

RESONANT SWITCH MODEL OF HF QPOs AND EQUATIONS OF STATE OF NEUTRON STARS AND QUARK STARS

ZDENĚK STUHLÍK*, MARTIN URBANEC, ANDREA KOTRLOVÁ,
GABRIEL TÖRÖK, KATEŘINA GOLUCHOVÁ

Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, CZ-74601 Opava, Czech Republic

* corresponding author: zdenek.stuchlik@fpf.slu.cz

ABSTRACT. The mass and spin estimates of the 4U 1636–53 neutron star obtained by the Resonant Switch (RS) model of high-frequency quasi-periodic oscillations (HF QPOs) are tested by a large variety of equations of state (EoS) governing the structure of neutron stars. Neutron star models are constructed under the Hartle–Thorne theory of slowly rotating neutron stars calculated using the observationally given rotational frequency $f_{\text{rot}} = 580$ Hz (or alternatively $f_{\text{rot}} = 290$ Hz) of the neutron star at 4U 1636–53. It is demonstrated that only two variants of the RS model are compatible with the parameters obtained by modelling neutron stars for the rotational frequency $f_{\text{rot}} = 580$ Hz. The variant giving the best fit with parameters $M \sim 2.20 M_{\odot}$ and $a \sim 0.27$ agrees with high precision with the prediction of one of the Skyrme EoS [1]. The variant giving the second best fit with parameters $M \sim 2.12 M_{\odot}$ and $a \sim 0.20$ agrees with lower precision with the prediction of the Gandolfi EoS [2].

KEYWORDS: neutron stars, X-ray variability, theory, observations.

1. INTRODUCTION

A new alternative to the standard models of HF QPOs has been proposed recently in [3, 4]. The Resonant Switch (RS) model of twin-peak HF QPOs observed in low-mass X-ray binaries (LMXBs) containing a neutron star is based on the switch of twin oscillations at a resonant point, where one pair of oscillating modes changes to some other pair due to non-linear resonant phenomena. The RS model has been applied to the atoll source 4U 1636–53, where we assume two resonant points observed at frequency ratios $\nu_U : \nu_L = 3 : 2$ and $5 : 4$ [3]. The range of allowed values of dimensionless spin a and mass M of the neutron star was determined by fitting the pairs of oscillatory modes admitted by the RS model to the observed data in the regions related to the resonant points [15]. Among acceptable variants of the RS model the most promising are those combining relativistic precession and total precession frequency relations or modifications to them, when the precision of the fits increases strongly (the χ^2 test is improved by almost one order) in comparison to the fits realized by individual frequency pairs along the whole data range [15]. Here we present preliminary results of testing the RS model by various models of EoS.

2. RESONANT SWITCH MODEL

The RS model [3, 4] is based on the idea that the twin oscillatory modes creating the sequences of lower and upper HF QPOs can switch at a resonant point where

the frequencies of the upper and lower oscillations ν_U and ν_L become commensurable. It is expected that at the resonant point non-linear resonant phenomena will excite a new oscillatory mode (or two new oscillatory modes) and dump one of the previously acting modes (or both the previously acting modes), i.e., switching from one pair of oscillatory modes (corresponding to a specific model of HF QPOs) to the other pair, which will act up to the next relevant resonant point.

In the simplest version of the RS model, we assume two resonant points at disc radii r_{out} and r_{in} , with observed frequencies $\nu_U^{\text{out}}, \nu_L^{\text{out}}$ and $\nu_U^{\text{in}}, \nu_L^{\text{in}}$, being in commensurable ratios $p^{\text{out}} = n^{\text{out}} : m^{\text{out}}$ and $p^{\text{in}} = n^{\text{in}} : m^{\text{in}}$. Observations put restrictions on $\nu_U^{\text{in}} > \nu_U^{\text{out}}$ and $p^{\text{in}} < p^{\text{out}}$. In the region covering the resonant point at r_{out} we assume twin oscillatory modes with the upper (lower) frequency determined by the function $\nu_U^{\text{out}}(r, M, a)$ ($\nu_L^{\text{out}}(r, M, a)$). Near the inner resonant point at r_{in} different oscillatory modes generally occur with the upper and lower frequency relation functions $\nu_U^{\text{in}}(r, M, a)$ and $\nu_L^{\text{in}}(r, M, a)$. We assume all the frequency functions to be given by combinations of the orbital and epicyclic frequencies of the geodesic motion in the Kerr backgrounds. Such a simplification is correct with high precision for near-maximum-mass neutron (quark) stars in a slow rotation regime related to all known atoll sources [5, 6].

In the Kerr spacetime, the epicyclic frequencies ν_{θ} and ν_r and the Keplerian (orbital) frequency ν_K

Model	Relations	
RP	$\nu_L = \nu_K - \nu_r$	$\nu_U = \nu_K$
RP1	$\nu_L = \nu_K - \nu_r$	$\nu_U = \nu_\theta$
TP	$\nu_L = \nu_\theta - \nu_r$	$\nu_U = \nu_\theta$
TP1	$\nu_L = \nu_\theta - \nu_r$	$\nu_U = \nu_K$
TD	$\nu_L = \nu_K$	$\nu_U = \nu_K + \nu_r$
WD	$\nu_L = 2(\nu_K - \nu_r)$	$\nu_U = 2\nu_K - \nu_r$

TABLE 1. Frequency relations corresponding to individual QPO models.

depend only on the spacetime parameter M (mass) and a (spin) [7–10].

The frequency-relation functions have to meet the observationally given resonant frequencies that can be determined by the “energy switch effect” [3, 11]. In the framework of the simple RS model this requirement enables direct determination of the Kerr background parameters describing the exterior of the neutron (quark) star [3, 4]. Independence of the frequency ratio on the mass parameter M implies that the conditions

$$\nu_U^{\text{out}}(x; a) : \nu_L^{\text{out}}(x; a) = p^{\text{out}}, \quad (1)$$

$$\nu_U^{\text{in}}(x; a) : \nu_L^{\text{in}}(x; a) = p^{\text{in}} \quad (2)$$

determine the relations for spin a in terms of the dimensionless radius $x = r/(GM/c^2)$ and the resonant frequency ratio p . They can be expressed in the form $a_p^{\text{out}}(x)$ and $a_p^{\text{in}}(x)$, or in an inverse form $x_p^{\text{out}}(a)$ and $x_p^{\text{in}}(a)$. At the resonant radii, the conditions

$$\nu_U^{\text{out}} = \nu_U^{\text{out}}(x; M, a), \quad \nu_U^{\text{in}} = \nu_U^{\text{in}}(x; M, a) \quad (3)$$

are satisfied along the functions $M_{p_{\text{out}}}^{\text{out}}(a)$ and $M_{p_{\text{in}}}^{\text{in}}(a)$ which can be obtained by using the functions $x_p^{\text{out}}(a)$ and $x_p^{\text{in}}(a)$. The parameters of the neutron (quark) star are then given by the condition [3, 4]

$$M_{p_{\text{out}}}^{\text{out}}(a) = M_{p_{\text{in}}}^{\text{in}}(a). \quad (4)$$

Condition (4) determines M and a with precision given by the error in determining the resonant frequencies by the energy switch effect.

We consider the pairs of frequency relations given by the relativistic precession (RP) model [9], the total precession (TP) model [12], and their modifications RP1 and TP1, combined also with the tidal disruption (TD) model [13], and the warped disc oscillations (WD) model [14]. The frequency relations are summarized in Table 1. For each of the frequency relations under consideration, the frequency resonance functions and the resonance conditions determining the resonant radii $x_{n:m}(a)$ are given in [3].

Combination of models	χ_{min}^2	a	$M [M_\odot]$
RP1(3:2) – RP(5:4)	55	0.27	2.20
TP(3:2) – RP(5:4)	55	0.52	2.87
RP1(3:2) – TP1(5:4)	61	0.20	2.12
RP1(3:2) – TP(5:4)	62	0.45	2.46
TP(3:2) – TP1(5:4)	68	0.31	2.39
RP(3:2) – TP1(5:4)	72	0.46	2.81
WD(3:2) – TD(5:4)	113	0.34	2.84

TABLE 2. The best fits and the corresponding spin and mass parameters of the neutron star located in the 4U 1636–53 source.

3. APPLICATION TO THE ATOLL SOURCE 4U 1636–53

In [3], the RS model has been applied in the case of the atoll source 4U 1636–53, where the observational data clearly demonstrate the possible existence of two resonant points with frequency ratios 3:2 and 5:4, where the energy switch effect occurs. The mass M and spin a ranges of the 4U 1636–53 neutron star predicted by the RS model with resonant frequencies given by the energy switch effect are very large (see Table 1 in [3]). However, the ranges can be strongly restricted by fitting the observational data near the resonant points by the pairs of frequency relations corresponding to the twin oscillatory modes. In the fitting procedure we apply those switched twin frequency relations predicted by the RS model that are acceptable due to the neutron (quark) star structure theory [3]. In fitting the observational data we use the standard least-squares (χ^2) method. The resulting limits on the mass M and spin a of the 4U 1636–53 neutron star implied by the data fitting procedure realized in the framework of the RS model of HF QPOs are presented in Table 2. The fitting procedure is shown to be by almost one order of magnitude more precise than the fitting realized by individual pairs along the whole range of the observational data [15]. The best fit obtained for the RS model with the frequency relation pair RP1–RP gives $\chi^2 \sim 55$ and $\chi^2/\text{dof} \sim 2.5$ [15]. The results of the fitting procedure for the best fit are presented in Figure 1.

The best fit occurs for a combination of the RP1 and RP models, where the RP1 model has to be related to the outer resonant point, while the RP model is related to the inner resonant point and predicts neutron star parameters $M \sim 2.20 M_\odot$ and $a \sim 0.27$ which are quite acceptable according to the neutron star theory and can be considered as the best prediction of the RS model. The second best fit (with $\chi^2 = 61$) is obtained for the frequency pair RP1–TP1, where the RP1 model has to be related to the outer resonant point, while the TP1 model is related to the inner resonant point and predicts the parameters $M \sim 2.12 M_\odot$ and $a \sim 0.20$, which are again acceptable according to the neutron star structure theory.

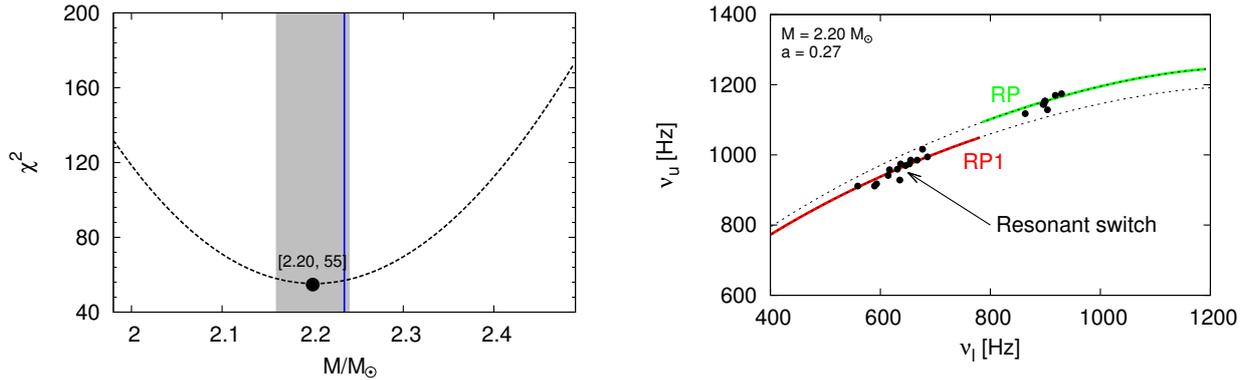


FIGURE 1. Results of the fitting the data of twin-peak HF QPOs in the atoll source 4U 1636–53 by the procedure of the RS model for the combination of the RP1(3:2) and RP(5:4) frequency relations. *Left panel*: Profile of the lowest χ^2 for a given M . Thick blue vertical lines give mean value of M as determined by the RS model from the frequency ratio governed by the energy switch effect. The grey region corresponds to the precision of the fit. *Right panel*: The pair of frequency relations RP1–RP obtained for the best fit to the observational data (with $\chi^2 \sim 55$).

4. EQUATIONS OF STATE FOR THE NEUTRON STAR IN SOURCE 4U 1636–53 TESTING THE RS MODEL

We compare results obtained in [15] with models of rotating neutron stars calculated using the Hartle–Thorne approximation [16, 17], which describes slowly rotating neutron stars. We construct models of rotating neutron stars using a large variety of acceptable EoS and with rotation frequency 580 Hz (or 290 Hz) observed for source 4U 1636–53 [18]. In Figure 2, the results of the Hartle–Thorne model are illustrated by appropriately denoted curves in the M – a plane that are calculated for the EoS under consideration.

We can see that no EoS enables a model of the neutron star that can fit the RS model data, if we assume the rotational frequency of the 4U 1636–53 neutron star $f_{\text{rot}} \sim 290$ Hz. For the rotational frequency $f_{\text{rot}} \sim 580$ Hz, neutron star models give very interesting restrictions that are in significant agreement with the results of applying the fitting the HF QPO data in the framework of the RS model. A neutron star model using one of the Skyrme EoS (SV) [1] meets with high precision the prediction of the RP1–RP version of the RS model that gives the best fit to the twin peak HF QPO data observed in the 4U 1636–53 source for neutron star parameters $M \sim 2.20 M_{\odot}$ and $a \sim 0.27$. The neutron star model based on the Gandolfi EoS [2] meets with acceptable precision the prediction of the RP1–TP1 version of the RS model that gives the second best fit to the observation data of the HF QPOs in 4U 1636–53 for a neutron star having parameters $M \sim 2.12 M_{\odot}$ and $a \sim 0.20$. Note that the second best RS model fit is marginally touched by another parameterized Skyrme EoS (Gs) [1] for the neutron star parameters $M \sim 2.12 M_{\odot}$ and $a \sim 0.20$. This result demonstrates that the 4U 1636–53 neutron star could be in a state very close to instability with respect to the radial perturbation,

corresponding to the maximum mass, predicted by the EoS.

All the other predictions of the RS model are located in M – a plane positions that are evidently outside the range of all the EoS considered in the present paper – we can expect that this is true even for all variants of the presently known EoS.

5. CONCLUSIONS

We can conclude that the EoS considered in our study strongly restrict the versions of the RS model. Only two of them (RP1–RP and RP1–TP1) are therefore acceptable. However, it is quite interesting that the RS model can put strong restrictions on the acceptable EoS, and it seems that only three of those considered here can be taken as plausible.

ACKNOWLEDGEMENTS

We would like to express our gratitude to the Czech Grant Agency for supporting project GAČR 202/09/0772, and for internal grants of the Silesian University in Opava SGS/11/2013 and SGS/23/2013. The authors further acknowledge the project on Supporting Integration with the International Theoretical and Observational Research Network in Relativistic Astrophysics of Compact Objects, reg. no. CZ.1.07/2.3.00/20.0071, supported by Operational Programme *Education for Competitiveness* funded by Structural Funds of the European Union and the state budget of the Czech Republic.

REFERENCES

- [1] J. R. Stone, et al. Nuclear matter and neutron-star properties calculated with the Skyrme interaction. *Phys Rev C* **68**(3):034324, 2003.
- [2] S. Gandolfi, et al. Microscopic calculation of the equation of state of nuclear matter and neutron star structure. *Monthly Notices Roy Astronom Soc* **404**:L35–L39, 2010. [arXiv:0909.3487\[nucl-th\]](https://arxiv.org/abs/0909.3487).
- [3] Z. Stuchlík, et al. Resonant switch model of twin peak HF QPOs applied to the source 4U 1636–53. *Acta*

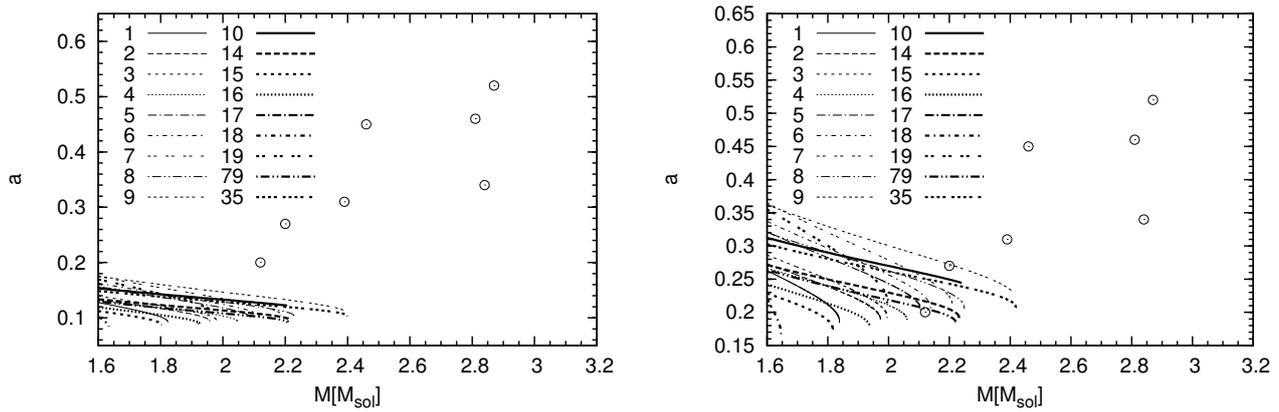


FIGURE 2. Hartle–Thorne models of neutron stars with a variety of EoS: 1–9: Skyrme [1], 10: UBS [19], 14: APR [20], 16: BBB2 [21], 17: BPAL12 [22], 18: BALBN1H1 [23], 19: GLENDNH3 [24], 35: APR2 [20], 79: Gandolfi [2]. The models are constructed for $f_{\text{rot}} \sim 290$ Hz (left panel), and $f_{\text{rot}} \sim 580$ Hz (right panel).

Astronom **62**(4):389–407, 2012.

arXiv:1301.2830 [astro-ph.HE].

- [4] Z. Stuchlík, et al. Multi-resonance orbital model of high-frequency quasi-periodic oscillations: possible high precision determination of black hole and neutron star spin. *Astronomy and Astrophysics* **552**(A10), 2013. arXiv:1305.3552 [astro-ph.HE].

- [5] G. Török, et al. On Mass Constraints Implied by the Relativistic Precession Model of Twin-peak Quasi-periodic Oscillations in Circinus X-1. *Astrophys J* **714**(1):748–757, 2010. arXiv:1008.0088 [astro-ph.HE].

- [6] M. Urbanec, et al. Quadrupole moments of rotating neutron stars and strange stars. *Monthly Notices Roy Astronom Soc* (published online), 2013. arXiv:1301.5925 [astro-ph.SR].

- [7] A. N. Aliev, et al. Radiation from relativistic particles in non-geodesic motion in a strong gravitational field. *Gen Relativity Gravitation* **13**:899–912, 1981.

- [8] S. Kato, et al. *Black-hole accretion disks*. Kyoto University Press, Kyoto, Japan, 1998.

- [9] L. Stella, et al. Lense–Thirring Precession and Quasi-periodic Oscillations in Low-Mass X-Ray Binaries. *Astrophys J Lett* **492**:L59–L62, 1998. arXiv:astro-ph/9709085.

- [10] G. Török, et al. Radial and vertical epicyclic frequencies of Keplerian motion in the field of Kerr naked singularities. Comparison with the black hole case and possible instability of naked singularity accretion discs. *Astronomy and Astrophysics* **437**(3):775–788, 2005. arXiv:astro-ph/0502127.

- [11] G. Török. Reversal of the amplitude difference of kHz QPOs in six atoll sources. *Astronomy and Astrophysics* **497**(3):661–665, 2009. arXiv:0812.4751 [astro-ph].

- [12] Z. Stuchlík, et al. On a multi-resonant origin of high frequency quasiperiodic oscillations in the neutron-star X-ray binary 4U 1636–53. *ArXiv e-prints*, 2007. arXiv:0704.2318 [astro-ph].

- [13] U. Kostić, et al. Tidal effects on small bodies by massive black holes. *Astronomy and Astrophysics* **496**(2):307–315, 2009. arXiv:0901.3447 [astro-ph.HE].

- [14] S. Kato. Resonant Excitation of Disk Oscillations by Warps: A Model of kHz QPOs. *Publ Astronom Soc Japan* **56**(5):905–922, 2004. arXiv:astro-ph/0409051.

- [15] Z. Stuchlík, et al. Test of the Resonant Switch model by fitting the data of twin-peak HF QPOs in the atoll source 4U 1636–53. *Acta Astronom* **64**(1):45–64, 2014.

- [16] J. B. Hartle, et al. Slowly rotating relativistic stars II. Model for neutron stars and supermassive stars. *Astrophys J* **153**:807–834, 1968.

- [17] S. Chandrasekhar, et al. On slowly rotating homogeneous masses in general relativity. *Monthly Notices Roy Astronom Soc* **167**:63–79, 1974.

- [18] T. E. Strohmayer, et al. Evidence for a Millisecond Pulsar in 4U 1636–53 during a Superburst. *Astrophys J* **577**:337–345, 2002. arXiv:astro-ph/0205435.

- [19] M. Urbanec, et al. Observational Tests of Neutron Star Relativistic Mean Field Equations of State. *Acta Astronom* **60**(2):149–163, 2010. arXiv:1007.3446 [astro-ph.SR].

- [20] A. Akmal, et al. Equation of state of nucleon matter and neutron star structure. *Phys Rev C* **58**:1804–1828, 1998. arXiv:nucl-th/9804027.

- [21] M. Baldo, et al. Microscopic nuclear equation of state with three-body forces and neutron star structure. *Astronomy and Astrophysics* **328**:274–282, 1997. arXiv:astro-ph/9707277.

- [22] I. Bombaci. An equation of state for asymmetric nuclear matter and the structure of neutron stars. In I. Bombaci, A. Bonaccorso, A. Fabrocini, et al. (ed.), *Perspectives on Theoretical Nuclear Physics*, pp. 223–237, 1995.

- [23] S. Balberg, et al. An effective equation of state for dense matter with strangeness. *Nuclear Phys A* **625**:435–472, 1997. arXiv:nucl-th/9704013.

- [24] N. K. Glendenning. Neutron stars are giant hypernuclei? *Astrophys J* **293**:470–493, 1985.