

COMPUTATIONAL SCHEMES FOR THE PROPAGATION OF ULTRA HIGH ENERGY COSMIC RAYS

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ABSTRACT. We discuss the problem of ultra high energy particles propagation in astrophysical backgrounds. We present two different computational schemes based on kinetic and Monte Carlo approaches. The kinetic approach is an analytical computation scheme based on the hypothesis of continuous energy losses while the Monte Carlo scheme takes into account also the stochastic nature of particle interactions. These schemes, which give quite reliable results, enable the computation of fluxes keeping track of the different primary and secondary components, providing a fast and useful workbench for studying Ultra High Energy Cosmic Rays.

KEYWORDS: particles astrophysics, ultra high energy cosmic rays, astrophysical backgrounds.

1. INTRODUCTION

Ultra High Energy Cosmic Rays (UHECR) are the most energetic particles observed in nature, with energies up to a several 10^{20} eV. Experimental studies of UHECR are currently being conducted in three different experiments: Auger in Argentina, HiRes and Telescope Array in the USA.

The propagation of UHECR from the source to the observer is conditioned by their interactions with astrophysical backgrounds: the Cosmic Microwave Background (CMB) and the Extragalactic Background Light (EBL). Understanding the key features of propagation is of paramount importance for interpreting experimental observations paving the way for the discovery of the astrophysical origin of these fascinating particles.

Several features of the observed spectrum can be linked directly to the chemical composition of UHECR and to their sources [1, 2, 7, 9, 13]. One of the particularly important features is the Greisin, Zatsepin and Kuzmin (GZK) suppression of the flux, an abrupt depletion of the observed proton spectrum, arising at energies $E \simeq 5 \times 10^{19}$ eV, due to the interaction of UHE protons with the CMB radiation field [9, 13]. GZK suppression, as follows from the original papers, refers to protons and it is due to the pion production process on the CMB radiation field ($p + \gamma_{\text{CMB}} \rightarrow \pi + p$). In the case of nuclei the expected flux also shows a suppression at the highest energies which, depending on the nucleus species, is due to the photo-disintegration process on the CMB and EBL radiation fields ($A + \gamma_{\text{CMB,EBL}} \rightarrow (A - nN) + nN$) [3]. Another important feature in the spectrum that can be directly linked with the nature of the primary particles and their origin (galactic/extra-galactic) is the pair-production dip [1, 7]. This feature is present only in the spectrum of UHE extragalactic protons and,

like GZK, is a direct consequence of the interaction of the proton with the CMB radiation field. In particular the dip brings a direct imprint of the pair production process $p + \gamma_{\text{CMB}} \rightarrow p + e^+ + e^-$ suffered by protons.

From the experimental point of view the situation is far from being clear with different experiments claiming contradictory results. The HiRes experiment, which is no longer taking data, showed a proton dominated spectrum till the highest energies [10, 11] while the Auger observations show a heavy mass composition at energies $E > 4 \times 10^{18}$ eV [5].

This puzzling situation, with different experiments favoring different scenarios, shows once more the importance of a systematic study of UHECR propagation in astrophysical backgrounds. In the present paper we will review the main points of two alternative computation schemes which enable the determination of the fluxes expected on earth fixing the injection spectrum and the distribution of sources. These two schemes are based on different approaches to modeling the interactions between particles and backgrounds: the continuum energy losses (CEL) approximation, which forms the basis of the kinetic approach, and the Monte Carlo (MC) technique.

As we will discuss in the following these two different schemes give reliable results that, in the framework of different assumptions, agree each other and offer a suitable theoretical framework to study experimental results unveiling the intimate nature of UHECR.

2. KINETIC EQUATIONS

The main assumption under which the kinetic theory is built is the CEL approximation [6], through which particle interactions are treated as a continuum process that continuously depletes the energy of the particles.

UHECR propagating through astrophysical backgrounds suffer different interaction processes:

- *protons* – UHE protons interact only with the CMB radiation field giving rise to the two processes of pair production and photo-pion production. Both of these reactions can be treated in the CEL hypothesis.
- *nuclei* – UHE nuclei interact with the CMB and EBL radiation fields, suffering the process of pair production, for which only CMB is relevant, and photo-disintegration, which involves both CMB and EBL backgrounds. While the first process can be treated in the CEL hypothesis, the nucleus species being conserved, the second cannot be, producing a change in the nucleus species. Following Aloisio et al. [3], in the framework of the kinetic approach, we will treat the photo-disintegration process as a “decaying” process that simply depletes the flux of the propagating nucleus.

Taking into account all energy loss processes we can describe the propagation of protons and nuclei through kinetic equations of the type:

$$\frac{\partial n_p(\Gamma, t)}{\partial t} - \frac{\partial}{\partial \Gamma} [b_p(\Gamma, t)n_p(\Gamma, t)] = Q_p(\Gamma, t) \quad (1)$$

$$\frac{\partial n_A(\Gamma, t)}{\partial t} - \frac{\partial}{\partial \Gamma} [n_A(\Gamma, t)b_A(\Gamma, t)] + \frac{n_A(\Gamma, t)}{\tau_A(\Gamma, t)} = Q_A(\Gamma, t) \quad (2)$$

where n is the equilibrium distribution of particles, b are the energy losses (adiabatic expansion of the Universe and pair/photo-pion production for protons or only pair-production for nuclei) Q is the injection of freshly accelerated particles and, in the case of nuclei, also the injection of secondary particles produced by photo-disintegration (see below).

The energy losses b for protons or nuclei depend only on the CMB field and in the CEL hypothesis they can be computed analytically [1, 3, 7].

The second process that affects nuclei propagation is photo-disintegration over CMB and EBL backgrounds. This process is treated as a decaying process that depletes the flux of nuclei. It enters in the kinetic equation (see Eq. 2) through a sort of “life-time” of the nucleus under the photo-disintegration process. This “life-time” corresponds to the mean time needed for a nucleus of Lorentz factor Γ and atomic mass number A to lose, at least, one of its nucleons:

$$\frac{1}{\tau_A} = \frac{c}{2\Gamma^2} \int_{\epsilon_0(A)}^{\infty} d\epsilon_r \sigma(\epsilon_r, A) \nu(\epsilon_r) \epsilon_r \int_{\epsilon_r/(2\Gamma)}^{\infty} d\epsilon \frac{n_{\text{bkg}}(\epsilon)}{\epsilon^2} \quad (3)$$

where $\sigma(\epsilon_r, A)$ is the photo-disintegration cross-section and $\nu(\epsilon_r)$ is the multiplicity associated with this process, namely the average number of nucleons extracted from the nucleus by a single interaction and $n_{\text{bkg}} = n_{\text{CMB}} + n_{\text{EBL}}$. The dependence on red-shift of τ_A follows directly from the evolution with red-shift of the background photon densities n_{CMB} and n_{EBL} .

In the case of CMB this dependence is known analytically while for EBL one should refer to evolution models (in our computations we have used the model by Stecker et al. [12]).

One important feature of the photo-disintegration process is that it starts to contribute to the propagation of nuclei at a Lorentz factor that is almost independent of the nuclei species $\Gamma_{\text{cr}} \simeq 2 \times 10^9$ [3]. This is an important general characteristic of nuclei photo-disintegration process from which we can immediately deduce the dependence on the nuclei species of the energy corresponding to the photo-disintegration suppression of the flux: $E_{\text{cut}}^A = Am_N \Gamma_{\text{cr}}$. A being the atomic mass number of the nucleus and m_N the proton mass. From this expression for E_{cut}^A it is evident how the flux behavior could provide informations on the chemical composition of UHECR. In the case of Helium ($A = 4$), suppression is expected around energies $E \simeq 10^{19}$ eV while in the case of Iron ($A = 56$) suppression is expected at higher energies $E \simeq 10^{20}$ eV.

Let us now discuss the generation function $Q_A(\Gamma, t)$ on the right hand side of Eq. 2. One should distinguish between primary nuclei, i.e. nuclei accelerated at the source and injected in the intergalactic space, and secondary nuclei and nucleons, i.e. particles produced as secondaries in the photo-disintegration chain. In the case of primaries the injection function is an assumption of the source model, while the injection of secondaries should be modeled taking into account the characteristics of the photo-disintegration process. The dominant process of photo-disintegration is one nucleon (N) emission, namely the process $(A + 1) + \gamma_{\text{bkg}} \rightarrow A + N$. This follows directly from the behavior of the photo-disintegration cross-section (see [3, and references therein]) which shows the giant dipole resonance corresponding to one nucleon emission. Moreover, at the typical energies of UHECR ($E > 10^{17}$ eV) one can safely neglect the nucleus recoil so that photo-disintegration will conserve the Lorentz factor of the particles. The production rate of secondary A -nucleus and A -associated nucleons will therefore be given by

$$Q_A(\Gamma, z) = Q_p^A(\Gamma, z) = \frac{n_{A+1}(\Gamma, z)}{\tau_{A+1}(\Gamma, z)} \quad (4)$$

where τ_{A+1} is the photo-disintegration life-time of the nucleus father ($A + 1$) and n_{A+1} is its equilibrium distribution, the solution of the kinetic equation (Eq. 2).

Using Eq. 4 we can build a system of coupled differential equations that starting from the primary injected nuclei (A_0) follows the complete photo-disintegration chain for all secondary nuclei ($A < A_0$) and nucleons. Clearly secondary proton¹ propagation will be described by the proper kinetic equation (Eq. 1) with an injection term given by Eq. 4. The solution

¹Neutrons decay very fast into protons, so we will always refer to secondary protons.

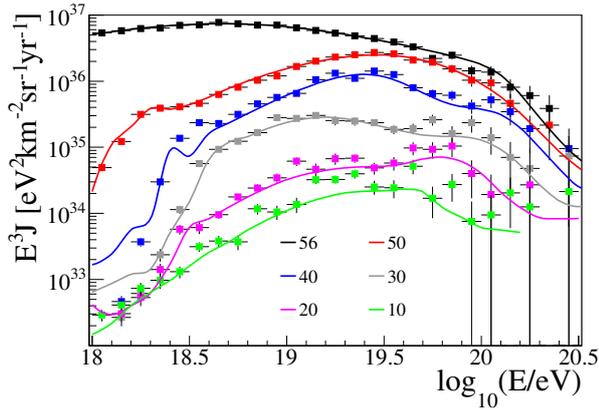


FIGURE 1. Flux of iron and secondary nuclei ($A = 50, 40, 30, 20, 10$) at $z = 0$ in the case of pure iron injection at the source with a power law injection index $\gamma = 2.2$. Full squares correspond to the *SimProp* result [4] while continuous lines correspond to the solution of the nuclei kinetic equation of [3].

of the kinetic equation for protons and nuclei can be worked out analytically. In the case of protons:

$$n_p(\Gamma, z) = \int_z^{z_{\max}} \frac{dz'}{(1+z')H(z')} Q_p(\Gamma', z) \frac{d\Gamma'}{d\Gamma}, \quad (5)$$

being Q_p the injection of primary protons or secondary protons (Eq. 4) and $\Gamma' = \Gamma'(z)$ is the characteristic function of the kinetic equation [3]. In the case of nuclei:

$$\begin{aligned} n_A(\Gamma, z) &= \\ &= \int_z^{z_{\max}} \frac{dz'}{(1+z')H(z')} Q_A(\Gamma', z) \frac{d\Gamma'}{d\Gamma} e^{-\eta_A(\Gamma', z)}, \end{aligned} \quad (6)$$

being, again, Q_A the injection of primary or secondary (Eq. 4) nuclei. The exponential term in Eq. 6 represents the survival probability during the propagation time $t' - t$ for a nucleus with fixed A and can be computed according to Aloisio et al. [3]. The derivative term $d\Gamma'/d\Gamma$ present in both solutions Eq. 5 and Eq. 6 is analytically given [3].

3. MONTE CARLO

The kinetic approach outlined above neglects interactions fluctuations considering an (average) continuum loss of energy suffered by particles. In the case of protons, this approximation has a limited effect on the flux computation only at the highest energies ($E > 100$ EeV) [1, 7, 8].

In order to evaluate the effects of fluctuations on the expected nuclei flux, we have built a computation scheme alternative to the kinetic one, which uses the MC technique to simulate nuclei interactions. First of all, let us remark that fluctuations could be relevant only in the case of nuclei photo-disintegration. This follows from the fact that the pair-production process involving nuclei can be considered as an interaction

process of the inside nucleon, therefore fluctuations in proton pair-production are irrelevant [8], and the same holds for nuclei. The *SimProp* MC simulation scheme that we have developed [4] is mono-dimensional: it does not take into account spatial distributions tagging sources only through their distance from the observer (red-shift). The MC simulation propagates particles in steps of red-shift following the injected nucleus, the secondary nuclei and protons produced at each photo-disintegration interaction and calculates their losses up to the observer, placed at red shift zero. The nuclear model on which *SimProp* is based is the same as is used for the kinetic approach (see [3, 4, and references therein]). The stochastic nature of the nuclei photo-disintegration process is modeled through the survival probability of a nucleus of atomic mass number A and Lorentz factor Γ

$$P(\Gamma, z) = \exp \left(- \int_z^{z^*} \frac{1}{\tau_A(\Gamma, z')} \left| \frac{dt}{dz'} \right| dz' \right) \quad (7)$$

where z and z^* are the values of the redshift of the current step (from z^* to z).

The *SimProp* code is designed in such a way that any red-shift distribution of sources and any injection spectrum can be simulated. This is achieved by drawing events from a flat distribution in the red-shift of the sources and of the logarithm of the injection energy. Once the event is recorded at $z = 0$ the actual source/energy distribution is recovered through a proper weight attributed to the event [4].

We will now compare the spectra obtained using *SimProp* [4] with the spectra calculated solving the kinetic equation associated to the propagation of nuclei [3]. To pursue this comparison, a pure iron injection with a power law injection of the type $\propto E_g^{-\gamma}$ with $\gamma = 2.2$ has been assumed. The sources have been assumed to be homogeneously distributed in the red-shift range $0 < z < 3$. In Fig. 1 the fluxes expected at $z = 0$ are shown for iron and secondary nuclei produced in the photo-disintegration chain suffered by primary injected irons. The points refer to the *SimProp* results, while the continuous lines refer to the fluxes computed in the kinetic approach. Good agreement between the two schemes is clearly visible in Fig. 1. At the highest energies the path-length of iron nuclei is very short. Therefore, to achieve good sampling in the MC simulation, higher statistics is needed; this is the reason for the larger errors bars in the *SimProp* results at the highest energies.

Let us conclude by discussing why it is useful to go beyond the kinetic approach. The kinetic approach has the important feature of being analytical: the fluxes are computed mathematically by solving a first principles equation [3]. This means that the flux of primaries and secondaries is expressed in terms of several integrals that can be computed numerically, once the injection spectrum and the sources distribution are specified. In particular, the flux of secondary nuclei and nucleons produced in the photo-disintegration

chain of primary A_0 is obtained by the numerical computation of A_0 nested integrals and this computation should be repeated each time the hypothesis on sources (injection and distribution) is changed. This computation, while it is always feasible numerically, takes some time. However the time can be substantially reduced by using a MC computation scheme. This follows from the fact that, as discussed above, it is possible within the *SimProp* approach to simulate different source distributions and injection spectra without repeating the overall propagation of particles. In this sense, a faster computation scheme is provided by the MC approach presented here, which is the minimal stochastic extension of the kinetic approach.

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DISCUSSION

Carlo Gustavino — The difference between Auger and the other experiments can be due to the fact they are looking from different hemispheres?

Roberto Aloisio — This is an hypothesis that was recently put forward. I personally do not believe in such explanation because of the simple reason that at energies around $(2 \div 3) \times 10^{19}$ eV, where already the difference between Auger and HiRes starts, the universe visible in UHECR has a huge scale of the order of Gpc. Therefore it is very unlikely to have differences between observations carried out from the southern and northern hemispheres.