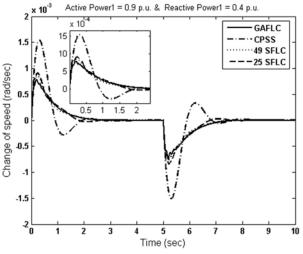


c) Speed change of Generator  $2\,$ 

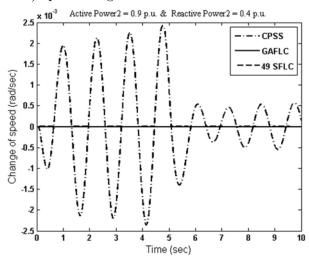
Fig. 13: Dynamic response of the synchronous generator equipped with SFLC-PSS, GAFLC-PSS and CPSS. Gen 1 is subjected to a step increase/decrease in Tm



a) The angular displacement between the two machine rotors under a heavy load



b) Speed change of Generator 1



c) Speed change of Generator 2

Fig. 14: Dynamic response of the synchronous generator equipped with SFLC-PSS, GAFLC-PSS and CPSS. Gen 1 is subjected to a step increase/decrease in Tm

rity of GAFLC compared with the other controllers, except PSS-FLC using seven fuzzy sets. The rising time, settling time and damping coefficient of the overall system is better than PSS-using FLC with static FS with 25 rules. The simulation results also show that GAFLC has a lower percentage overshoot than CPSS. Figures 6 to 8 show the normalized membership function (MFs) before and after training using the AGAPOP algorithm for the input and output variables of the fuzzy controller.

#### B. Medium load conditions

The second case studied is when each generator is loaded with Pe = 0.65 pu, Qe = 0.45 pu and is subjected to the same torque disturbance as in case study A. Figures 13a, b, c show the simulation results for this case, including the power angle displacement between the two rotors  $(\delta)$ , in radians, and the speed deviation,  $\Delta\omega$ , in rad/sec for Gen 1 and Gen 2, respectively.

### C. Heavy load conditions

The third case studied is when each generator is loaded with  $P_e = 0.9$  pu,  $Q_e = 0.4$  pu and is subjected to the same torque disturbance as in case study A. Figures 14a, b, c show the simulation results for this case, including the power angle displacement between the two rotors ( $\delta$ ) in radians, and the speed deviation,  $\Delta\omega$  in rad/sec for Gen 1 and Gen 2; respectively.

#### 6 Conclusion

This paper has presented a new fuzzy logic control power system stabilizer for the supervisory power system stabilizers of a two-machine system. The adaptive fuzzy set is introduced and tested through a simulation program. The proposed adaptive fuzzy controller driven by a genetic algorithm improves the settling time and the rise time, and decreases the damping coefficient of the system under study. The simulation results show the superiority of the adaptive fuzzy controller, driven by a genetic algorithm, in comparison with other controllers. The results also show the effectiveness of the proposed GAFLC with an adaptive fuzzy set scheme as a promising technique. The specifications of the parameter constraints related to the input/output reference fuzzy sets are based on 2nd order fuzzy sets. The problem of constrained nonlinear optimization is solved on the basis of a genetic algorithm with variable crossover and mutation probability rates. The proposed GAFLC using AFS also reduced the computational time of the FLC, where the number of rules is reduced from 49 to 25 rules. In addition, the proposed adaptive FLC technique driven by a genetic algorithm reduced the complexity of the fuzzy model.

## Appendix A

All parameters and data are given in per-unit values The machine# 1 parameters are as follows:

$$P_e=0.8,\ Q_e=0.6,\ V_t=1.05,\ X_d=1.2,\ X_d'=0.19,\ X_q=0.743,\ H=4.63,\ T_{do'}=7.76,\ D=2,\ \xi=0.3$$
 The machine#2 parameters are as follows:

 $P_e = 0.75, \ Q_e = 0.55, \ V_t = 1.0, \ X_d = 1.15, \ X_d' = 0.13, \ X_q = 0.643, \ H = 3.63, \ T_{do'} = 7.00,$ 

 $D = 1.8, \, \xi = 0.27$ 

Local load data:

Load#1: connected to machine #1 G1 = 0.449, B1 = 0.262

Load#2: connected to machine #2 G2 = 0.249, B2 = 0.221

Line data:

 $X_{T.L.} = 0.997$  $R_{T.L} = 0.034,$ 

AVR data:

Machine#1 and Machine#2:  $K_{A1}=400,\ K_{A2}=$  $370,\,T_{A1}=0.02,\,T_{A2}=0.015$ 

## Appendix B

#### Determining Constraints of Gaussian MFs

The membership function  $\mu(x)$  of a fuzzy set is frequently approximated by a Gaussian. A Gaussian shape is formed by two parameters: center c and width  $\sigma$ , as in formula (B.1):

$$\mu_{G1}(x; c_j, \sigma_j) = e^{-\frac{(x - c_j)^2}{2\sigma_j^2}}$$
 (B.1)

The idea of a 2<sup>nd</sup> order fuzzy set was introduced by Melikhov to obtain a boundary of Gaussian shape of the membership function [13]. The 2<sup>nd</sup> order fuzzy set of a given MF(x) is the area between  $d^+$  and  $d^-$ , where  $d^+$ , and  $d^-$  are the upper and lower crisp boundaries of 2<sup>nd</sup> order fuzzy sets, respectively, as shown in Figure B.1 The expressions for determining the crisp boundaries are (B.2), and (B.3):

$$d_i^+(x_i) = \min(1, MF_j(x_i) + \delta)$$
 (B.2)

$$d_i^-(x_i) = \max(0, MF_i(x_i) - \delta)$$
 (B.3)

Formulas (B.2) and (B.3) are based on the assumptions that the height of the slice of the 2<sup>nd</sup> order fuzzy region, bounded by  $d^+$  and  $d^-$ , at point x is equal to  $2\delta$  where  $\delta \in [0, 0.3679]$  and these boundaries are equidistant from MF(x). To obtain the ranges for the shape forming parameters of the MFs, it should be assumed that these 2<sup>nd</sup> order fuzzy sets are MF search spaces. All MFs with acceptable parameters should therefore be inside the area. In the general case, the intervals of acceptable values for every MF shape forming parameter (e.g.,  $\Delta c = [c_{11}, c_{22}]$ , and  $\Delta \sigma = [\sigma_{11}, \sigma_{22}]$  for Gaussian) may be determined by solving formulas (B.1), (B.2) and (B.3). In practice, this may be done approximately, considering  $d^+$  and  $d^-$  as soft constraints. For example,  $c_{11}$  and  $c_{22}$  for the Gaussian may be found as the maximum root and the minimum root of the equation  $d^+ = 1$ , which can easily be calculated. This equation is based on the assumption that a fuzzy set represented by the Gaussian must have a point where it is absolutely true.  $\sigma_{11}$  and  $\sigma_{22}$  can easily be found from the following four equations:

$$\mu_{G1}((c+\sigma); c, \sigma) + \delta = \mu_{G1}((c+\sigma); c, \sigma_{22}); \text{ (B.4)}$$

$$\mu_{G1}((c+\sigma); c, \sigma) - \delta = \mu_{G1}((c+\sigma); c, \sigma_{11})$$

$$\mu_{G1}((c-\sigma); c, \sigma) + \delta = \mu_{G1}((c-\sigma); c, \sigma_{22}); \text{ (B.5)}$$

$$\mu_{G1}((c-\sigma); c, \sigma) - \delta = \mu_{G1}((c-\sigma); c, \sigma_{11})$$

where we choose  $\sigma_{11}$  as the minimum and  $\sigma_{22}$  as the maximum from the roots. These equations are based on the assumption that the acceptable Gaussian with  $[\sigma_{11}, \sigma_{22}]$  should cross the 2<sup>nd</sup> order fuzzy region slices at points  $x = (c \pm \sigma)$ . There are two options for finding the constraints of Gaussian parameters. First, we consider the constraints as hard constraints, and it follows that the lower and upper bounds of the center of the Gaussian membership function will be chosen as  $c_{\min}$ , and  $c_{\max}$  should be lower than the values of  $c_{11}$  and  $c_{22}$  to satisfy the search space constraint conditions of  $2^{\rm nd}$  order fuzzy sets, as shown in Figure B.2. The lower and upper bounds for the width of Gaussian membership function  $\sigma_{\min}$  and  $\sigma_{\max}$  will be equal to  $\sigma_{11}$  and  $\sigma_{22}$ , respectively, to satisfy the search space constraint conditions of 2<sup>nd</sup> order fuzzy sets, as shown in Figure B.1. A second option is to consider these constraints as soft constraints, i.e.,  $[c_{\min}, c_{\max}]$  equal to  $[c_{11}, c_{22}]$ , and  $[\sigma_{\min}, \sigma_{\max}]$  equal to  $[\sigma_{11}, \sigma_{22}]$ .

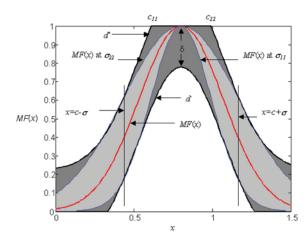


Fig. B.1: Upper and lower boundaries of width  $\sigma$ , using a 2<sup>nd</sup> order fuzzy set

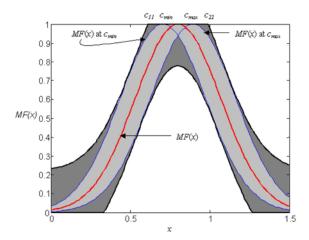


Fig. B.2: Upper and lower boundaries of center c, using a  $2^{\text{nd}}$  order fuzzy set

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