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GENERAL MODEL OF RADIATIVE AND CONVECTIVE HEAT TRANSFER IN BUILDINGS: PART I: ALGEBRAIC MODEL OF RADIATIVE HEAT TRANSFER

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Abstract. Radiative heat transfer is the most effective mechanism of energy transport inside buildings. One of the methods capable of computing the radiative heat transport is based on the system of algebraic equations. The algebraic method has been initially developed by mechanical engineers for a wide range of thermal engineering problems. The first part of the present serial paper describes the basic features of the algebraic model and illustrates its applicability in the field of building physics. The computations of radiative heat transfer both in building enclosures and also in open building envelopes are discussed and their differences explained. The present paper serves as a preparation stage for the development of a more general model evaluating heat losses of buildings. The general model comprises both the radiative and convective heat transfers and is presented in the second part of this serial contribution.

Keywords: Radiative heat transfer, view factor, radiosity, room envelope, radiative heat in interiors, heat loss.

1. Introduction

Heat radiation represents a dominant transfer mechanism of heat energy inside buildings. Estimating this transfer is, therefore, a useful indicator of effectiveness of heating systems, especially those based on radiant panels.

So far, several algebraic methods for determining radiative heat transfer have been published. Probably, the pioneering work in the field of algebraic models can be ascribed to Hottel [1], who introduced the concept of the total-view factor $\underline{F_{ij}}$. Soon afterwards, Hotell and Sarofim [2] improved the method by introducing the total-exchange area $\overline{S_iS_j}$. Their model is based on the so-called radiosity heat flux and, thus, it is often referred to as a radiosity method. Besides the radiosity method, there are some other modifications of the algebraic approach. Gebhart [3, 4] introduced a method utilizing the so-called absorption factor. Among other methods dealing with radiative heat transfer in enclosures, it is possible to mention the methods by Sparrow [5], Sparrow and Cess [6], Oppenheim [7], Ecker and Drake [8], Love [9], Wiebelt [10], and Siegel and Howell [11]. A comprehensive analysis of the methods that were introduced by Hottel and Sarofim [1, 2] and Gebhart [3] was published by Clark and Korybalski [12]. These two authors showed that the methods of Hottel, Sarofim and Gebhart, although written in different forms, were mathematically equivalent. Liesen and Pedersen [13] published an overview of many radiant exchange models that range from the exact models using uniform radiosity networks and exact view factors to mean radiant temperature and area-weighted view factors. These authors compared the radiant exchange models to each other for a simple zone with varying aspect ratios. Since that time, many various applications of algebraic methods have been published in the field of mechanical engineering, but *in the field of building thermal technology, the algebraic methods has not actually been used.* The reason may be related to the fact that a complete solution of the combined radiative-convective transport leads to the system of transcendent non-linear equations whose solutions seem to be problematic in some cases.

Recent studies of radiative heat transport in the inner spaces of buildings (i.e., in enclosures) avoid algebraic methods and prefer differential transport equations [14–18]. The main interest is focused on floor heating systems [14–17], but radiant panel systems situated on walls or ceilings are also investigated [18–21]. Coefficients of heat transfers at interiors or exteriors along with heat losses are often measured or calculated as well [20, 22–25].

As has been mentioned above, the radiative heat transfer is the most effective mechanism of energy transport as compared to the free convection or conduction. This fact has been verified many times both theoretically and experimentally. For example, Rahimi and Sabernaeemi [21] have experimentally investigated these transports between the radiant ceiling surface and other internal surfaces of a room and have found that more than 90 % of the heat is transferred by radiation.

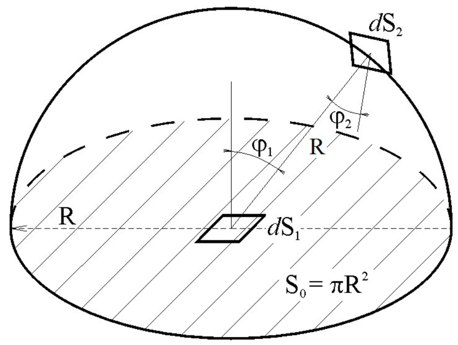

FIGURE 1. Heat exchange by radiation between two small surface elements dS_1 and dS_2.

As is well-known, the radiative heat is formed by electromagnetic waves containing many different wavelengths. When the room under investigation contains glazed windows, it is necessary to consider the capability of glass to transmit these waves. Glass is transparent for wavelengths between 0 and 4 μm but above this range, it behaves as an opaque matter. The wavelengths of solar radiation near the surface of the Earth assume values less than 2.5 μm and thus they may pass through the windows into interiors. Temperatures of inner furnishings usually do not exceed 30 °C, i.e. 300 K, and, according to Wien's displacement law, the wavelength of maximum radiation of such furnishings is

$$\lambda_{max} = \frac{2898\,\mu\text{mK}}{300\,\text{K}} = 9.66\,\mu\text{m}.$$

This value is sufficiently above the critical transmittance range, which makes the glazed window an opaque barrier for *inner radiation* similarly as a wall [14]. However, like with walls, heat is transferred by convection and radiation in the vicinity of the internal and external sides of windows, but inside the glass panes, solely by conduction. When the windows are double or triple glazed, the cavities between glass panes filled with an inert gas represent a narrow space where the heat is transferred by convection, radiation and often the conduction may participate as well.

Since the heat radiation *within opaque enclosures* represents a dominant heat transfer between inner surfaces, it is desirable to have a reliable computational tool for its quantification. The algebraic method is certainly such a tool. Its theoretical formalism is presented in the following section.

2. Basics of radiosity method

The first step in performing the radiosity method consists in forming the matrix of view factors. These factors facilitate redistributing the radiative energy among the surfaces of the enclosure under study.

2.1. View factors

A view factor F reduces the total energy irradiated by a surface to that part of energy that reaches the neighboring surface (Figure 1). View factors ares dimensionless, assume only positive values and are restricted to the interval $F \in \langle 0, 1 \rangle$. They are defined as follows [26–28]:

$$F_{ij} = \frac{1}{S_i} \int_{(S_i)} \left[\int_{(S_j)} \frac{\cos\varphi_i \cos\varphi_j}{\pi R^2} dS_j \right] dS_i \tag{1}$$

$$F_{ji} = \frac{1}{S_j} \int_{(S_j)} \left[\int_{(S_i)} \frac{\cos\varphi_j \cos\varphi_i}{\pi R^2} dS_i \right] dS_j \tag{2}$$

From Equations (1) and (2), the symmetry relation between F_{ij} and F_{ji} can be immediately deduced:

$$S_i \cdot F_{ij} = S_j \cdot F_{ji} \tag{3}$$

This *symmetry rule* holds quite *generally* regardless of the types of surfaces and their geometrical positions.

There is another important property of view factors. If a radiant surface is incapable of irradiating itself, its view factor is zero:

$$F_{ii} = 0 \tag{4}$$

For example, the perfect planes or the external surfaces of spheres belong to this class of surfaces. It should be highlighted that *zero rule* (4) does not hold generally, but it is restricted to special surfaces.

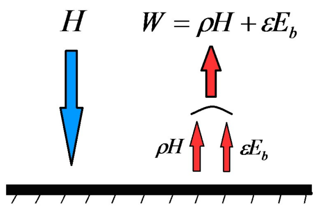

FIGURE 2. Scheme of surface energy exchange.

The third property concerns the closed envelopes that consist of different surfaces numbered as $1, 2, 3, \ldots, n$:

$$\sum_{j=1}^{n} F_{ij} = 1 \tag{5}$$

Relation (5) may be called the *summation rule*. It should be stressed that *its validity is restricted solely to the closed envelopes*. These may be, for example, inner spaces of rooms.

The view factors can be ordered into a matrix whose elements have to fulfil the basic rules (3) - (5):

$$\begin{pmatrix} F_{11} & F_{12} & \ldots & F_{1n} \\ F_{21} & F_{22} & \ldots & F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ F_{n1} & F_{n2} & \ldots & F_{nn} \end{pmatrix} \tag{6}$$

2.2. Algebraic equations of radiative heat transfer

The radiosity method relies on *diffuse and grey surfaces*. These surfaces emit or reflect radiation that is directional and wavelength independent. So the term ‘grey’ has little to do with real colours of surfaces. The two assumptions (diffuse and grey) are reasonable for most engineering applications. The emissivities ε of *grey diffuse Lambert’s surfaces* assume the values lying between zero and one. The radiative heat transfer between such surfaces is based on two laws, namely the Stefan-Boltzmann law and the Kirchhoff law [26–28]. The grey diffuse solid surfaces fulfil two conditions:

i) Transmittance is zero $\tau = 0$.

ii) Reflectance ρ and absorbance α (or emissivity ε) are independent of wavelengths and their sum equals one, i.e. $\rho + \varepsilon = 1$.

In Figure 2, there is a scheme of energy exchange occurring on the surface of a radiant body. The symbol H represents the total radiative heat flux (W/m^2) coming from all neighbouring surfaces whereas the symbol W represents **radiosity**, which is the total heat flux emitted from the surface (W/m^2). Radiosity is the sum of the Stefan-Boltzmann radiation ($\varepsilon E_b = \varepsilon \sigma T^4$, $\sigma = 5.67 \cdot 10^{-8}$ W/(m^{2}K^4)) and the reflected heat flux (ρH), i.e.

$$W = \varepsilon E_b + \rho H \tag{7}$$

$$H = \frac{W - \varepsilon E_b}{\rho}. \tag{8}$$

The resulted (net) radiative heat flux q related to the investigated surface is determined as the difference between the emitted (W) and coming (H) fluxes:

$$q = W - H = \varepsilon E_b - \varepsilon H = \begin{cases} > 0 \Longrightarrow \text{surface emits energy} \\ < 0 \Longrightarrow \text{surface absorbs energy} \end{cases} \tag{9}$$

By replacing H in Eq. (9) by the fraction from Eq. (8), we obtain:

$$q = W - \frac{W - \varepsilon E_b}{\rho} = \frac{(\rho - 1)W + \varepsilon E_b}{\rho} = \frac{-\varepsilon W + \varepsilon E_b}{\rho} = \frac{\varepsilon}{\rho}(E_b - W) \tag{10}$$

Thus, the density heat flux q_i of the i-th surface is given as follows

$$q_i = \frac{\varepsilon_i}{\rho_i}(E_{bi} - W_i) \qquad (\mathrm{W/m^2}) \tag{11}$$

From this equation, it is clear that the density heat flux q_i can be positive or negative. The *positive value* indicates that the surface emits the heat energy whereas *negative value* means that the surface absorbs the energy. To determine the density heat flux q_i from Eq. (11), it is necessary to know radiosity W_i. The equations for radiosities are specified in the following paragraphs.

2.3. Radiosity

The i-th surface of area S_i is supplied by the energy H_i coming from all the n surfaces (including the i-th surface itself if it is curved):

$$S_i H_i = \sum_{j=1}^{n} S_j F_{ji} W_j \qquad \text{(W)} \tag{12}$$

Taking into account the rule of symmetry $S_j \cdot F_{ji} = S_i \cdot F_{ij}$, the following equations may be obtained

$$S_i H_i = \sum_{j=1}^{n} S_i F_{ij} W_j \qquad \text{(W)} \tag{13}$$

$$H_i = \sum_{j=1}^{n} F_{ij} W_j \qquad (\text{W/m}^2) \tag{14}$$

By considering Equations (7) and (14), the system of n linear algebraic equations emerges:

$$W_i = \varepsilon_i E_{b_i} + \rho_i \sum_{j=1}^{n} F_{ij} W_j \quad i, j = 1, 2, 3, \dots, n \qquad (\text{W/m}^2) \tag{15}$$

The solution of this system offers n values of radiosities $\{W_i\}_{i=1}^{n}$ by means of which the total radiative heat flux q_i for each of the n surfaces may be calculated according to Eq. (11). If the heat flux q_i is determined, it is easy to calculate the net heat flow ϕ_i corresponding to i-th surface:

$$\phi_i = S_i q_i \qquad \text{(W)} \tag{16}$$

If this quantity is positive, the surface emits energy but negative quantity determines the absorption of the energy.

Equations (11), (15), and (16) are the basic algebraic relations that specify a radiative heat transfer between grey surfaces not only within the closed systems of surfaces, i.e. enclosures, but also in the open systems of surfaces.

2.4. Heat flux of absolutely black surface

In previous section 2.3, the basic relation for heat flux (11) associated with grey surfaces has been introduced. By applying Eq. (11) to absolutely black surfaces ($\tau = 0,\ \rho = 0,\ \varepsilon = 1$), it leads to an uncertain expression 0/0 and thus it is necessary to derive another expression that does not suffer from such drawback. From Eqs. (9) and (14), it follows

$$q_i = W_i - \sum_{j=1}^{n} F_{ij} W_j \tag{17}$$

which is a convenient alternative for calculating heat fluxes associated with both the *black and grey surfaces*. Relation (17) does not lead to any uncertain expression but in comparison with (11), it requires more numerical work and, for this reason, expression (11) is a preferable choice when dealing with *grey surfaces*.

2.5. Energy exchange between couples of surfaces

Heat exchange between two surfaces i and j, i.e. $\phi_{i\leftrightarrow j}$, will be calculated as a difference between those portions of heat that are absorbed by these surfaces

$$\phi_{i\leftrightarrow j} = \phi_{i\leftarrow j} - \phi_{i\rightarrow j} \tag{18}$$

The absorbed heat $\phi_{i\leftarrow j}$ will be calculated under the condition requiring that the surface j may emit energy while other surfaces emit nothing, i.e. they are ascribed by zero temperatures.

The absorbed heat $\phi_{i\leftarrow j}$ may be determined by means of relations (11) and (16) in which $E_{bj} = 0$:

$$\phi_{i\leftarrow j} = \frac{S_i \varepsilon_i}{\rho_i} (-W_i^{(j)}) \quad i \neq j \tag{19}$$

where $W_i^{(j)}$ is a radiosity when the surface j irradiates energy while others emit nothing because they are ascribed by zero temperatures.

Similarly, $\phi_{i\to j}$ will be calculated as the difference between absorbed portions of heat when the surface i irradiates energy while others do not emit any energy, i.e. they have zero temperatures:

$$\phi_{i\to j} = \frac{S_j\varepsilon_j}{\rho_j}(-W_j^{(i)}) \quad i \neq j \tag{20}$$

The expressions for $\phi_{i\leftarrow i}$ ($=\phi_{i\to i}$) assume a bit different forms. Their derivation can be started from a general definition (9):

$$\begin{aligned} q &= W - H \\ q &= \varepsilon E_b + \rho H - H \\ q &= \varepsilon E_b + (\rho - 1)H \\ q &= \varepsilon E_b - \varepsilon H \\ -\varepsilon H &= q - \varepsilon E_b \end{aligned} \tag{21}$$

The term $-\varepsilon E_b$ is actually the energy (W/m^2) absorbed by the surface under the investigation. To obtain power in Watts, this term has to be multiplied by the area S of the surface:

$$\begin{aligned} -S\varepsilon H &= Sq - S\varepsilon E \\ -S\varepsilon H &= \phi - S\varepsilon E \\ -S\varepsilon H &= \frac{S\varepsilon}{\rho}(E - W) - S\varepsilon E \\ -S\varepsilon H &= \frac{S\varepsilon}{\rho}\left[E - W - \rho E\right] \\ -S\varepsilon H &= \frac{S\varepsilon}{\rho}\left[(1-\rho)E - W\right] \\ -S\varepsilon H &= \frac{S\varepsilon}{\rho}\left[\varepsilon E - W\right] \end{aligned} \tag{22}$$

As seen from Eqs. (22), the expressions $\phi_{i\leftarrow i}$ ($=\phi_{i\to i}$) assume the following form

$$\phi_{i\leftarrow i} = \phi_{i\to i} = -S_i\varepsilon_i H_i = \frac{S_i\varepsilon_i}{\rho_i}(\varepsilon_i E_i - W_i) \tag{23}$$

Combining Eqs. (19), (20) and (23) more general expressions emerge:

$$\phi_{i\leftarrow j} = \frac{S_i\varepsilon_i}{\rho_i}(\varepsilon_i E_i \delta_{ij} - W_i^{(j)}) = \begin{cases} \dfrac{S_i\varepsilon_i}{\rho_i}(-W_i^{(j)}) & \text{for } i \neq j \\ \dfrac{S_i\varepsilon_i}{\rho_i}(\varepsilon_i E_{bi} - W_i^{(j)}) & \text{for } i = j \end{cases} \tag{24}$$

$$\phi_{i\to j} = \frac{S_j\varepsilon_j}{\rho_j}(\varepsilon_j E_j \delta_{ij} - W_j^{(i)}) = \begin{cases} \dfrac{S_j\varepsilon_j}{\rho_j}(-W_j^{(i)}) & \text{for } i \neq j \\ \dfrac{S_j\varepsilon_j}{\rho_j}(\varepsilon_j E_{bj} - W_j^{(i)}) & \text{for } i = j \end{cases} \tag{25}$$

where δ_{ij} is the Kronecker symbol

$$\delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}.$$

A general expression for the energy exchange between couples of surfaces may be obtained by using Eqs. (18) and (24)/(25)

$$\phi_{i\leftrightarrow j} = \phi_{i\leftarrow j} - \phi_{i\to j} = \begin{cases} 0 & \text{for } i = j \\ \dfrac{S_j\varepsilon_j}{\rho_j}W_j^{(i)} - \dfrac{S_i\varepsilon_i}{\rho_i}W_i^{(j)} & \text{for } i \neq j \end{cases} \quad (W) \tag{26}$$

Hottel and Sarofim [2] published a different expression for $\phi_{i\leftrightarrow j}$:

$$\dot{Q}_{j\leftrightarrow i} = \frac{S_i\varepsilon_i}{\rho_i}\left(\frac{{}_jW_i}{E_j} - \delta_{ij}\varepsilon_j\right)(E_j - E_i) = \frac{S_j\varepsilon_j}{\rho_j}\left(\frac{{}_iW_j}{E_i} - \delta_{ij}\varepsilon_i\right)(E_j - E_i) \tag{27}$$

In fact, Eqs. (26) and (27) are only two equivalent alternatives since they both yield the same numerical results. However, expression (26) seems to be *more instructive*.

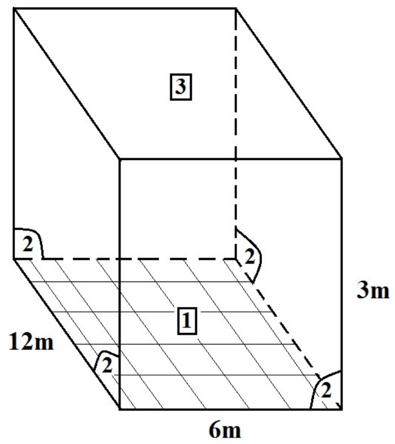

Figure 3. A simple room with three surfaces: heated floor no. 1, side walls no. 2, and ceiling no. 3.

Parameters	Surface 1	Surface 2	Surface 3
ε	0.95	0.80	0.75
ρ	0.05	0.20	0.25
T (K)	300	295	290
S (m^2)	72	108	72
εE_{ij} (W/m^2)	436.3065	343.5272	300.7712

Table 1. Input data for the simple room shown in Fig. 3.

As seen from Eq. (26), the expression $\phi_{i\leftrightarrow j}$ represents a *quasi-symmetric matrix* ($\phi_{i\leftrightarrow j} = -\phi_{j\leftrightarrow j}$). Each row of the matrix describes the energy exchange between a particular surface and the remaining neighbouring surfaces. For example, the i-th row of the matrix contains energies that are exchanged between the i-th surface and all the remaining ones. The matrix $\phi_{i\leftrightarrow j}$ of energy exchange assumes the following form

$$\begin{pmatrix} \phi_{1\leftrightarrow 1} & \phi_{1\leftrightarrow 2} & \cdots & \phi_{1\leftrightarrow n} \\ \phi_{2\leftrightarrow 1} & \phi_{2\leftrightarrow 2} & \cdots & \phi_{2\leftrightarrow n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n\leftrightarrow 1} & \phi_{n\leftrightarrow 2} & \cdots & \phi_{n\leftrightarrow n} \end{pmatrix} \tag{28}$$

3. Application of radiosity method

In this section, the algebraic radiosity method is applied to three particular cases. The first application presented in sub-section 3.1 illustrates the functionality of the method within an enclosure, which is represented by a simple room.

The second application described in sub-sections 3.2 is focused on an open system created paradoxically as a closed room envelope in which some parts of the envelope completely transmit heat radiation (quasi-enclosure).

The third application in sub-section 3.3 concerns a real open system in which some constructional parts of the room envelope are completely missing.

In open systems, the radiative heat energy is not conserved as in the closed systems but a large portion of the heat disappears in the open space. The radiosity method is capable of determining the escaped energy independently whether the system of surfaces is closed with some transmitting parts or some completely missing parts.

3.1. Application to enclosure

To illustrate the functionality of the radiosity method, a simple room with heated floor has been chosen as shown in Fig. 3.

The matrix of view factors F_{ij} determined by means of the graphs published in the technical literature [26–28] and the rules specified by Eqs. (3), (4), and (5) reads

$$\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.\overline{3} & 0.\overline{3} & 0.\overline{3} \\ 0.5 & 0.5 & 0 \end{pmatrix} \tag{29}$$

Radiosities (Eq. (15)):

$$\begin{aligned} W_1 &= 436.3065 + 0.05\,(0.5W_2 + 0.5W_3) \\ W_2 &= 343.5727 + 0.2(0.\bar{3}W_1 + 0.\bar{3}W_2 + 0.\bar{3}W_3) \\ W_3 &= 300.7712 + 0.25(0.5W_1 + 0.5W_2) \end{aligned} \tag{30}$$

$$W_1 = 457.352710, \quad W_2 = 430.140549, \quad W_3 = 411.707857\ \mathrm{W/m^2} \tag{31}$$

Heat flows (Eqs. (11) and (16)):

$$\begin{aligned} \phi_1 &= S_1 q_1 = +2\,622.85272\ \mathrm{W} \\ \phi_2 &= S_2 q_2 = -316.029168\ \mathrm{W} \\ \phi_3 &= S_3 q_3 = -2\,306.791512\ \mathrm{W} \end{aligned} \tag{32}$$

$$\sum_{i=1}^{3} \phi_i = 0.032040 \approx 0\ \mathrm{W} \tag{33}$$

The power $\phi_1 = +2\,622.85272$ W represents the energy flow from the heated floor. The floor has the highest temperature and thus emits the heat power from its surface into the room. The walls and the ceiling have lower temperatures as compared to the floor and from this reason, they assume the heat powers $\phi_2 = -316.029168$ W and $\phi_3 = -2\,306.791512$ W. These energies are absorbed into the volumes of the walls and the ceiling. In enclosures, the emitted energy is redistributed among the cooler surfaces and thus no portion of radiative energy can be lost ($\sum_{i=1}^{n} \phi_i = 0$). In our case, the sum of heat flows shows a tiny deviation from zero ($\sum_{i=1}^{n} \phi_i = 0.032$), but this is caused by rounding errors during the computations and inaccurately reading the values of view factors from their published graphs. The zero sum of radiative heat energies is the general property of closed systems and follows from the first and second laws of thermodynamics. However, since the algebraic radiosity model is not a 'product of nature' but a product of human creativity, it is desirable to verify whether this artificial model satisfies the laws of thermodynamics. For this reason, the property $\sum_{(i)} \phi_i = 0$ (which may be termed as *compensation theorem*) will be verified mathematically:

The proof is based on three properties related to view factors and specified by definition (1), symmetry property (3), and summation property (5):

$$\begin{aligned} \sum_{i=1}^{n} \phi_i &= \sum_{i=1}^{n} S_i \left(W_i - \sum_{j=1}^{n} F_{ij} W_j \right) = \\ &= \sum_{i=1}^{n} S_i W_i - \sum_{i=1}^{n} S_i \left(\sum_{j=1}^{n} F_{ij} W_j \right) = \\ &= \sum_{i=1}^{n} S_i W_i - \sum_{i=1}^{n} \sum_{j=1}^{n} S_i F_{ij} W_j = \\ &= \sum_{i=1}^{n} S_i W_i - \sum_{i=1}^{n} \sum_{j=1}^{n} S_j F_{ji} W_j = \\ &= \sum_{i=1}^{n} S_i W_i - \sum_{j=1}^{n} \sum_{i=1}^{n} S_j F_{ji} W_j = \\ &= \sum_{j=1}^{n} S_j W_j - \sum_{j=1}^{n} S_j W_j \cdot \left(\sum_{i=1}^{n} F_{ji} \right) = \\ &= \sum_{j=1}^{n} \left[(S_j W_j) - (S_j W_j) \left(\sum_{i=1}^{n} F_{ji} \right) \right] = \\ &= \sum_{j=1}^{n} (S_j W_j) \cdot \left[1 - \sum_{i=1}^{n} F_{ji} \right] = \\ & \begin{cases} = 0 & \text{(for closed envelopes only)} \\ \neq 0 & \text{(for opened envelopes only)} \end{cases} \end{aligned} \tag{34}$$

The expression $[1 - \sum_{i=1}^{n} F_{ji}]$ in Eq. (34) is zero only if $\sum_{i=1}^{n} F_{ji} = 1$, that holds solely for *enclosures (closed envelopes)*, as follows from *summation rule* (5). However, in *open envelopes, summation rule (5) does not hold,*

Surface 1 radiates others not - $W_i^{(1)}$ (W/m^2)	$W_1^{(1)} = 438.677644$	$W_2^{(1)} = 35.565381$	$W_3^{(1)} = 59.280378$
Surface 2 radiates others not - $W_i^{(2)}$ (W/m^2)	$W_1^{(2)} = 10.501990$	$W_2^{(2)} = 372.237215$	$W_3^{(2)} = 47.842401$
Surface 3 radiates others not - $W_i^{(3)}$ (W/m^2)	$W_1^{(3)} = 8.173076$	$W_2^{(3)} = 22.337953$	$W_3^{(3)} = 304.585078$

Table 2. Special radiosities $W_i^{(j)}$ for computing the matrix $\phi_{i\leftrightarrow j}$.

i.e. $\sum_{i=1}^{n} F_{ji} \neq 1$, and thus the sum of radiative heat flows $\sum_{i=1}^{n} \phi_i$ *assumes non-zero values*, as shown in Eq. (34).

At first sight, it might seem that the values of radiosities, heat fluxes and heat flows shown in various places of the present paper include a too large number of figures after the decimal points. This is because the computations are performed in the regime of double precision using the input data containing more figures after the decimal points in order *to suppress rounding errors* and meet the requirement of the *compensation theorem* ($\sum_{(i)} \phi_i = 0$) as accurate as possible.

The tested room shown in Fig. 3 has been chosen as a very simple room possessing only three surfaces with different temperatures and emissivities. In reality, rooms may have a much larger number of surfaces with different geometries, temperatures and emissivities (windows, doors, furnishings, wooden or artificial decorations, carpets, textiles, etc.). The possible geometric complexity of rooms concerns solely the matrix of view factors. Although there are many tables and formulae for determining view factors in the literature, e.g. in Ref. [26–28], many general cases are missing. Since the analytical derivation of view factors in these cases may be difficult, the numerical computations of double integrals (Eq. (1)) seem to be the only way to overcome this problem. An interesting method for the numerical evaluation of view factors has been presented only recently [29]. However, for common geometries of internal rooms, there is a sufficient number of formulae, graphs or tables to easily determine the corresponding view factors. As soon as the matrix of view factors is formed, other computational steps related to radiosites, heat flux and heat flows are the matter of routine numerical operations for which a large number of various surfaces with different temperatures or emissivities does not represent a larger problem. In addition, it is clear that quite small surfaces compared to the area of the room envelope have small influences on results and thus neglecting some of them will not introduce an essential inaccuracy.

Finally, as the heat losses of the room under investigation are concerned, some parts of the absorbed energies $\phi_2 = -316.029168\,\mathrm{W}$ and $\phi_3 = -2\,306.791512\,\mathrm{W}$ may be propagated by conduction through the solid constructions and at the external sides they may be transferred by convection and radiation into the exterior. But the real amount of heat loss depends on the quality of thermal insulation, the temperature difference between interior and exterior, external coefficients of heat transfer and external emissivities. The complete computations of heat losses that include inner convective and radiative transfers along with external convective and radiative transfers are accomplished in the second serial paper [30], which thematically continues the present paper.

3.1.1. Matrix of heat exchange

In the following paragraphs, a detail analysis of radiative heat exchange between couples of surfaces of the investigated simple room (Fig. 3) is presented. Although such an analysis is not required for determining heat losses, it could be useful for understanding the mechanism of energy exchange between surfaces. For this purpose, the matrix of energy exchange $\phi_{i\leftrightarrow j}$ will be computed according to relation (26). This relation requires special radiosities $W_i^{(j)}$ determined when some surfaces (i-surfaces) have zero temperatures and only one surface (j-surface) has its own correct non-zero temperature. These radiosities have been computed and their values are gathered in Tab. 2.

The matrix $\phi_{i\leftrightarrow j}$ of energy exchange (Eq. (26)):

$$\begin{pmatrix} 0_{1\leftrightarrow 1} & +997.522272_{1\leftrightarrow 2} & +1\,623.793680_{1\leftrightarrow 3} \\ -997.522272_{2\leftrightarrow 1} & 0_{2\leftrightarrow 2} & +683.962920_{2\leftrightarrow 3} \\ -1\,623.793680_{3\leftrightarrow 1} & -683.962920_{3\leftrightarrow 2} & 0_{3\leftrightarrow 3} \end{pmatrix} \Longrightarrow \begin{pmatrix} \sum_1 = +2\,621.315952 \approx \phi_1 \\ \sum_2 = -313.559352 \approx \phi_2 \\ \sum_3 = -2\,307.756600 \approx \phi_3 \end{pmatrix} \quad \mathrm{W} \qquad (35)$$

$$\sum\nolimits_1 + \sum\nolimits_2 + \sum\nolimits_3 = 0 \quad \text{(Precisely)}$$

As seen from matrix (35), the sum of numbers in each row approximately equals the net heat power ϕ computed previously (see Eqs. (32)).

Parameters	Floor no. 1	Side walls no. 2	Ceiling no. 3
ε	0.95	1	0.75
ρ	0.05	0	0.25
$T\,(\mathrm{K})$	300	0	290
$S\,(\mathrm{m}^2)$	72	108	72
$\varepsilon E_b\,(\mathrm{W/m}^2)$	436.3065	0	300.7712

TABLE 3. Input data for the quasi-enclosure of the simple room.

From the first row of matrix (35), it is obvious that surface no. 1 (floor) has emitted $\sim 998\,\mathrm{W}$ towards surface 2 (side walls) and $\sim 1\,624\,\mathrm{W}$ towards surface no. 3 (ceiling). Both these transfers result in $\sim 2\,621\,\mathrm{W}$ of the net heat power emitted by the floor (no. 1).

From the second row of matrix (35), it follows that surface no. 2 (side walls) has absorbed ~ 998 W coming from the floor (no. 1) but, simultaneously, surface no. 2 has sent $\sim$ 684 W towards the ceiling (no. 3). By summing these two transfers, the net heat power absorbed by the side walls (no. 2) has amounted to $\sim 314\,\mathrm{W}$.

Similarly, the third row of matrix (35) has let us know that the ceiling (no. 3) has absorbed $\sim 1\,624\,\mathrm{W}$ coming from the floor (no. 1) and also $\sim 684\,\mathrm{W}$ that has been sent from the walls (no. 2). Thus, the net absorbed power of the ceiling (no. 3) is $\sim 2\,308\,\mathrm{W}$.

The radiative heat energies redistributed within the room enclosure and specified by matrix (35) fulfil precisely the compensation theorem in accordance with the first and the second laws of thermodynamics, namely, each transfer of energy is directed from a warmer surface to cooler one and all the transferred radiative energies are conserved inside the enclosure without any possibility to escape, i.e. their sum is precisely zero ($\sum_{i=1}^{3} \phi_i = 0$).

Since the majority of basic monographs, e.g. [26–28], do not discuss behaviour of the radiosity method within *open room envelopes*, the following two sections 3.2 and 3.3 are devoted to this problem. Section 3.2 illustrates the behaviour of the radiosity method within the quasi-enclosure whereas section 3.3 explores the properties of this method within a real open envelope. We would like to mention that these two open structures do not serve as the prototypes of window openings since the glazed windows under the acting of *long-wave room thermal radiation* can be treated as non-transparent structures undertaking only *surface absorption* without surface transparency just as with solid walls. This fact has been explained in the introduction. Sections 3.2 and 3.3 that discuss open systems along with section 3.1 that explores closed systems provide a *complete treatise* concerning the functionality of the radiosity method applied to a variety of surface arrangements.

3.2. Application to quasi- enclosure

The quasi-enclosure is realized like a simple room shown in Fig. 3 whose *four side walls no. 2 are completely transparent by heat radiation.* These side walls are considered as *ideally black bodies with zero temperatures*, i.e. $\varepsilon = 1,\ \rho = 0,\ T_2 = 0$. In fact, such an enclosure behaves as an open system in which heat escapes through the ideally absorptive side walls. The input data are summarized in Tab. 3.

The matrix of view factors F_{ij} is the same as in section 3.1:

$$\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.\overline{3} & 0.\overline{3} & 0.\overline{3} \\ 0.5 & 0.5 & 0 \end{pmatrix} \tag{36}$$

Radiosities (Eq. 15):

$$\begin{aligned} W_1 &= 436.3065 + 0.05\,(0.5W_2 + 0.5W_3) \\ W_2 &= 0 \\ W_3 &= 300.7712 + 0.25\,(0.5W_1 + 0.5W_2) \end{aligned} \tag{37}$$

$$W_1 = 445.2170,\ W_2 = 0,\ W_3 = 356.4233\,(\mathrm{W/m}^2) \tag{38}$$

Heat flows (Eqs. (11), (16) and (17)):

$$\begin{aligned} \phi_1 &= S_1 q_1 = 72 \cdot \frac{1}{0.05} \cdot (436.3065 - 0.95 \cdot 445.21708) = +19\,224\,\mathrm{W} \quad \text{(see Eq. (11))} \\ \phi_2 &= S_2 q_2 = 108 \cdot \left[0 - \left(0.\overline{3}W_1 + 0.\overline{3}W_3\right)\right] = -28\,859\,\mathrm{W} \quad \text{(see Eq. (17))} \\ \phi_3 &= S_3 q_3 = 72 \cdot \frac{1}{0.25} \cdot (300.7712 - 0.75 \cdot 356.4233) = +9\,635\,\mathrm{W} \quad \text{(see Eq. (11))} \end{aligned} \tag{39}$$

Surface 1 radiates others not - $W_i^{(1)}$ (W/m^2)	$W_1^{(1)} = 437.67423$	$W_2^{(1)} = 0$	$W_3^{(1)} = 54.70928$
Surface 2 "radiates" others not - $W_i^{(2)}$ (W/m^2)	$W_1^{(2)} = 0$	$W_2^{(2)} = 0$	$W_3^{(2)} = 0$
Surface 3 radiates others not - $W_i^{(3)}$ (W/m^2)	$W_1^{(3)} = 7.54285$	$W_2^{(3)} = 0$	$W_3^{(3)} = 301.71410$

TABLE 4. Special radiosities $W_i^{(j)}$ for computing the matrix $\phi_{i\leftrightarrow j}$ of the quasi-enclosure.

$$\sum_{i=1}^{3} \phi_i = 0\,\mathrm{W} \tag{40}$$

In the case of the *quasi-enclosure*, the radiosity method provides results that resemble the results achieved with the real *closed envelope*, i.e. the heat powers transferred are 'conserved' since their sum is zero ($\sum_{i=1}^{3} \phi_i = 0$) in a complete accordance with the *compensation theorem*. In addition, the transfers of energies are directed from wormer surfaces (floor and ceilings) to cooler surface (side walls). Yet, there is some difference. The side walls (no. 2) are completely transparent for heat radiation and thus their *total net absorbed power* 28 859 W represents the heat that escapes into the open space beyond the quasi-enclosure. This result corresponds to that of a *real open enclosure* of a room whose side walls are completely removed. The question is whether the radiosity method will determine the same heat loss (28 859 W) if the real open enclosure consisting only of the heated floor and the ceiling is considered. The answer can be found in section 3.3.

Prior to starting section 3.3, it would be instructive to explore the energy exchanges between surfaces within this quasi-enclosure. The corresponding results are presented in Tab. 4 and within the matrix $\phi_{i\leftrightarrow j}$.

The matrix $\phi_{i\leftrightarrow j}$ of energy exchange (Eq. (26)):

$$\begin{pmatrix} 0_{1\leftrightarrow 1} & +17\,725.8_{1\leftrightarrow 2} & +1\,498.6_{1\leftrightarrow 3} \\ -17\,725.8_{2\leftrightarrow 1} & 0_{2\leftrightarrow 2} & -11\,133.2_{2\leftrightarrow 3} \\ -1\,498.6_{3\leftrightarrow 1} & +11\,133.2_{3\leftrightarrow 2} & 0_{3\leftrightarrow 3} \end{pmatrix} \Longrightarrow \begin{pmatrix} \sum_1 = +19\,223.4 = \phi_1 \\ \sum_2 = -28\,859.0 = \phi_2 \\ \sum_3 = +9\,634.6 = \phi_3 \end{pmatrix} \quad \mathrm{W} \tag{41}$$

From the first row of matrix (41) it is obvious that floor (no. 1) has sent 17 725.8 W to the transmittable side walls (no. 2) and this energy escapes into the open space. In addition, the floor (no. 1) has also sent 1 498.6 W to the ceiling (no. 3).

The second row of matrix (41) summarizes the total energy coming from the floor (17 725.8 W) and the ceiling (11 133.2 W). These energies are absorbed by the transmittable side walls (no. 2) and represent the total heat losses going into the open space (28 859.0 W).

The third row of matrix (41) describes the energy exchanges between the ceiling (no. 3) and the remaining surfaces. The ceiling has absorbed 1 498.6 W from the floor (no. 1) but it has emitted 11 133.2 W towards the transmittable side walls (no. 2) and this energy represents the heat loss escaping into the open space. The net power of the ceiling is 9 634.6 W.

By comparing heat exchanges within the quasi-open enclosure (matrix (41)) and the regular enclosure (matrix (35)), it is obvious that the 'quasi-open' envelopes of rooms enormously increase heat losses. This is due to the energy that escapes through the transparent (open) parts of envelopes.

In the next section, the case of the quasi-open enclosure will be replaced by the real open system and the problem will be recalculated. This will enable us to compare results of the radiosity method applied to differently arranged surfaces.

3.3. Application to regularly open envelope

Let us consider the arrangement shown in Fig. 3, in which the side walls have been completely removed. The corresponding input data can be found in Tab. 1. The remaining surfaces are associated with the **floor (no. 1)** and the **ceiling (no. 2)**. - the ceiling has been **renumbered**. This arrangement corresponds to the quasi-enclosure in which the side walls have really been removed. The matrix of view factors of this two-dimensional problem may be easily derived from matrix (36):

$$\begin{pmatrix} 0 & 0.5 \\ 0.5 & 0 \end{pmatrix} \tag{42}$$

Radiosities (Eq. 15):

$$\begin{aligned} W_1 &= 436.3065 + 0.05 \cdot (0.5W_2) \\ W_2 &= 300.7712 + 0.25 \cdot (0.5W_1) \end{aligned} \tag{43}$$

Surface 1 radiates other not - $W_i^{(1)}$ (W/m^2)	$W_1^{(1)} = 437.67423$	$W_2^{(1)} = 54.70928$
Surface 2 radiates other not - $W_i^{(2)}$ (W/m^2)	$W_1^{(2)} = 7.54285$	$W_2^{(2)} = 301.71406$

TABLE 5. Special radiosities $W_i^{(j)}$ for computing the matrix $\phi_{i\leftrightarrow j}$ of the regularly open envelope.

$$W_1 = 445.2170,\ W_2 = 0,\ W_3 = 356.4233\,(\mathrm{W/m^2}) \tag{44}$$

Heat flows (Eqs. (11), (16) and (17)):

$$\begin{aligned} \phi_1 &= S_1 q_1 = 72 \cdot \frac{1}{0.05} \cdot (436.3065 - 0.95 \cdot 445.21708) = +19\,224.4\,\mathrm{W} \quad \text{(see Eq. (11))} \\ \phi_2 &= S_2 q_2 = 72 \cdot \frac{1}{0.25} \cdot (300.7712 - 0.75 \cdot 356.4233) = +9\,634.6\,\mathrm{W} \quad \text{(see Eq. (11))} \end{aligned} \tag{45}$$

$$\sum_{i=1}^{3} \phi_i = +28\,859\,\mathrm{W} \tag{46}$$

As seen from results (45) and (46), the floor and the ceiling emit energies (they provide only positive values), i.e. no portion of energy is absorbed by these surfaces (no negative values are present). This means that the emitted energies (**28 859 W**) represent heat losses directed into the open space. This conclusion is in a full agreement with the foregoing computations performed within the quasi-open enclosure presented in section 3.2.

The total heat flow (46) of the regularly open system cannot be zero as in the case of the enclosure (40) or (33) since a large portion of energy escapes into the open space.

Let us now investigate the matrix of energy exchange associated with the two-dimensional radiosity method (see Tab. 5).

The matrix $\phi_{i\leftrightarrow j}$ of energy exchange (Eq. (26)):

$$\begin{pmatrix} 0_{1\leftrightarrow 1} & +1\,498.6_{1\leftrightarrow 2} \\ -1\,498.6_{2\leftrightarrow 1} & 0_{2\leftrightarrow 2} \end{pmatrix} \Longrightarrow \begin{pmatrix} \sum_1 = +1\,498.6 \\ \sum_2 = -1\,498.6 \end{pmatrix} \mathrm{W} \tag{47}$$

Matrix (47) specifies the energy transferred between the floor and the ceiling. The energy of 1 498.6 W has been emitted by the floor (no. 1) and the same energy has been absorbed by the ceiling (no. 2). This is in agreement with the computations performed within the quasi-enclosure in sub-section 3.2. This energy exchange obeys the principle of the second law of thermodynamics that declares that, during natural processes, the heat always flows from the wormer body to the cooler one. The elements $\phi_{1\leftrightarrow 2}$ and $\phi_{2\leftrightarrow 1}$ of matrix (47) are in agreement with the elements $\phi_{1\leftrightarrow 3}$ and $\phi_{3\leftrightarrow 1}$ of matrix (41).

However, the *two-dimensional* modification of radiosity model is not capable of providing such detailed information of energy exchanges between surfaces as in the case of thre-dimensional radiosity method. Although numerically more demanding, the *three-dimensional* radiosity model used in the previous section is more informative.

4. Conclusions

The discussed computational model for the estimation of radiative heat transfer is applicable both to the open and closed systems of surfaces. In the first computational step, the matrix of view factors is formed on the basis of the graphs or formulae published in the technical literature and by means of the three auxiliary rules termed as the symmetry rule (Eq. 3), the zero rule (Eq. 4) and the summation rule (Eq. 5). The radiosities are then determined from the system of linear algebraic equations (Eqs. 15). The radiosities enable to compute heat fluxes q_i (Eqs. 11/17) and heat flows ϕ_i (Eq. 16) associated with particular surfaces. In addition, the radiative heat portions that are exchanged between couples of surfaces may be identified as the elements of the heat exchange matrix $\phi_{i\leftrightarrow j}$.

The radiative portions of heat that are transferred in the open and closed systems of surfaces differ as to the estimation of energy losses. In the open system, the large portion of the heat is emitted irreversibly into the open space whereas in the closed system, some portion of the heat is reflected back into the interior, which ensures a more economical performance of enclosures. In addition, the stationary thermal state in the *interior of the enclosure* is characterized by the equilibrium between emitted and absorbed portions of heat, i.e. the sum of heat flows is zero ($\sum_{(i)} \phi_i = 0$). This is in an agreement with the *compensation theorem* and the basic laws of thermodynamics. On the contrary, the total heat flow of the *regularly open system* cannot be zero as in the case of the *closed system* since a large portion of energy escapes into the open space.

The open system of surfaces may be investigated either as the quasi-enclosure or the regularly open envelopes. Although the radiosity method applied to both these arrangements provides equivalent results, the concept of quasi-enclosure seems to be more informative.

The present paper has summarized the algebraic computational model based on radiosity. Different properties and behaviours of that model when applied to various systems of room envelopes have been analysed and discussed. All this should serve as a preparation stage for the development of a more general model for the estimation of heat losses of inner spaces of buildings, in which the heat transfer is realized by radiation and convection. The general model of combined radiative and convective heat transfer is formulated and applied in the separate serial paper [30] thematically related to the present contribution.

The performed analysis of the algebraic radiosity model enables to draw several summarizing conclusions:

(1.) The algebraic radiosisty model is capable of a correct functioning not only in closed systems but also in *open enclosures*. The analysis of its properties in open enclosures is usually missing in monographs related to this model.

(2.) The validity of the *compensation theorem* related to the overall radiative heat flow in enclosures has been proven on the rigorous mathematical basis (Eq. 34).

(3.) A new alternative formula for heat exchange between the couples of surfaces has been derived (Eq. (26)).

(4.) So far, the algebraic radiosity model has been preferably used for thermal applications in the field of mechanical engineering where it was originally formed. The present paper has illustrated that this model may also be useful in the field of thermal building technology and building physics since it is capable of straightforwardly computing the radiative heat energy produced by heating systems.

(5.) Although the presented application of the radiosity method has been aimed at heated large-area floors, the method may also be applicable to radiant heat energy emitted by small-area radiant panels often used in housing dwellings. Such applications are in progress.

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