Fractality of Electrostatic Microdischarges on the Surface of Polymers

T. Ficker, V. Kapička, J. Macur, P. Slavíček, P. Benešovský

Ramified Lichtenberg figures caused by electrostatic microdischarges on the surface of polymeric polyethylene terephthalate have been studied. They occurred in consequence of the previous electret forming of the polymeric sheets and were initiated in the air gap between the grounded electrode and the polymeric sheets. Multifractal image analysis was used to determine the fractal dimensions of the Lichtenberg patterns in dependence on the loading voltages used for electret forming.

Keywords: Electrostatic microdischarges, Lichtenberg figures, multifractal formalism, fractal dimension, electret saturated states.

1 Introduction

Electrostatic erosive microdischarges accompany triboelectric charging of various objects during their industrial production as well as during their use in everyday life. It is especially polymeric solids that are liable to surface charge accumulation to such an extent that they can even cause electric breakdown of an adjacent gas environment. Usually, this phenomenon is undesirable because of its disruptive effects in industrial production, laboratory research, and civil working activities. The study of these surface microdischarges is therefore useful from various points of view.

Surface microdischarges often assume the form of branched channel structures whose particular channels represent streamer discharges propagated along the surface of the object. Such branched patterns can be visualised by using photographic or powder techniques. These visible tracks are called Lichtenberg figures and have often been used to study gaseous discharges on the surface of dielectrics [1–6].

To visualise Lichtenberg figures, it was necessary to chose a transparent, highly resistive material in the form of thin layers (foils). Polyethylene terephthalate (PET) seems to be a convenient material for such a purpose. It is in broad use in industrial, academic, as well as civil applications. PET is a hard, stiff, strong, dimensionally stable material that absorbs very little water. It has good gas barrier properties and good chemical resistance except to alkalis. Its crystallinity varies from amorphous to fairly high crystalline; it can be highly transparent and colourless but thicker sections are usually opaque. It is widely known as a biaxially oriented and transparent and colourless but thicker sections are usually opaque. It is widely known as a biaxially oriented and colourless but thicker sections are usually opaque. It is widely known as a biaxially oriented and colourless but thicker sections are usually opaque. It is widely known as a biaxially oriented and colourless but thicker sections are usually opaque. It is widely known as a biaxially oriented and transparent, highly resistive material in the form of thin layers (foils). Polyethylene terephthalate (PET) seems to be a convenient material for such a purpose. It is in broad use in industrial, academic, as well as civil applications. PET is a hard, stiff, strong, dimensionally stable material that absorbs very little water. It has good gas barrier properties and good chemical resistance except to alkalis. Its crystallinity varies from amorphous to fairly high crystalline; it can be highly transparent and colourless but thicker sections are usually opaque. It is widely known as a biaxially oriented and transparent, highly resistive material in the form of thin layers (foils). Polyethylene terephthalate (PET) seems to be a convenient material for such a purpose. It is in broad use in industrial, academic, as well as civil applications. PET is a hard, stiff, strong, dimensionally stable material that absorbs very little water. It has good gas barrier properties and good chemical resistance except to alkalis. Its crystallinity varies from amorphous to fairly high crystalline; it can be highly transparent and colourless but thicker sections are usually opaque. It is widely known as a biaxially oriented and transparent, highly resistive material in the form of thin layers (foils). Polyethylene terephthalate (PET) seems to be a convenient material for such a purpose. It is in broad use in industrial, academic, as well as civil applications. PET is a hard, stiff, strong, dimensionally stable material that absorbs very little water. It has good gas barrier properties and good chemical resistance except to alkalis. Its crystallinity varies from amorphous to fairly high crystalline; it can be highly transparent and colourless but thicker sections are usually opaque. It is widely known as a biaxially oriented and transparent, highly resistive material in the form of thin layers (foils). Polyethylene terephthalate (PET) seems to be a convenient material for such a purpose. It is in broad use in industrial, academic, as well as civil applications. PET is a hard, stiff, strong, dimensionally stable material that absorbs very little water. It has good gas barrier properties and good chemical resistance except to alkalis. Its crystallinity var...
same dimension $E$. The basic cell of the grid is of a linear size $\epsilon$. Using probability moments $p_i$ a partition sum $m_{q \mid i}$ is defined on the grid

$$m(\epsilon, q) = \sum_{i=1}^{\infty} p_i^q(\epsilon), \quad p_i(\epsilon) = \left(\frac{n_i}{N}\right), \quad \sum_{i=1}^{\infty} p_i(\epsilon) = 1, \quad q \in (-\infty, \infty)$$

where $n_i$ is a number of points in the $i$-th cell and $N$ represents a number of all points of the complex system.

The spectrum of generalised dimensions $D_q$ is usually used for the purpose of the analysis

$$D_q = \lim_{q \to -1} \frac{\partial \ln m(\epsilon, q)}{\partial \ln \epsilon}.$$  \hspace{1cm} (2)

The complex systems under investigation are in the form of graphical bitmap files that were created by means of a scanner. In this way the object is converted into a set of graphical pixels that are covered with a two-dimensional grid with the basic cell $\epsilon$. On this grid the partition sum (1) is determined. To avoid a dependence on the orientation of the grid, several positions of the grid in the Euclidean space are realised and an average partition sum $M_{q \mid i}(\epsilon, q)$ is calculated

$$M_{q \mid i}(\epsilon) = \frac{1}{\epsilon^2} \sum_{j=1}^{\infty} m_{j \mid i}(\epsilon, q).$$

The averages $M_{q \mid i}(\epsilon)$ are determined for a series of $\epsilon$-grids and the slopes within the coordinate system $(\ln \epsilon, \ln M_{q \mid i}(\epsilon))$ are estimated using the linear regression. The slopes divided by $(q - 1)$ give the generalised dimensions $D_q$. An object which produces identical $D_q$ for all $q$ is called ‘monofractal’ while different $D_q$-values indicate a ‘multifractal’ object.

### 3 Results and discussion

In this study 150 PET samples were investigated. Three different forming voltages ($8$ kV, $4$ kV, $2$ kV) and ten different forming intervals ($480$ min., $240$ min., $120$ min., $60$ min., $30$ min., $15$ min., $5$ min., $1$ min., $0.5$ min., $0.25$ min.) were used to create corresponding electret states each of which was represented mostly by five PET samples. However, some of the samples were removed from the statistical ensemble because their Lichtenberg structures were not developed or because of other experimental problems. The remaining 110 samples were analysed and their fractal dimensions de-

<table>
<thead>
<tr>
<th>$t$/min.</th>
<th>8 kV</th>
<th>4 kV</th>
<th>2 kV</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>–</td>
<td>1.384</td>
<td>1.253</td>
</tr>
<tr>
<td>240</td>
<td>–</td>
<td>1.387</td>
<td>1.404</td>
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<tr>
<td>120</td>
<td>1.404</td>
<td>1.296</td>
<td>1.205</td>
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<tr>
<td>60</td>
<td>1.505</td>
<td>–</td>
<td>1.260</td>
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<tr>
<td>30</td>
<td>1.522</td>
<td>1.214</td>
<td>1.133</td>
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<tr>
<td>15</td>
<td>1.613</td>
<td>1.175</td>
<td>1.214</td>
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<tr>
<td>5</td>
<td>1.514</td>
<td>1.267</td>
<td>1.098</td>
</tr>
<tr>
<td>1</td>
<td>1.494</td>
<td>1.268</td>
<td>discharges disappear</td>
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<tr>
<td>0.5</td>
<td>1.290</td>
<td>1.204</td>
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<tr>
<td></td>
<td>–</td>
<td>1.085</td>
<td></td>
</tr>
</tbody>
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Average dimensions $D_t$ of saturated states and their probability errors

$$\begin{align*}
1.477 \pm 0.026 & \quad 1.253 \pm 0.022 & \quad 1.224 \pm 0.025
\end{align*}$$

Fig. 1: Lichtenberg figures generated at 8 kV and 30 min., $(D = 1.486$, sample no. 27)
The resulting Hausdorf-Besicovitch dimensions of the Lichtenberg figures generated from the saturated electret states are listed in Table 1. The experiments were performed at normal atmospheric conditions with small variations (less than 5 %) around the average values: \( p_2 = 102.19 \text{ kPa} \), \( \Theta_0 = 24.31 ^\circ \text{C} \), \( \varphi_0 = 69.7 \% \text{ RH} \). The processing of one typical sample (cleaning, conditioning, forming, separating, developing, digitising, filtering, analysing) took about 30 hours.

Comparing our experimental conditions (forming electric fields and times) with those of normally used [12], it is possible to assume that for all our combinations of voltages and times (Table 1) the saturated electret states were reached. These saturated electret states are dependent only on forming voltages \( U \) and not on times \( t \), since the chosen times \( (t \geq 0.25 \text{min.}) \) are sufficiently long for reaching electret saturation with PET samples at 2 kV and higher voltages \( (U \geq 2 \text{ kV}) \).

Therefore, if the fractal dimension \( D \) of the Lichtenberg figures risen from the saturated electret states is dependent on the stage of saturation, then the fractal dimension \( D \) should be dependent on the forming voltages \( U \) and has to show an increasing dependence \( D(U) \). This hypothesis has proved to be sound and is illustrated by Figs. 4–6 in which the increasing linear dependence \( D_a U + b \) is well recognisable. In addition, if our forming times are sufficiently long for reaching saturation states, the average dimensions \( D_s \), calculated from those belonging to the same columns of Table 1, i.e. for the voltages of 8 kV, 4 kV, and 2 kV (dimensions 1.477, 1.253 and 1.223), also have to be an increasing function of voltage.
Ds(U), which is documented in Fig. 7. In addition, this trend of branching structures with increasing voltages is visible in Figs. 1–3 which present three different Lichtenberg figures in dependence on voltage (the forming times were chosen sufficiently long to reach saturated electret states for the respective voltages). All these experimental findings lead us to the opinion that the morphology of Lichtenberg figures (i.e. their fractal dimension) is not a consequence of purely stochastic processes but rather it is dependent on the stage of the electret saturation which is determined solely by the forming voltages $U$, provided sufficiently long forming times have been used.

4 Conclusions

The performed experiments have shown that the branching of Lichtenberg figures caused by electrostatic microdischarges on the surface of polyethylene terephthalate is not quite a stochastic phenomenon but rather a process that is governed by the stage of electret forming. Since the attainability of a saturated electret state is dependent on the forming voltage $U$ and time $t$, the fractal dimension $D$ of the corresponding Lichtenberg figures should be a function of these parameters, i.e. $D(U, t)$. Nevertheless, for sufficiently long times $t > t_{\text{min}}$ the dimension $D$ is a function of voltage only, i.e. $D(U)$. It seems to be natural that the $D(U)$ function cannot increase its values to infinity but rather it should be restricted by some maximum $D_{\text{max}}$ and minimum $D_{\text{min}}$ values characterising a particular electret ($0 \leq D_{\text{min}} < D_{\text{max}} < 2$). However, a verification of this idea would require further experiments.

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References


