

## PROPOSAL OF NEW OPTICAL ELEMENTS

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**ABSTRACT.** A overview of our patented proposals of new optical elements is presented. The elements are suitable for laser pulse analysis, telescope, X-ray microscopy and X-ray telescope. They are based on the interference properties of light: a special grating for a double slit pattern, parabolic strip imaging for a telescope, and Bragg's condition for X-ray scattering on a slice of a single crystal for X-ray microscopy and X-ray telescope.

**KEYWORDS:** Optical elements, interference of light, telescopes, X-ray microscopy, X-ray telescope.

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### 1. INTRODUCTION

Over the last twelve years we have patented four new optical elements and systems. Prof. M. Havlíček has been interested in our work all this time. We would like to dedicate this summary to him.

We will describe the following proposals:

- A two-diffraction system;
- A parabolic strip telescope;
- An optical element for X-ray microscopy;
- An X-ray Bragg's telescope.

The principles of each proposal will be described in the next four sections.

### 2. TWO-DIFFRACTION SYSTEM

The two-diffraction system was patented [1] and published in [5].

The idea of the two-diffraction system is based on the two-slit experiment, with two channels coupled with alternating slits, and light passes through both of them. When a measurement is made or a photon is identified in one of the channels, we can distinguish which channel the light passed through, and the interference of light between the two channels is lost. Otherwise, when light passes through the two channels, and nobody can distinguish which channel the light passed through, the interference of the light between the channels remains. The new feature of the proposal is the combination of a two-slit arrangement with an interference grid. This allows for better measurement options. Let us provide an example.

One possible realization of the 2-diffraction system is shown in Fig. 1. Light source 1 is monochromatic. The light is divided into two channels 3 by a 1:1 beam splitter 2, and is brought into the 2-diffraction grating 5. 2-diffraction grating generally consists of a large number of equally spaced slits, which allows the connection of one channel with odd slits and the other

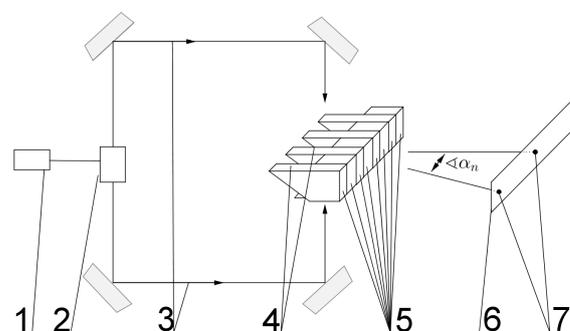


FIGURE 1. Let 1 be a monochromatic light source. The light is divided into two channels 3, by a 1:1 beam splitter 2, and is brought into the two-diffraction gratings 5. A two-diffraction grating generally consists of a large number of equally spaced slits, which allows the connection of one channel with odd slits and the other channel with even slits 4. Then spots (interference maxima) appear on the screen 6.

channel with even slits 4. Then spots 7 — interference maxima — appear on the screen 6. The positions of the maxima depend on the possibility of distinguishing the paths.

**Case 1.** If it is possible to distinguish the photon paths between the two channels, then

$$\sin \alpha_n = \frac{n\lambda}{2d}, \quad (1)$$

where  $d$  is the spacing of the slits (or optical segments as in Fig. 1),  $\lambda$  is the wavelength, and  $n = 0, \pm 1, \pm 2, \dots$

**Case 2.** If it is not possible to distinguish the path through which of the channels the photon has passed and has the same phase in even and odd slits, then

$$\sin \alpha_n = \frac{n\lambda}{d}, \quad (2)$$

and half of the spots will not appear. Of course, the intensity of the spots is different.

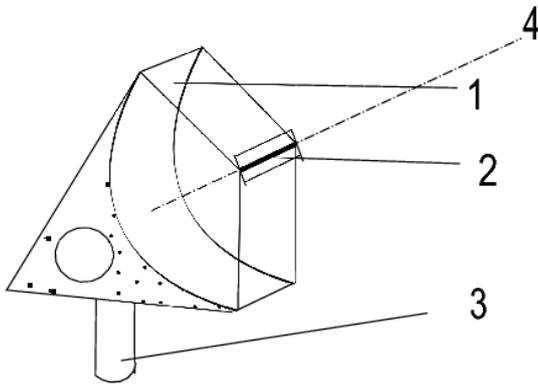


FIGURE 2. Principle of the new telescopic system. It consists of a parabolic strip mirror 1, a CCD camera 2 in the image plane, supported by the mounting 3, and it rotates around the optical axis 4. The shots are stored in a computer, where the image is reconstructed.

**Case 3.** If it is possible to distinguish the photon paths between the two channels only for some of the photons (for example for the interacting photons) and it is not possible for the other photons, the intensity of the spots depends on the ratio of these parts.

The reliability of the information transmission depends on the efficiency of the detectors. Moreover, entry into the transmitted information also needs a detector. A small part of the light is turned aside and detected there. If the efficiency of this detector is comparable with the detector used in the 2-diffraction system, the entry will be detected.

The intensity of the light passing through a diffraction grating with  $n$  slits of the same width  $a$ , spaced at an equal distance  $a + b$  in Cases 1 or 2, depends on angle  $\alpha$

$$I_\gamma(\alpha) = I_{\gamma 0} \left( \frac{\sin(\frac{\pi a}{\lambda} \sin \alpha)}{\frac{\pi a}{\lambda} \sin \alpha} \frac{\sin(\frac{n\pi\gamma(a+b)}{\lambda} \sin \alpha)}{\sin(\frac{\pi\gamma(a+b)}{\lambda} \sin \alpha)} \right)^2, \quad (3)$$

where  $\gamma = 1$  for the Case 1 and  $\gamma = 2$  for the Case 2,  $I_{10}$  and  $I_{20}$  are the integral intensities of the light passed through the diffraction system in Cases 1 or 2,  $\lambda$  is the wavelength of the light, and  $n$  is the number of slits.

In Case 3, for the same diffraction grating the intensity is the sum of the intensities of Case 1 and Case 2. Let  $p$  percent of the photons belong to Case 1 and  $100 - p$  percent belong to Case 2. Then the intensity is

$$I_3(\alpha) = \frac{p}{100} I_0 \left( \frac{\sin(\frac{\pi a}{\lambda} \sin \alpha)}{\frac{\pi a}{\lambda} \sin \alpha} \frac{\sin(\frac{n\pi(a+b)}{\lambda} \sin \alpha)}{\sin(\frac{\pi(a+b)}{\lambda} \sin \alpha)} \right)^2 + \frac{100 - p}{100} I_0 \left( \frac{\sin(\frac{\pi a}{\lambda} \sin \alpha)}{\frac{\pi a}{\lambda} \sin \alpha} \frac{\sin(\frac{n\pi 2(a+b)}{\lambda} \sin \alpha)}{\sin(\frac{\pi 2(a+b)}{\lambda} \sin \alpha)} \right)^2,$$

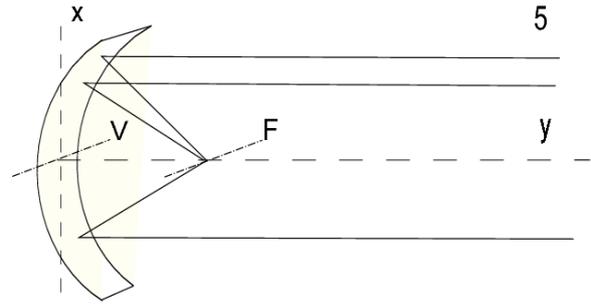


FIGURE 3. The paraxial beams 5 are reflected by the strip on focus line F, and V is the vertex line of the strip.

where  $I_0$  is the integral intensity of the light passed through the system.

### 3. TELESCOPE WITH A ROTATING OBJECTIVE ELEMENT

The parabolic strip telescope was patented [2] and published in [6].

The basic idea of our new system was inspired by X-ray computer tomography (CT). The integral absorptions of X-rays at different angles step by step during the rotation of the sample are measured. The total absorption of all photons coming along different lines perpendicular to the camera are registered as points of a one-dimensional picture. Finally, the inverse Radon transform is used to reconstruct the magnitude of the absorption of X-rays in different points of the media.

The same approach can be used when the parabolic strip is the primary mirror of a telescope, following the scheme shown in Figures 2 and 3. The images of the observed objects are the lines. Each line represents the integral intensity of the light incoming from the object or objects perpendicular to the strip (parallel to the focal line) located inside the field of view which is guaranteed by the geometry. Making a series of photos step by step during the rotation of the telescope around its optical axis, one can use the inverse Radon transform to reconstruct the image with the above-mentioned angular resolution. Fig. 5 shows a series of images of five illuminated circular apertures, after rotation of the telescope by 0, 45, 90 and 135 degrees.

The angular resolution of a parabolic strip telescope is

$$\delta_L = \frac{\lambda}{L},$$

where  $\lambda$  is the wavelength of the light,  $L$  is the length of the projection of the parabolic strip on the  $x$  axis.

The main advantages of such telescopes are:

- good angular resolution;
- low cost;



FIGURE 4. Two parabolic strips of lengths 20 and 40 cm; for the proof-of-principle experiment, the 40 cm strip was used.

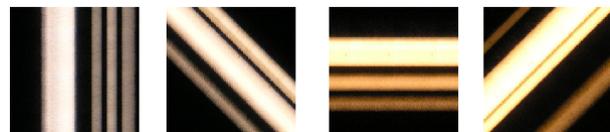


FIGURE 5. Series of images of the five illuminated circular objects after rotations the telescope by 0, 45, 90 and 135 degrees.

- simple technological development;
- possibility to install a grid of large telescopes across the Earth;
- lower weight for use on satellites.

The only major complication is that one more rotational movement is needed to reconstruct the image with the same angular resolution in all directions.

The good angular resolution can be used for direct observation of bright objects. Additional rotation is not necessary for some purposes, for example when the plane of rotation of the rotating objects is known, and changes of orbits are observed.

#### 4. OPTICAL ELEMENT FOR X-RAY MICROSCOPY

The optical element for X-ray microscopy was patented [3] and published in [7].

Our proposal exploits a single crystal to display monochromatic X-ray radiation with wavelength  $\lambda$ . Bragg's condition is guaranteed by applying stress to the single crystal.

Let us consider one dislocation-free single crystal strip with atomic planes oriented in parallel with the optical axis, which is the line connecting an imaging point with the center of its image, see 6c). The mutual distance of the atomic planes in a state of rest, i.e. without stress, is  $d_0$ . The cross section of the single crystal is variable, in order to use stress to change the distances of the atomic planes. With respect to

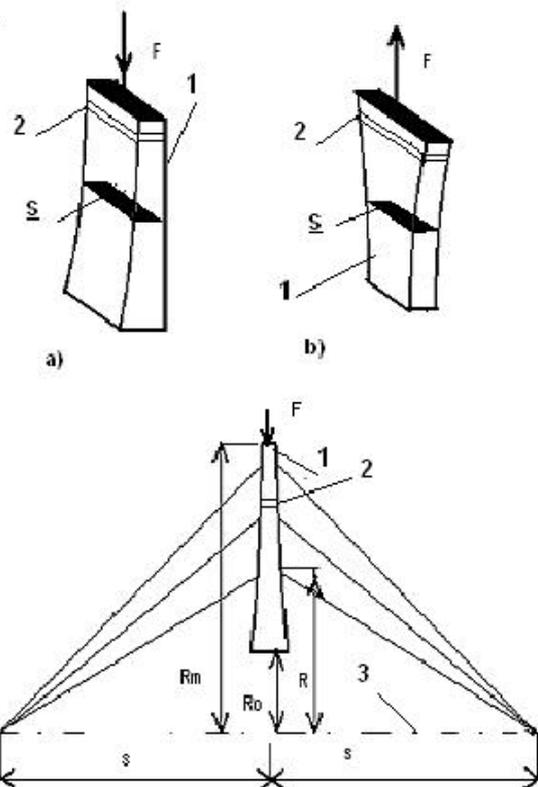


FIGURE 6. a) and b) show two examples of the possible shape of the single crystal 1, designed for shifting the atomic planes 2 by push and pull force  $F$ . c) shows an example of an optical set-up diffracting X-ray radiation with wavelength  $\lambda$ . It consists of a single crystal 1 with atomic planes 2 which are parallel to the optical axis 3. The mutual distance of the atomic planes 2 in resting state is  $d_0$  and the single crystal's 1 cross section  $S$  is variable. It is equipped with a push device to maintain a push force  $F$  in the direction orthogonal to the atomic planes 2 of this single crystal 1, located between distances  $R_0$  and  $R_m$ .

the optical axis, the farther or closer side of the single crystal, orthogonal to this optical axis, is equipped with a device to create a push 6a) or pull 6b) force and to maintain a pull or push force in the direction orthogonal to the atomic planes of this single crystal. We have to determine the cross section  $S$  of the single crystal at a distance  $R$  from optical axis.

For the constructive interference at the image plane it is necessary for the difference in the paths between two neighboring reflected rays an integer multiple of the wavelength. Since the planes are parallel, we will study the effect of spatial grating via Bragg's condition for X-ray diffraction. Bragg's condition of constructive interference is given for the distance between two atomic planes  $d$  by the expression

$$n\lambda = 2d \sin \alpha,$$

where  $n$  is an integer,  $\lambda$  is the wavelength of the X-ray, and  $\alpha$  is the angle between the ray and the optical

axis

$$\sin \alpha = \frac{R}{\sqrt{R^2 + s^2}}.$$

The distance  $d$  can be expressed via the distance between the atomic layers without any stress  $d_0$ , i. e.  $d = d_0 \pm \Delta d$  and Hooke's law gives

$$\frac{\Delta d}{d_0} = \frac{F}{ES},$$

where  $S$  is a cross section,  $F$  is the push or pull force, and  $E$  is the Young modulus of elasticity of the single crystal in the direction of the acting force. The single crystal's cross section is given by the following formulas

$$S = \frac{F}{E} \left( \frac{1}{\frac{n\lambda}{2Rd_0} \sqrt{R^2 + s^2} - 1} \right),$$

$$S = \frac{F}{E} \left( \frac{1}{1 - \frac{n\lambda}{2Rd_0} \sqrt{R^2 + s^2}} \right), \quad (4)$$

for the pull and push force, respectively. The cross section has to be positive in both cases, and is restricted by the limits of Hooke's law. The force  $F$  is defined by the equation

$$F = \pm S_0 E \left( \frac{n\lambda}{2R_0 d_0} \sqrt{R_0^2 + s^2} - 1 \right),$$

where  $s$ , with respect to the single crystal's longitudinal axis, is an object and also an image distance and  $S_0$  is a pre-selected cross section of a single crystal based on requirements of the application at a distance  $R_0$  from the optical axis,  $\lambda$  is the X-ray radiation wavelength and  $n$  is a natural number. In both the equations, the plus sign applies for the pull force, the minus sign applies for the push force. The cross section is then

$$S = S_0 \frac{\sqrt{R^2 + s^2} - 1}{\sqrt{R_0^2 + s^2} - 1}.$$

The calculated cross section  $S$  corresponds to a stress-free state, but  $R$  is the distance from the optical axis under stress. In order to calculate the shape of the strip for manufacturing, it is necessary to calculate the cross sections corresponding to the resting distances by the following procedure:

Express an inverse function  $R(x)$  of the function  $x(R)$

$$x = d_0 \frac{R - R_0}{\frac{n\lambda}{2rd_0} \sqrt{R^2 + s^2}}$$

( $x$  is the distance from  $R_0$ ). Then substitute  $R(x)$  in (4) and the cross section  $S$  is expressed as a function of  $x$  — the distance of the corresponding cross section from the end of the single crystal in a stress-free state. A correction according to the applicable Poisson ratio can be included. Numerical calculation methods are applicable. The maximal  $R_m$  is then given by the

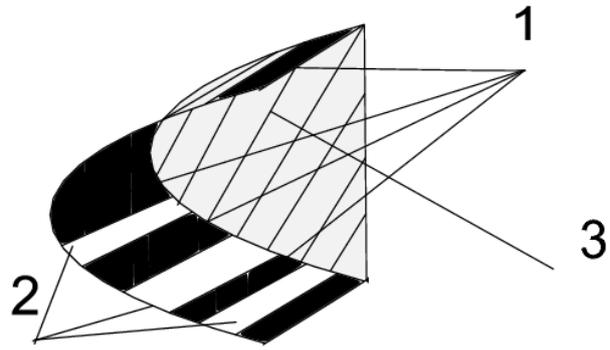


FIGURE 7. A parabolic cylinder consisting of bent single crystal strips of two types 1 and 2. Each strip has a different distance between the atomic planes parallel with its surface. The temperature at each point determines that the Bragg condition for X-ray reflection is satisfied there. Bent single crystal strips are fixed on the sides by brackets 3.

Young modulus of the crystal in the direction of the stress.

An advantage of this device is that it is relatively simple to manufacture its components. Another advantage is that it also works for shorter wavelengths, i.e. below 1 nm.

## 5. X-RAY BRAGG TELESCOPE

We have proposed a structure based on Bragg's X-ray reflection from single crystals arranged along a truncated parabolic cylinder. The basic idea presented in [7] and [6] is applied in telescopes. The X-ray reflector is composed of bent rectangular single crystal strips. The strips are cut so that the atomic planes are parallel to their surfaces, see Fig. 7. These bent single crystal strips are attached by their ends fixed in two parallel brackets, ensuring parabolic geometry of the assembly. The reflected incoming X-rays are focused roughly to a line.

The essence of the arrangement is that the bent strips form a parabolic cylinder. This geometry ensures that the incidence and reflection angles are equal at each point of the parabolic cylinder. The conditions for Bragg's reflection have to be guaranteed by the structure satisfying the following conditions:

- (1.) the cylinder is assembled from single crystal strips made from various materials and cut along different planes, so that the mutual distances between the atomic planes at one end of each strip fulfill Bragg's condition;
- (2.) the mutual distance between the atomic planes of each single crystal strip is changed by suitable thermal expansion at different places on the strip.

It is also possible to control the concentration of the impurities in order to change the mutual distances of the atomic planes.

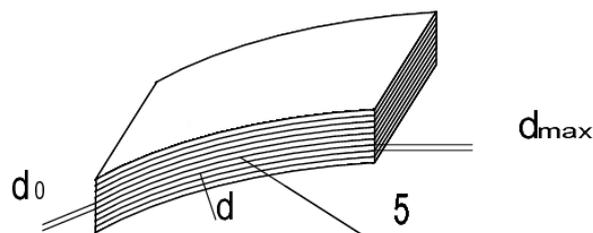


FIGURE 8. One bent single crystal strip displaying atomic planes 5 with mutual distance  $d_0$  at one end and  $d_{\max}$  at the opposite end.

The geometry is the same as in Fig. 3. The vertex line of a given parabolic cylinder goes through the origin ( $x = 0, y = 0, z$ ), and the focal line has the coordinates ( $x = 0, y = p/2, z$ ). The position of a strip can then be parameterized by its  $x$  coordinate. A strip is located between  $x_{\min}$  and  $x_{\max}$ . Each dislocation-free single crystal strip has atomic planes in parallel with its surface. Let the mutual distance of the atomic planes at temperature  $T$  be  $d$ . The Bragg condition is fulfilled at all points on the strip due to the changing mutual distance between the atomic planes parallel with the surface at different points.

The mutual distance of the atomic planes in different points of the strip is properly changed by the local changes in temperature. The smallest distance  $d_0$  at temperature  $T_{x_{\min}}$  is at the end of the strip closer to the vertex line of the parabolic cylinder  $x_{\min}$ , and the maximal distance between planes  $d_{\max}$  is at the opposite end, at  $x_{\max}$ . The mutual distance  $d$  of the atomic planes between the ends of the strip according to Bragg's condition depends on  $x$ ,

$$d = \frac{n\lambda}{2p} \sqrt{x^2 + p^2},$$

where  $n$  is a natural number indicating the order of Bragg's reflection from the two adjacent atomic planes, and  $p$  is a parameter of the parabola. The temperature along the strip at points in the direction of the  $x$  axis has to be a function of the  $x$ -coordinate

$$T(x) = T(x_{\min}) + \frac{1}{\gamma} \left( \frac{n\lambda}{2pd_0} \sqrt{x^2 + p^2} - 1 \right),$$

where  $\gamma$  is the coefficient of thermal expansion of the single crystal strip in the direction perpendicular to its surface.

For example, for a germanium single crystal strip cut along plane (100), for temperatures between  $T_{\min} = 250$  K and  $T_{\max} = 1000$  K, for wavelength of X-rays 0.5 nm the ratio is  $\frac{x_{\max}}{x_{\min}} = 1.023$ , and for wavelength 0.55 nm the ratio is  $\frac{x_{\max}}{x_{\min}} = 1.08$ , i.e. for example if  $x_{\min} = 10$  cm, then  $x_{\max} = 10.8$  cm. The ratio given by

$$\frac{x_{\max}^2}{x_{\min}^2} = \frac{4d_{\max}^2 - n^2\lambda^2}{4d_0^2 - n^2\lambda^2}$$

will be enhanced when the difference between  $2d_0$  and  $n\lambda$  is smaller. It is also possible to make very small changes in wavelength by a change in the temperature distribution on the surface of the single crystal strip.

## 6. CONCLUSION

We have proposed four optical elements, They are suitable for use in the examining of the interference properties of two beams, in optical astronomy, in X-ray microscopy, and in X-ray astronomy.

## ACKNOWLEDGEMENTS

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