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ON THE SPEED OF SOUND IN STEAM

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Abstract. A study of the speed of sound in a pure water substance is presented here. The IAPWS data on the state of water and steam are applied only for investigating the speed of sound for a one-phase medium. A special numerical model for investigating the parameters of shock waves in steam is presented here and is applied for investigating extremely weak waves to obtain velocities representing the speed of sound in both one-phase and two-phase steam. Problems with the speed of sound in two-phase steam are discussed, and three types of speed of sound are derived for the metastable region of wet steam.

Keywords: speed of sound; steam; IAPWS-IF97; shockwave.

1. Introduction

The speed of sound is a physical quantity closely connected with the compressibility of a medium. Physically, the speed of sound expresses the speed of the advance of undulation in a given medium. Generally, the speed of sound a in a continuum is defined as the square root of an infinitesimal pressure disturbance ∂p related to an infinitesimal change in density $\partial \varrho$ in an isentropic process

$$a = \sqrt{\left(\frac{\partial p}{\partial \varrho}\right)_s}. \tag{1}$$

Thee fundamental definition of the speed of sound is expressed by (1). There is no problem in evaluating the speed of sound when the state equation in the form $f(p, \varrho, s) = 0$ is known. The well-known relation of the speed of sound can be derived for an ideal gas as

$$a = \sqrt{\kappa p/\varrho} = \sqrt{\kappa r T}. \tag{2}$$

From (2), it follows that the speed of sound in an ideal gas depends on temperature only, because $\kappa =$ const. is the ratio of the heat capacities (the Poisson constant) and r is the specific gas constant

$$r = R/M, \tag{3}$$

$R = 8314.41\,\mathrm{J\,kmol^{-1} 1\,K^{-1}}$ is the universal gas constant, and M is the molar mass of the gas.

Some considerations on the speed of sound in steam will be presented in this paper. In their previous publications [1, 2], the authors pointed out problems with the propagation of waves in steam. One point is that the speed of sound in steam depends on state parameters in a more complicated manner than in (2). This is obvious, because the equation of state for steam according to the data of the International Association for Properties of Water and Steam (IAPWS) is complex. IAPWS has released two formulations for water and steam: the first is the Formulation for General and Scientific Use IAPWS-95 [3], and the second is the Industrial Formulation IAPWS-IF97 [4].

It should be mentioned here that the data on the speed of sound in steam are available only for a one-phase medium. The available tools are tables or calculators. The data on the speed of sound in wet steam have not yet been integrated. Papers [5, 6] provide much stimulation for further studies of the speed of sound in wet steam, because they deal with the propagation of waves in wet steam. However, no measured data have in fact been published.

2. Speed of sound in steam

The formulation of IAPWS-95 is a fundamental equation for specific Helmholtz free energy $f(\varrho, T)$. Its dimensionless form is separated into two parts

$$\frac{f(\varrho, T)}{rT} = \phi(\delta, \tau) = \phi^o(\delta, \tau) + \phi^r(\delta, \tau), \tag{4}$$

where $\delta = \varrho/\varrho_\mathrm{c}$ and $\tau = T_\mathrm{c}/T$. For water substance reference constants, IAPWS defined in [3] the critical temperature $T_\mathrm{c} = 647.096\,\mathrm{K}$, the critical density $\varrho_\mathrm{c} = 322\,\mathrm{kg\,m^{-3}}$, and the specific gas constant $r = 461.51805\,\mathrm{J\,kg^{-1}\,K^{-1}}$. Functions $\phi^o(\delta, \tau)$ and $\phi^r(\delta, \tau)$ are defined by IAPWS [2]. The speed of sound is then calculated from

$$a(\delta, \tau) = \left[rt\left(1 + 2\delta \frac{\partial \phi^r}{\partial \delta} + \delta^2 \frac{\partial^2 \phi^r}{\partial \delta^2} - \frac{\left(1 + \delta \frac{\partial \phi^r}{\partial \delta} - \delta\tau \frac{\partial^2 \phi^r}{\partial \delta\, \partial \tau}\right)^2}{\tau^2 \left(\frac{\partial^2 \phi^o}{\partial \tau^2} + \frac{\partial^2 \phi^r}{\partial \tau^2}\right)} \right) \right]^{1/2}. \tag{5}$$

The formulation of IAPWS-IF97 is divided into 5 regions, where different fundamental equations are

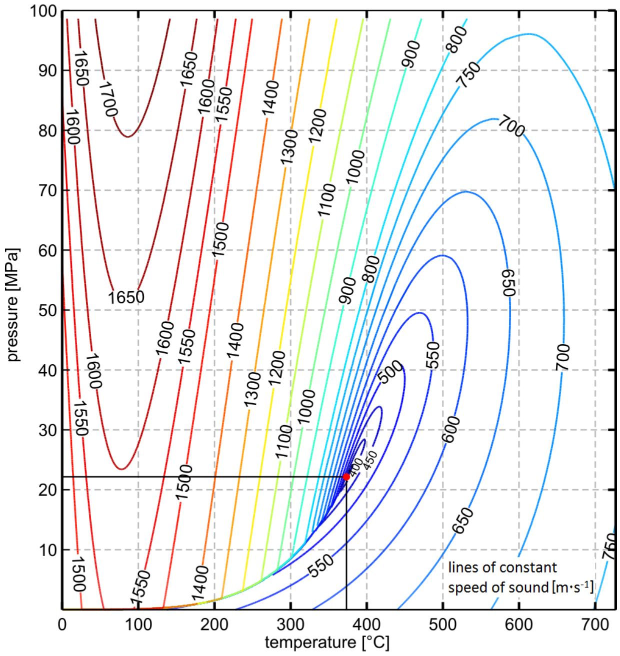

FIGURE 1. Values for the speed of sound [m s^{-1}] in water and in steam in a p–t diagram.

defined: Gibbs free energy $g(p, T)$ and Helmholtz free energy $f(\varrho, T)$. The fundamental equation for Gibbs free energy is expressed in dimensionless form as

$$\frac{g(p,T)}{rT} = \gamma(\pi, \tau), \tag{6}$$

where π is dimensionless reduced pressure, and τ is dimensionless reduced temperature. The function $\gamma(\pi, \tau)$ is defined by IAPWS [4], and then the speed of sound is solved from

$$a(\pi,\tau) = \sqrt{\frac{rT\left(\frac{\partial\gamma}{\partial\pi}\right)^2 \tau^2 \frac{\partial^2\gamma}{\partial\tau^2}}{\left(\frac{\partial\gamma}{\partial\pi} - \tau\frac{\partial^2\gamma}{\partial\pi\,\partial\tau}\right)^2 - \frac{\partial^2\gamma}{\partial\pi^2}\tau^2\frac{\partial^2\gamma}{\partial\tau^2}}}. \tag{7}$$

Values for the speed of sound were evaluated according to (7), and were presented in [1] in a p–t (pressure-temperature) phase diagram. The values are shown in Fig. 1. It is evident that ideal gas theory cannot be applied for water and steam.

3. The speed of sound in steam solved by means of the model for solving the thermodynamic parameters of steam downstream from a normal shock wave

An equilibrium model of a shock wave in steam was formulated in [1]. The theoretical approach for calculating steam parameters is based on balance equations for steam passing the infinitesimally thin control volume on a normal shock wave (Fig. 2). The three modified balance equations are: balance of mass, balance of momentum, and balance of energy (under the assumption of constant total enthalpy $h_{01} = h_{02} = h_0$):

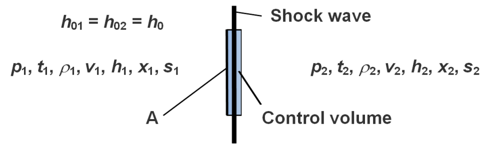

FIGURE 2. Scheme of a shock wave, control volume, and parameters on a normal shock wave in steam.

- balance of mass

$$\frac{\dot{m}}{A} = \varrho_1 v_1 = \varrho_2 v_2; \tag{8}$$

- balance of momentum $p_1 - p_2 = \frac{\dot{m}}{A}(v_2 - v_1)$, hence

$$p_1 + \varrho_1 v_1^2 = p_2 + \varrho_2 v_2^2; \tag{9}$$

- balance of energy

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2} = h_{01} = h_{02} = h_0; \tag{10}$$

- equation of state

$$1/\varrho = f_{v,ph}(p, h). \tag{11}$$

Index 1 is upstream from the shock wave, index 2 is downstream from the shock wave, 0 indicates a total value.

All thermodynamic parameters upstream from the normal shock wave are given. In paper [1], the calculation procedure for a given pressure downstream from the shock wave p_2 (when $p_2 > p_1$) is derived. The iterative procedure is based on the balance equations and the equation of state of steam according to IAPWS-IF97, and all thermodynamic parameters downstream from the normal shock wave are calculated. The wet steam model assumes isobaric separation of the two-phase medium, for further details, see [1].

Relations for velocities as functions of the calculated thermodynamic parameters can be derived from the balance equations. The velocity v_1 of steam upstream from the normal shock wave can be calculated according to the relation

$$v_1 = \frac{\frac{p_2 - p_1}{\varrho_1}}{\sqrt{2\frac{p_2-p_1}{\varrho_1} - 2(h_2 - h_1)}}. \tag{12}$$

The velocity v_2 downstream from the normal shock wave is also derived from the balance equations, and can be calculated according to the relation

$$v_2 = v_1 - \frac{p_2 - p_1}{\varrho_1 v_1}. \tag{13}$$

The model for calculating the shock wave parameters was applied successfully for superheated, saturated and wet steam for various ratios of pressures p_2/p_1. A special case is for $p_2 = p_1$; then $v_1 = v_2 = a$, (a is the speed of sound).

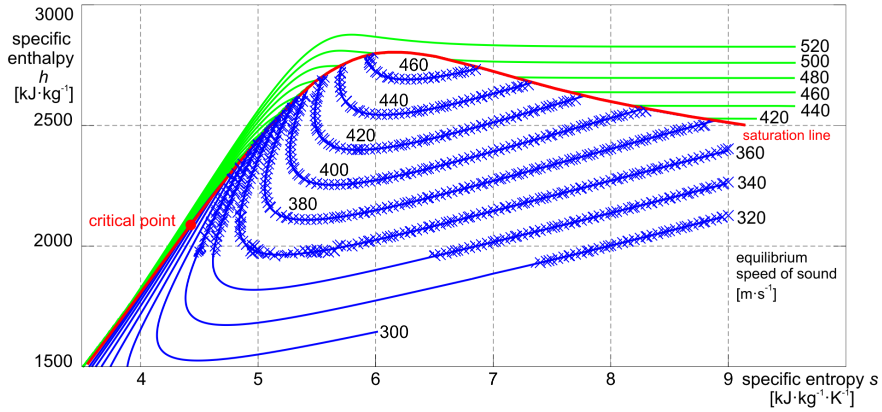

Figure 3. The h–s diagram of water and steam with curves of the constant speed of sound in steam a [m s^{-1}]. The wet steam region blue lines, are from definition (1) and the blue cross values are calculated by means of the model for normal shock waves. Superheated steam a values according to IAPWS-IF97 are depicted by green curves.

3.1. Results of the solution for the speed of sound

The numerical procedure for the model for calculating the thermodynamic parameters of steam downstream from a normal shock wave was performed for $p_2/p_1 \gg 1$. The values obtained for the speed of sound proved to be very close to the IAPWS data, according to (7) for superheated and saturated steam. We developed calculation tool in MatLab, which proved to be well performing. The results for wet steam are very close (lower than 2 % difference) to the values for the speed of sound in wet steam, derived according to (1), see Fig. 3, where a detail of the h–s (enthalpy–entropy) diagram for water and steam is depicted. The assumption of an infinitesimally weak normal shock wave defines the equilibrium conditions for wet steam, so the speed of the sound values obtained here corresponds to the speed of sound for the thermodynamic equilibrium state of wet steam. A numerical model for calculating the speed of sound in steam is developed and verified.

Figure 3 shows some special effect of the speed of sound on the steam saturation line – there is considerable discontinuity of the values. This effect can be explained by a discontinuity of the first derivative of lines of constant temperature on the steam saturation line in the h–s (enthalpy–entropy) diagrams and in the p–v (pressure–specific volume) diagrams, and perhaps by a discontinuity of the first derivative of the lines of constant pressure on the saturation line in the T–s (temperature–entropy) diagram for steam. The state parameters — namely pressure p, density ϱ and entropy s — defined in the two-phase region by means of the Maxwell rule as equilibrium parameters. From the thermodynamic point of view, wet steam in the equilibrium state is therefore considered to be a continuum, which contains both phases of water (saturated water liquid and saturated steam) according to the dryness value. The speed of sound obtained from this numerical model for wet steam should be referred to as the *equilibrium speed of sound*.

4. The speed of sound in wet steam in the metastable region

4.1. Equilibrium speed of sound

The equilibrium speed of sound can be evaluated for the whole wet steam region, see Fig. 3. Low equilibrium speed of sound values near the saturation line of water are not acceptable. This is shown in Fig. 4, where the speed of sound values are depicted as dependencies of dryness x for constant pressures p.

4.2. Frozen speed of sound

The assumption is often made that isobars and isotherms are identical in the wet steam region when calculating the speed of sound in wet steam. According to (2), the value of the speed of sound is therefore calculated using the relation

$$a_{\mathrm{F}} = \sqrt{\kappa r T''}. \tag{14}$$

where T'' is temperature of saturated steam. The lines for the constant speed of sound are therefore identical to the isobars in wet steam. The speed of sound calculated using this assumption should be referred to as the *frozen speed of sound*. This approach can be applied for approximate calculations in the region of temperatures up to 100 °C and dryness of wet steam from 0.98 to 1.00. However, it is more convenient to determine the frozen speed of sound from the IAPWS data for saturated steam.

4.3. Metastable speed of sound

A further approach is based on a continuum model, so that the dependence of the speed of sound on the specific enthalpy is extended from the region of superheated steam into the region of wet steam. If superheated steam is considered as an ideal gas, the dependence of speed of sound on specific enthalpy is parabolic. It can be derived from (2):

$$a_{\mathrm{MC}} = \sqrt{(\kappa - 1) h_{\mathrm{id}}}, \tag{15}$$

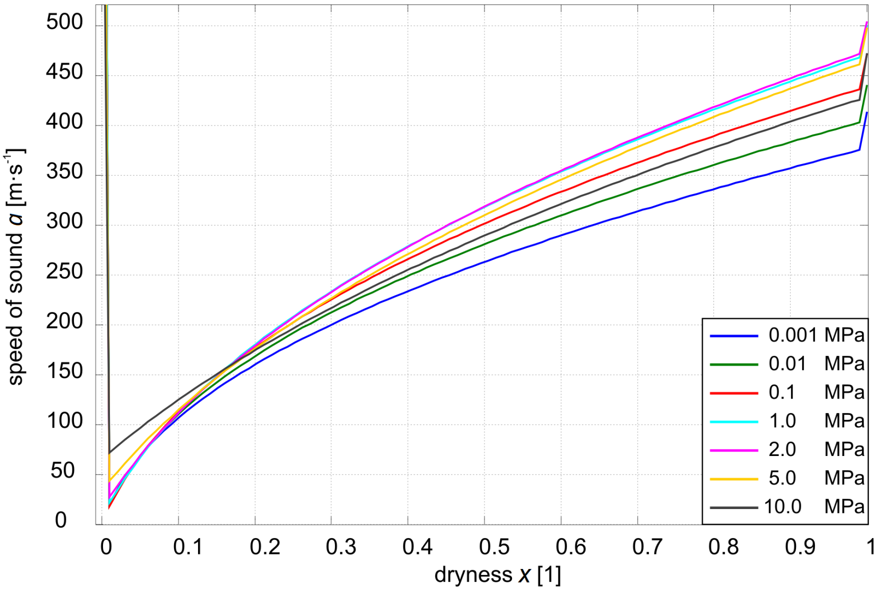

FIGURE 4. Dependence of values for the equilibrium speed of sound a on the dryness of wet steam at constant pressures p.

Pressure p [MPa]	Specific enthalpy h [$\mathrm{kJ\,kg^{-1}}$]	Dryness x [$\mathrm{kg_{sat}\,kg_{wet}}^{-1}$]
0.001	2421.89	0.9629
0.010	2473.12	0.9534
0.100	2542.29	0.9411
1.000	2645.18	0.9350
2.000	2658.32	0.9264
5.000	2660.51	0.9186
10.000	2555.87	0.8699

TABLE 1. Parameters of wet steam where the metastable and equilibrium speeds of sound are equal.

where κ is the ratio of specific heat capacities and h_{id} is the specific enthalpy of an ideal gas defined as a zero value at zero total temperature. The IAPWS-IF97 data are depicted in Fig. 5 in the diagram showing speed of sound vs. specific enthalpy for constant pressures. In the region of superheated steam the dependencies are similar curves, but the equilibrium speed of sound has a discontinuity on the saturation line. It should be mentioned here that the specific enthalpy of steam is applied according to the IAPWS definition – in the triple point, the enthalpy of steam is $h_{\mathrm{t.p.}} = 2500.9\,\mathrm{kJ\,kg^{-1}}$. In the new model, the curves are prolonged into the region of wet steam. The speed of sound defined by these prolonged curves should be referred to as the metastable speed of sound. The region from the saturation line of steam to the intersection of the prolonged curves with the dependencies of the equilibrium speed of sound determines the region of metastable speed of sound. We estimated the numerical uncertainty of the calculation to be lower than 5 % and can be further improved. The conditions for equality values of the metastable and equilibrium speeds of sound are determined and introduced in Table 1.

It is remarkable that the dryness values in Table 1 are very close to the Wilson line for condensation of steam into droplets. The speed of sound in the metastable region can acquire any value between the metastable speed of sound and the equilibrium speed of sound. The values for the frozen speed of sound are rather overvalued. For lower values of the dryness of wet steam given in Table 1, continuum models lose validity, and it is necessary to investigate the propagation of waves in a heterogeneous (two-phase) environment. Examples can be found in [5, 6]. The metastable-vapor region is defined by IAPWS [4, 7] from the saturation line to the 95 % equilibrium dryness line for pressures from the triple-point pressure up to 10 MPa.

5. CONCLUSIONS

A numerical model for calculating the speed of sound in steam based on balances of mass, momentum and energy and an equation of state for steam based on IAPWS-IF97 has been developed and verified. Our results are in very good accordance with the IAPWS data in the region of superheated steam, where the relative error is ranging from 0.3 % up to the maximum of 1.5 % in the critical point surrounding. Proper method for determining the uncertainty of the calculated values of the speed of sound in wet steam was not yet determined. A detailed study of the speed of sound in wet steam has attempted to determine the metastable region and to define the speed of sound in this region. It should be pointed out here that no measured data are available for the speed of sound in wet steam, and there are also only limited theoretical resources.

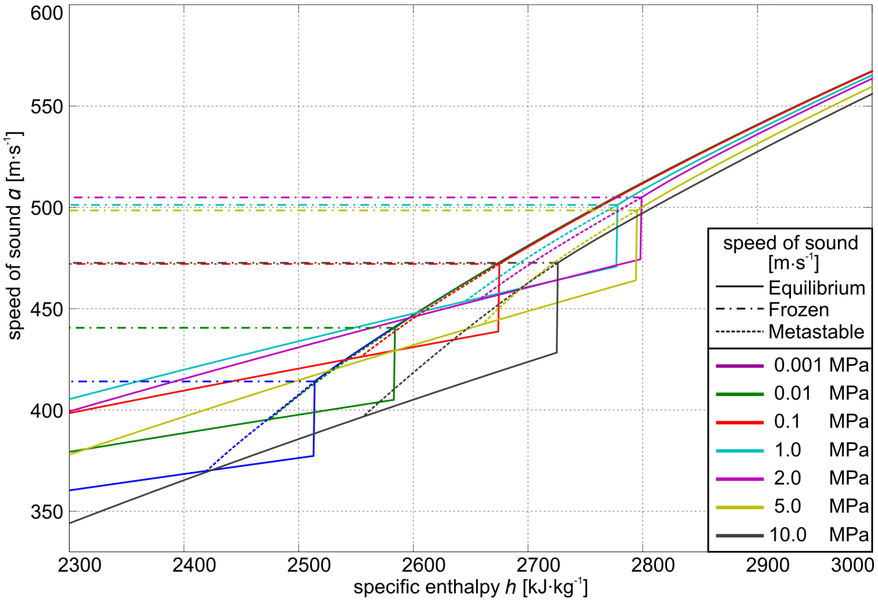

FIGURE 5. Dependence of speed of sound on specific enthalpy for constant pressures.

List of symbols

p pressure [Pa]
a speed of sound $[\mathrm{m\,s^{-1}}]$
ϱ density $[\mathrm{kg\,m^{-3}}]$
T temperature [K]
t temperature [°C]
κ ratio of heat capacities (Poisson constant) [1]
r specific gas constant $[\mathrm{J\,kg^{-1}\,K^{-1}}]$
v velocity $[\mathrm{m\,s^{-1}}]$
h specific enthalpy $[\mathrm{kJ\,kg^{-1}}]$
s specific entropy $[\mathrm{kJ\,kg^{-1}\,K^{-1}}]$
x dryness $[\mathrm{kg_{sat}\,kg_{wet}^{-1}}]$
A area $[\mathrm{m^2}]$
$\dot{m}$ mass flux $[\mathrm{kg\,s^{-1}}]$

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