

ACCRETION DISKS WITH A LARGE SCALE MAGNETIC FIELD AROUND BLACK HOLES

GENNADY BISNOVATYI-KOGAN^{a,b,*}, ALEXANDR S. KLEPNEV^{a,b},
RICHARD V.E. LOVELACE^c

^a Space Research Institute Rus. Acad. Sci., Moscow, Russia

^b Moscow Engineering Physics Institute, Moscow, Russia

^c Cornell University, Ithaca, USA

* corresponding author: gkogan@iki.rssi.ru

ABSTRACT. We consider accretion disks around black holes at high luminosity, and the problem of the formation of a large-scale magnetic field in such disks, taking into account the non-uniform vertical structure of the disk. The structure of advective accretion disks is investigated, and conditions for the formation of optically thin regions in central parts of the accretion disk are found. The high electrical conductivity of the outer layers of the disk prevents outward diffusion of the magnetic field. This implies a stationary state with a strong magnetic field in the inner parts of the accretion disk close to the black hole, and zero radial velocity at the surface of the disk. The problem of jet collimation by magneto-torsion oscillations is investigated.

KEYWORDS: accretion, black holes, jets, magnetic field.

1. INTRODUCTION

Quasars and AGN contain supermassive black holes, about 10 HMXR contain stellar mass black holes – microquasars. Jets are observed in objects with black holes: collimated ejection from accretion disks.

The standard model for accretion disks of Shakura and Sunyaev [20] is based on several simplifying assumptions. The disk must be geometrically thin and rotate at the Kepler angular velocity. These assumptions make it possible to neglect radial gradients and to proceed from differential to algebraic equations. For low accretion rates \dot{M} , this assumption is fully appropriate. However, for high accretion rates, the disk structure may differ from the standard model. To solve the more general problem, advection and a radial pressure gradient have been included in the analysis of the disk structure by Paczynski & Bisnovaty-Kogan [19]. It was shown by Artemova et al. [1], that for large accretion rates there are no local solutions that are continuous over the entire region of existence of the disk and undergo Kepler rotation. A self-consistent solution for an advective accretion disk with a continuous description of the entire region between the optically thin and optically thick regions has been obtained by Artemova et al. [3], and Klepnev and Bisnovaty-Kogan [13].

Early work on disk accretion to a black hole argued that a large-scale magnetic field of, for example, the interstellar medium would be dragged inward and greatly compressed by the accreting plasma [11, 12, 14]. Subsequently, analytic models of the field advection and diffusion in a turbulent disk suggested, that the large-scale field diffuses outward rapidly [15, 17], and prevents a significant amplifica-

tion of the external poloidal field. This has led to the suggestion that special conditions (non-axisymmetry) are required for the field to be advected inward [21]. The question of the advection/diffusion of a large-scale magnetic field in a turbulent plasma accretion disk was reconsidered by Bisnovaty-Kogan & Lovelace [6, 7], taking into account its non-uniform vertical structure. The high electrical conductivity of the surface layers of the disk, where the turbulence is suppressed by the radiation flux and the high magnetic field, prevents outward diffusion of the magnetic field. This leads to a strong magnetic field in the inner parts of accretion disks.

2. BASIC EQUATIONS FOR ACCRETION DISK STRUCTURE

We use equations describing a thin, steady-state accretion disk, averaged over its thickness [3]. These equations include advection and can be used for any value of the vertical optical thickness of the disk. We use a pseudo-newtonian approximation for the structure of the disk near the black hole, where the effects of the general theory of relativity are taken into account using the Paczyński & Wiita [18] potential $\Phi(r) = -\frac{GM}{r-2r_g}$, where M is the mass of the black hole, and $2r_g = 2GM/c^2$ is the gravitational radius. The self-gravitation of the disk is neglected, the viscosity tensor $t_{r\phi} = -\alpha P$. The conservation of mass is expressed in the form $\dot{M} = 4\pi r h \rho v$, where \dot{M} is the accretion rate, $\dot{M} > 0$, and h is the half thickness of the disk. The equilibrium in the vertical direction $\frac{dP}{dz} = -\rho z \Omega_K^2$ is replaced by the algebraic relation in the form $h = \frac{c_s}{\Omega_K}$, where $c_s = \sqrt{P/\rho}$ is the isothermal

sound speed. The equations of motion in the radial and azimuthal directions are, respectively, written as

$$v \frac{dv}{dr} = -\frac{1}{\rho} \frac{dP}{dr} + (\Omega^2 - \Omega_K^2)r, \quad (1)$$

$$\frac{\dot{M}}{4\pi} \frac{d\ell}{dr} + \frac{d}{dr}(r^2 h t_{r\phi}) = 0,$$

where Ω_K is the Kepler angular velocity, given by $\Omega_K^2 = GM/r(r-2r_g)^2$; $\ell = \Omega r^2$ is the specific angular momentum. Other components of the viscosity tensor are assumed negligibly small. The vertically averaged equation for the energy balance is $Q_{\text{adv}} = Q^+ - Q^-$, where

$$Q_{\text{adv}} = -\frac{\dot{M}}{4\pi r} \left[\frac{dE}{dr} + P \frac{d}{dr} \left(\frac{1}{\rho} \right) \right],$$

$$Q^+ = -\frac{\dot{M}}{4\pi} r \Omega \frac{d\Omega}{dr} \left(1 - \frac{l_{\text{in}}}{l} \right), \quad (2)$$

$$Q^- = \frac{2aT^4 c}{3(\tau_\alpha + \tau_0)h} \left[1 + \frac{4}{3(\tau_0 + \tau_\alpha)} + \frac{2}{3\tau_*^2} \right]^{-1}$$

are the energy fluxes ($\text{erg cm}^{-2} \text{s}^{-1}$) associated with advection, viscous dissipation, and radiation from the surface, respectively, τ_0 is the Thomson optical depth, and $\tau_0 = 0.4\rho h$ for the hydrogen composition. We have introduced the optical thickness for absorption, $\tau_\alpha \simeq 5.2 \times 10^{21} \frac{\rho^2 T^{1/2} h}{acT^4}$, and the effective optical thickness $\tau_* = [(\tau_0 + \tau_\alpha)\tau_\alpha]^{1/2}$. The equation of state for a mixture of a matter and radiation is $P_{\text{tot}} = P_{\text{gas}} + P_{\text{rad}}$. The gas pressure is given by the formula $P_{\text{gas}} = \rho RT$, R is the gas constant, and the radiation pressure is given by

$$P_{\text{rad}} = \frac{aT^4}{3} \frac{1 + \frac{4}{3(\tau_0 + \tau_\alpha)}}{1 + \frac{4}{3(\tau_0 + \tau_\alpha)} + \frac{2}{3\tau_*^2}}. \quad (3)$$

The specific energy of the mixture of the matter and radiation is determined as $\rho E = \frac{3}{2}P_{\text{gas}} + 3P_{\text{rad}}$. Expressions for Q^- and P_{rad} , valid for any optical thickness, were obtained by Artemova et al. [1].

3. METHOD OF SOLUTION AND NUMERICAL RESULTS

The system of differential and algebraic equations can be reduced to two ordinary differential equations,

$$\frac{x}{v} \frac{dv}{dx} = \frac{N}{D}, \quad (4)$$

$$\frac{x}{v} \frac{dc_s}{dx} = 1 - \left(\frac{v^2}{c_s^2} - 1 \right) \frac{N}{D} + \frac{x^2}{c_s^2} \left(\Omega^2 - \frac{1}{x(x-2)^2} \right) + \frac{3x-2}{2(x-2)}. \quad (5)$$

Here the numerator N and denominator D are algebraic expressions depending on x , v , c_s , and l_{in} , the equations are written in dimensionless form with

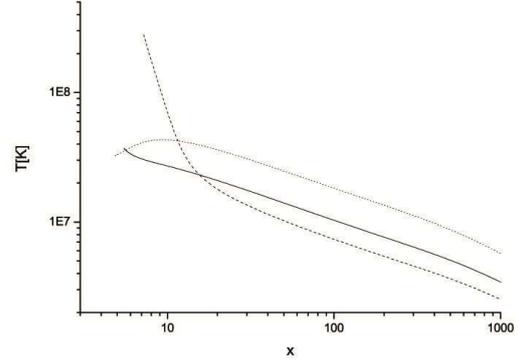


FIGURE 1. The radial dependence of the temperature of the accretion disk for an accretion rate $\dot{m} = 50$, and viscosity parameters $\alpha = 0.01$ (dotted curve), $\alpha = 0.1$ (smooth curve), and $\alpha = 0.4$ (dashed curve).

$x = r/r_g$, $r_g = GM/c^2$. The velocities v and c_s have been scaled by the speed of light c , and the specific angular momentum l_{in} by the value c/r_g . This system of differential equations has two singular points, defined by the conditions $D = 0$, $N = 0$. The inner singularity is situated near the last stable orbit with $r = 6r_g$. The outer singularity, lying at distances much greater than r_g , is an artifact arising from our use of the artificial parametrization $t_{r\phi} = -\alpha P$ of the viscosity tensor. The system of ordinary differential equations was solved by a finite difference method discussed by Artemova et al. [2]. The method is based on reducing the system of differential equations to a system of nonlinear algebraic equations which are solved by an iterative Newton-Raphson scheme, with an expansion of the solution near the inner singularity and using l_{in} as an independent variable in the iterative scheme [2]. The solution is almost independent of the outer boundary condition. The numerical solutions have been obtained for the structure of an accretion disk over a wide range of the parameters \dot{m} ($\dot{m} = \frac{Mc^2}{L_{\text{EDD}}}$) and α . For low accretion rates, $\dot{m} < 0.1$, the solution for the advection model has $\tau_* \gg 1$, $v \ll c_s$, and the angular velocity is close to the Kepler velocity everywhere, except a very thin layer near the inner boundary of the disk. As the accretion rate increases, the situation changes significantly. The changes show up primarily in the inner region of the disk. The calculations made by Klepnev & Bisnovaty-Kogan [8] are presented in Fig. 1, where there are given the radial dependences of the temperature of the accretion disk for the accretion rate $\dot{m} = 50$, and different values of the viscosity parameter $\alpha = 0.01$, 0.1 and 0.4 . Clearly, for large \dot{m} and α the inner part of the disk becomes optically thin. Because of this, a sharp increase in the temperature of the accretion disk is observed in this region.

Two distinct regions can be seen in the plot of the radial dependence of the temperature of the accretion

disk. This is especially noticeable for a viscosity parameter $\alpha = 0.4$, where one can see the inner optically thin region with a dominant non-equilibrium radiation pressure P_{rad} , and an outer region which is optically thick with dominant equilibrium radiation pressure. Things are different when the viscosity parameter is small. Only a small (considerably smaller than for $\alpha = 0.4$) inner region becomes optically thin for accretion rates of $\dot{m} \approx 30 \div 70$. Meanwhile, in the case of $\alpha = 0.01$, there are no optically thin regions at all.

4. THE FULLY TURBULENT MODEL

There are two limiting accretion disk models which have analytic solutions for a large-scale magnetic field structure. The first was constructed by Bisnovatyi-Kogan & Ruzmaikin [12] for a stationary non-rotating accretion disk. A stationary state is maintained by the balance between magnetic and gravitational forces, and a local thermal balance is maintained by Ohmic heating and radiative heat conductivity for optically thick conditions. The mass flux to the black hole in the accretion disk is determined by the finite electrical conductivity of the disk matter and the diffusion of matter across the large-scale magnetic field. It is widely accepted that the laminar disk is unstable to different hydrodynamic, magneto-hydrodynamic and plasma instabilities, which implies that the disk is turbulent. In X-ray binary systems the assumption of a turbulent accretion disk is necessary for construction of realistic models [20]. The turbulent accretion disks were constructed for non-rotating models with a large-scale magnetic field. A formula for turbulent magnetic diffusivity was derived by Bisnovatyi-Kogan and Ruzmaikin [12], similar to the scaling of the shear α -viscosity in a turbulent accretion disk in binaries [20]. Using this representation, the expression for the turbulent electrical conductivity σ_t is written as

$$\sigma_t = \frac{c^2}{\tilde{\alpha} 4\pi h \sqrt{P/\rho}}. \quad (6)$$

Here, $\tilde{\alpha} = \alpha_1 \alpha_2$. The characteristic turbulence scale is $\ell = \alpha_1 h$, where h is the half-thickness of the disk, and the characteristic turbulent velocity is $v_t = \alpha_2 \sqrt{P/\rho}$. The large-scale magnetic field threading a turbulent Keplerian disk arises from external electrical currents and currents in the accretion disk. The magnetic field may become dynamically important, influencing the accretion disk structure, and leading to powerful jet formation, if it is strongly amplified during the radial inflow of the disk matter. This is possible only when the radial accretion speed of matter in the disk is larger than the outward diffusion speed of the poloidal magnetic field due to the turbulent diffusivity $\eta_t = c^2/(4\pi\sigma_t)$. Estimates by Lubow, Papaloizou & Pringle [17] have shown that for turbulent conductivity (Eq. 6), the outward diffusion speed is larger than the accretion speed, and there is no large-scale magnetic field amplification. The numerical

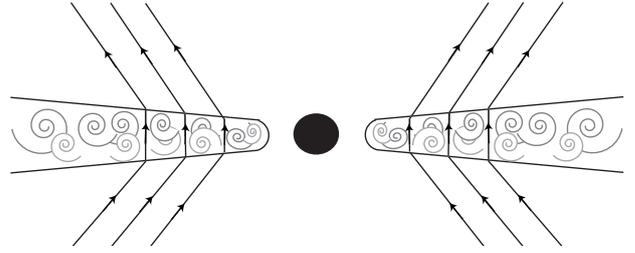


FIGURE 2. Sketch of the large-scale poloidal magnetic field threading a rotating turbulent accretion disk with a radiative outer boundary layer. The toroidal current flows mainly in the highly conductive radiative layers. The large-scale (average) field in the turbulent region is almost vertical.

calculations of Lubow, Papaloizou & Pringle [17] are reproduced analytically for the standard accretion disk structure by Bisnovatyi-Kogan & Lovelace [6, 7]. The characteristic time t_{visc} of the matter advection due to the shear viscosity is $t_{\text{visc}} = \frac{r}{v_r} = \frac{j}{\alpha v_s^2}$. The time of the magnetic field diffusion is $t_{\text{diff}} = \frac{r^2}{\eta} \frac{h}{r} \frac{B_z}{B_r}$, $\eta = \frac{c^2}{4\pi\sigma_t} = \tilde{\alpha} h v_s$. In the stationary state, the large-scale magnetic field in the accretion disk is determined by the equality $t_{\text{vis}} = t_{\text{diff}}$, which determines the ratio $\frac{B_r}{B_z} = \frac{\alpha}{\tilde{\alpha}} \frac{v_s}{v_K} = \frac{\alpha}{\tilde{\alpha}} \frac{h}{r} \ll 1$, $v_K = r\Omega_K$ and $j = rv_K$ for a Keplerian disk. In a turbulent disk, matter penetrates through magnetic field lines, almost without field amplification: the field induced by the azimuthal disk currents has $B_{zd} \sim B_{rd}$.

5. TURBULENT DISK WITH RADIATIVE OUTER ZONES

Near the surface of the disk, in the region of low optical depth, the turbulent motion is suppressed by the radiative and magnetic fluxes, similar to the suppression of the convection over the photospheres of stars with outer convective zones. The presence of the outer radiative layer does not affect the characteristic time t_{visc} of the matter advection in the accretion disk, determined by the main turbulent part of the disk. The time of the field diffusion, however, is significantly changed, because the electrical current is concentrated in the radiative highly conductive regions, which generate the main part of the magnetic field.

The structure of the magnetic field with outer radiative layers is shown in Fig. 2.

Inside the turbulent disk the electrical current is negligibly small, so that the magnetic field there is almost fully vertical, with $B_r \ll B_z$. In the outer radiative layer, the field diffusion is very small, so that the matter advection leads to strong magnetic field amplification. We suppose that in the stationary state the magnetic forces support the optically thin regions against gravity. When the magnetic force balances the gravitational force in the optically thin part of the disk of surface density Σ_{ph} , the relation

takes place [12]

$$\frac{GM\Sigma_{\text{ph}}}{r^2} \simeq \frac{B_z I_\phi}{2c} \simeq \frac{B_z^2}{4\pi}, \quad (7)$$

The surface density over the photosphere corresponds to a layer with effective optical depth close to 2/3 (see e.g. [5]). We estimate the lower limit of the magnetic field strength, taking κ_{es} (instead of the effective opacity $\kappa_{\text{eff}} = \sqrt{\kappa_{\text{es}}\kappa_{\text{a}}}$, $\kappa_{\text{a}} \ll \kappa_{\text{es}}$). Writing $\kappa_{\text{es}}\Sigma_{\text{ph}} = 2/3$, we obtain $\Sigma_{\text{ph}} = 5/3$ (g cm^{-2}), for the Thomson scattering opacity, $\kappa_{\text{es}} = 0.4 \text{ cm}^2 \text{ g}^{-1}$. We estimate the lower bound on the large-scale magnetic field in a Keplerian accretion disk as [6, 7]

$$B_z = \sqrt{\frac{5\pi}{3}} \frac{c^2}{\sqrt{GM_\odot}} \frac{1}{x\sqrt{m}} \simeq 10^8 \text{ G} \frac{1}{x\sqrt{m}}. \quad (8)$$

Here $x = \frac{r}{r_g}$, $m = \frac{M}{M_\odot}$. The maximum magnetic field is reached when the outward magnetic force balances the gravitational force on the surface with a mass density Σ_{ph} . In equilibrium, $B_z \sim \sqrt{\Sigma_{\text{ph}}}$. We find that B_z in a Keplerian accretion disk is about 20 times less than its maximum possible value from Bisnovaty-Kogan & Ruzmaikin [12], for $x = 10$, $\alpha = 0.1$, and $\dot{m} = 10$.

6. SELF-CONSISTENT NUMERICAL MODEL

Self-consistent models of the rotating accretion disks with a large-scale magnetic field require solution of the equations of magneto-hydrodynamics. The strong field solution is the only stable stationary solution for a rotating accretion disk. The vertical structure of the disk with a large-scale poloidal magnetic field was calculated by Lovelace, Rothstein & Bisnovaty-Kogan [16], taking into account the turbulent viscosity and diffusivity, and the fact that the turbulence vanishes at the surface of the disk. Coefficients of the turbulent viscosity ν , and magnetic diffusivity η are connected by the magnetic Prandtl number $P \sim 1$, $\nu = P\eta = \alpha \frac{c_{s0}^2}{\Omega_K} g(z)$, where α is a constant determining the turbulent viscosity [20]; $\beta = c_{s0}^2/v_{A0}^2$, where $v_{A0} = B_0/(4\pi\rho_0)^{1/2}$ is the midplane Alfvén velocity. The function $g(z)$ accounts for the absence of turbulence in the surface layer of the disk. In the body of the disk $g = 1$, whereas near the surface of the disk g tends over a short distance to a very small value, effectively zero. Smooth function with similar behavior is taken by Lovelace, Rothstein & Bisnovaty-Kogan [16] in the form $g(\zeta) = \left(1 - \frac{\zeta^2}{\zeta_s^2}\right)^\delta$, with $\delta \ll 1$.

In the stationary state the boundary condition on the disk surface is $u_r = 0$, and only one free parameter – magnetic Prandtl number P – remains in the problem. In a stationary disk, the vertical magnetic field has a unique value. An example of the radial velocity distribution for $P = 1$ is shown in Fig. 3 from Bisnovaty-Kogan & Lovelace [8, 9].

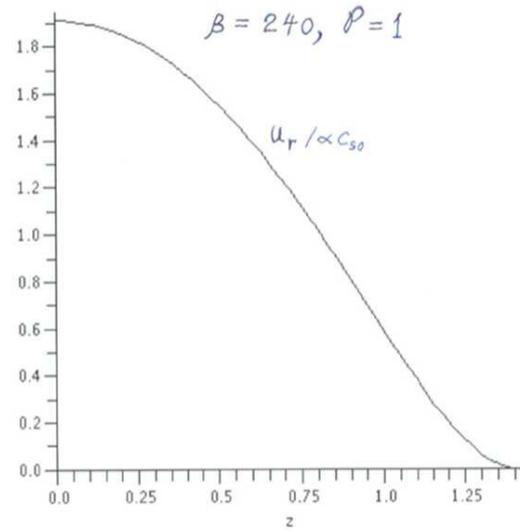


FIGURE 3. Distribution of the radial velocity over the thickness in the stationary accretion disk with a large scale poloidal magnetic field.

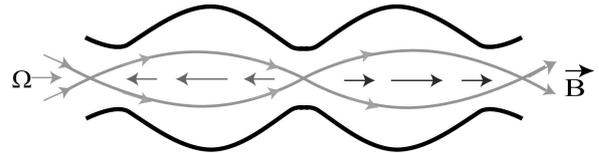


FIGURE 4. Qualitative picture of jet confinement by magneto-torsional oscillations.

7. JET COLLIMATION BY MAGNETO-TORSIONAL OSCILLATIONS.

Following Bisnovaty-Kogan [6, 7], we consider the stabilization of a jet by a magneto-hydrodynamic mechanism associated with torsional oscillations. We suggest that the matter in the jet is rotating, and different parts of the jet rotate in different directions, see Fig. 4. Such a distribution of the rotational velocity produces an azimuthal magnetic field, which prevents a disruption of the jet. The jet represents a periodical, or quasi-periodical, structure along the axis, and its radius oscillates with time all along the axis. The space and time periods of the oscillations depend on the conditions at jet formation: the length-scale, the amplitude of the rotational velocity, and the strength of the magnetic field. The time period of the oscillations can be obtained during the construction of the dynamical model, and the model should also show at what input parameters a long jet stabilized by torsional oscillations could exist.

Let us consider a long cylinder with a magnetic field directed along its axis. It is possible that a limiting value of the radius of the cylinder could be reached in a dynamic state, when the whole cylinder undergoes

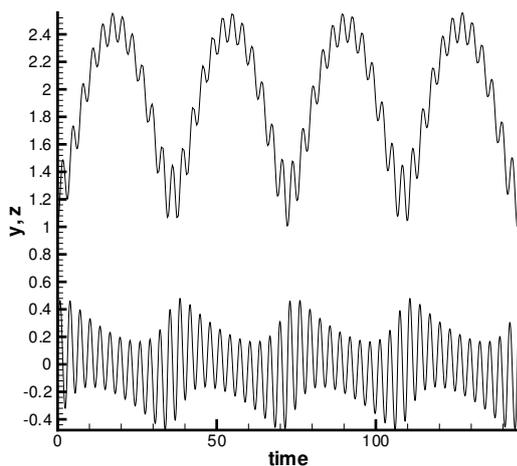


FIGURE 5. Time dependence of non-dimensional radius y (upper curve), and non-dimensional velocity z (lower curve), for $D = 2.1$, $y(0) = 1$.

magneto-torsional oscillations. Such oscillations produce a toroidal field, which prevents radial expansion. There is competition between the induced toroidal field, compressing the cylinder in the radial direction, and the gas pressure, together with the field along the cylinder axis (poloidal), tending to increase its radius. During magneto-torsional oscillations there are phases when either the compression force or the expansion force prevails, and, depending on the input parameters, there are three possible kinds of behavior of such a cylinder with negligible self-gravity.

- (1.) The oscillation amplitude is low, so the cylinder suffers unlimited expansion (no confinement).
- (2.) The oscillation amplitude is high, so the pinch action of the toroidal field destroys the cylinder and leads to the formation of separated blobs.
- (3.) The oscillation amplitude is moderate, so the cylinder, in the absence of any damping, survives for an unlimited time, and its parameters (radius, density, magnetic field etc.) change periodically, or quasi-periodically, in time.

A simplified equation describing the magneto-torsional oscillations of a long cylinder was obtained by Bisnovatyi-Kogan [6, 7].

It describes approximately the time dependence of the outer radius of the cylinder $R(t)$ in the symmetry plane, where the rotational velocity remains zero.

The equation contains a dimensionless parameter D , which determines the dynamic behavior of the cylinder. An example of the dynamically stabilized cylinder at $D = 2.1$ is given in Fig. 5, from Bisnovatyi-Kogan [6, 7], y and z are the non-dimensional radius and the radial velocity, respectively. The transition to a stochastic regime in these oscillations was investigated by Bisnovatyi-Kogan et al. [10].

8. DISCUSSION

We have obtained an unambiguous solution for the structure of an advection accretion disk surrounding a nonrotating black hole for different values of the viscosity parameter and the accretion rate. This solution is global, trans-sonic, and, for high \dot{m} and α , is characterized by a continuous transition of the disk from optically thick in the outer region to optically thin in the inner region. It has a temperature peak in the inner (optically thin) region, which might cause the appearance of a hard component in the spectrum.

For a rotating black hole, the peak temperature is so high that it may lead to the formation of electron-positron pairs and change the emission spectrum of the disk at energies of 500 keV and above. Preliminary calculations have been made for a disk around a rapidly rotating black hole, with quasi-newtonian gravitational potential, approximating the effects of the Kerr metric [4]. We obtain that, for a sufficiently large Kerr rotation parameter, the temperature in the optically thin inner region may substantially exceed 500 keV. A consideration with a self-consistent account of pair creation is under way. In the presence of a large scale magnetic field we may expect the formation of relativistic jets with a high lepton excess.

The inner optically thin region may exist only at $\alpha > \sim 0.01$. This is because at very high \dot{m} large optical thickness is associated with high density in the inner regions of the disk; at low \dot{m} large effective optical depth is connected with high density because of low temperature. Therefore, the effective optical depth has a minimum at intermediate values of \dot{m} , and for $\alpha \leq \sim 0.01$ this minimum turns out to be greater than unity.

The poloidal magnetic field is amplified during disk accretion, due to high conductivity in the outer radiative layers. A stationary solution is obtained corresponding to $\beta = 240$, for $Pr = 1$. Note that the value of β is obtained using the density of the disk in the symmetry plane. The local value of β in the outer radiative regions is much lower, and approximately corresponds to equipartition between the pressure of a gas and the magnetic field.

9. CONCLUSIONS

- (1.) A global, trans-sonic solution exists, which at high \dot{m} and α is characterized by a continuous transition of the disk from optically thick in the outer region to optically thin in the inner region.
- (2.) The model, with correct accounting for the transition between the optically thick and optically thin regions, reveals the existence of a temperature peak in the inner (optically thin) region, which may cause the appearance of a hard component in the spectrum. A high temperature in the inner region of an accretion disk may lead to the formation of electron-positron pairs (in the Kerr metric).

- (3.) When $\alpha = 0.5$, a very substantial optically thin region is observed, when $\alpha = 0.1$ we have a slight optically thin region, and when $\alpha = 0.01$ no optically thin region is seen at all.
- (4.) The magnetic field is amplified during disk accretion due to high conductivity in the outer radiative layers. The stationary solution corresponds to $\beta = 240$ for $Pr = 1$.
- (5.) The jets from the accretion disk are magnetically collimated in the presence of a large-scale poloidal magnetic field, by torsion oscillations, which may be regular or chaotic. Jets may be produced in magneto-rotational explosions (supernova, etc.).

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