Influence of Ring Stiffeners on a Steel Cylindrical Shell

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Shell structures are usually formed from concrete, steel and nowadays also from many others materials. Steel is typically used in the structures of chimneys, reservoirs, silos, pipelines, etc. Unlike concrete shells, steel shells are regularly stiffened with the help of longitudinal and/or ring stiffeners.

The authors of this paper investigated steel cylindrical shells and their stiffening with the use of ring stiffeners. The more complete the stiffening, the more closely the shell will act to beam theory, and the calculations will be much easier. However, this would make realization of the structure more expensive and more laborious.

The target of the study is to find the limits of ring stiffeners for cylindrical shells. Adequate stiffeners will eliminate semi-bending action of the shells in such way that the shell structures can be analyzed with the use of numerical models of the struts (e.g., by beam theory) without significant divergences from reality.

Recommendations are made for the design of ring stiffeners, especially for the distances between stiffeners and for their bending stiffness.

Keywords: shell, cylindrical shell, chimney, steel structure, wind load, ring stiffener, distance of stiffeners, stiffness of ring stiffener, design.

1 Introduction

Shell structures are a very broad topic. Shells differ in their shape (cylindrical, spherical, parabolic, etc.), in the way in which their walls are stiffened (laterally, longitudinally, with orthogonal stiffeners), by type of load action, by type of material used (concrete, steel), etc. This great variability and range of shell performance presents many practical difficulties in their design. In this paper we deal just with one type of steel cylindrical shell loaded by wind.

Large non-elastic deformations lead to buckling or plastic failure of steel cylindrical shells. This type of failure differs from that in the case, for example of slab components, where bending is predominant and the behaviour is easy to predict.

It is common knowledge that thin shell structures transfer their loading by means of the membrane tensional and compression forces that act in the walls of the shell. Also it is known that shells have very high efficiency under symmetrical loading and support. Transfer of asymmetrical loading and local load is not desirable.

For a symmetrical load, a simple structural geometry and simple boundary conditions (e.g., cylindrical shells with axis symmetrical loading), an analysis of the shell is not too difficult. But when at least one of these factors is missing, analysis becomes more complicated and the results are often unexpected.

In real life, shell structures are used mainly as chimneys, tanks, pipelines and silos. An analysis can be made with the use of simplifying methods if all the above mentioned conditions are fulfilled. More sophisticated methods for analysing, shell structures are necessary if the conditions are more complex. The level of analysis rises from the simplest calculations in linear structural analysis (LA) to stability calculations of ideal structures (without imperfections) and also geometrically and materially non-linear calculations with structural imperfections (GMNIA). All these methods are mentioned in the new European Standard for steel shells [27]. Another important shell analysis could study the shapes of oscillations of the structure, and with analysis of the structure for the basic dynamic loading.

Most of the types of analysis of the numerical models mentioned above are more or less accessible in recent practice, but during the preliminary design of structures, i.e., when only the basic dimensions of the structure need to be established, complex computer analysis is inapplicable for time and financial reasons. For this reasons in the doctoral thesis of the first author [7] of this paper the theme of cylindrical shells loaded by wind was thoroughly investigated under the supervision of the second author.

The main aim of the theoretical investigations was to obtain limits for stiffening of a cylindrical shell (by ring stiffeners only) such that the strut approach based on beam theory for calculating of a chimney shell would be realistic enough, i.e., when it would be realistic to neglect semi-bending [15]. The main goal of this work is to determine the distance of the stiffeners and to determinate their minimum stiffness.

2 Parametric studies

In the first stage of the work, the performance of the chimney shell was investigated in a parametric study on numerical models solved by FEM. The computational study focused on determining the limit distance of the stiffeners. Models were used in the calculation: a linear analysis of the structure (LA) and a geometrical non-linear analysis of the structure (GNA). The GNA model was based on the Newton–Raphson method. The comparative calculation made use of classical linear analysis of buckling.

The second part of the work contains parametric studies for determining the optimum stiffness of the ring stiffeners, making use of the optimum distance of the stiffeners determined in the first step of the work.

For an investigation of the interaction between the diameter of the cylindrical shell, its thickness, the distance of the stiffeners and their optimum stiffness (taking into account wind load only), two independent studies evaluated by regression analysis were carried out.

All calculations were done using ESA-Prima Win computer program, produced by SCIA with solvers developed by the FEM consulting company [13]. The software is suitable for static analysis (LA, GNA, and etc.) and also for dynamic and stability analysis of structures. In the numerical models, the stiffeners were simulated by curved beam elements.

The numerical models of the shells were made of quadrilateral elements formed from four triangular plane sub elements with one common node – the centre of the quadrilateral element [14]. The centre is defined as the intersection point of two straight lines connecting the mid-side points of the two opposite side of the element. This definition allows the quadrilateral element to have, if needed, a singular (in fact triangular) form with two nodes in one point, without numerical difficulties. Therefore the triangular sub element with $3\times6 = 18$ nodal deformation parameters forms the basis of the whole calculation. The physical sense of the parameters is in the case of 1D elements ($u, v, w, \varphi_x, \varphi_y, \varphi_z$), which ensures nodal compatibility between 1D and 2D elements.

In a quadrilateral element the deformation parameters of its centre ($u, v, w, \varphi_x, \varphi_y, \varphi_z$) are common for all four triangular sub elements, and are eliminated in advance and consistently by static condensation. In its final form a quadrilateral element possesses only $4\times6 = 24$ natural displacement parameters at its vertex nodes, and its stiffness matrix $\boldsymbol{K}_L$ is of the order (24, 24).

The size of the elements was chosen between 10 and 75 mm according to the diameter and length of the numerical model. In the introduction to the first parametric study, we detected that was it necessary to exploit at least 100 elements along the perimeter and length of the model. The size of the elements strongly influenced the precision of the parametric studies.

The wind load was taken according to the European standard [28], with the reference wind speed 24 m·s^{-1} and open terrain. The shell was assumed to be made from common S235 low carbon steel.

3 Determining the limit distance of the stiffeners

The numerical model of the shell used in the parametric study is shown in Fig.1. For the parametric study, four basic sets of calculations were used according to the diameter of the shell. The calculated diameters were 400 mm, 800 mm, 1600 mm and 2400 mm. The varying parameter was the length and thickness of the shell.

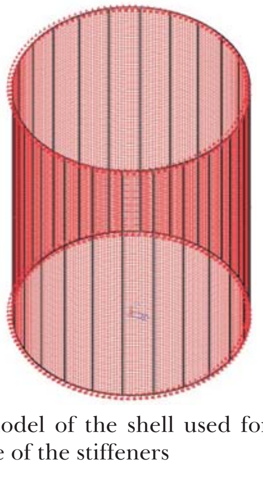


Fig. 1: Numerical model of the shell used for determining the limit distance of the stiffeners

The boundary conditions simulated an absolutely rigid ring stiffener of the cylindrical shell. The whole numerical model represented a segment of the shell between two rigid stiffeners. In the linear analysis of the structure (LA) and in the geometrical nonlinear analysis (GNA) of the structure, the following parameters were checked:

- Deformation U_y, which is the deformation of the shell measured in the wind direction. The maximum value is always situated in the middle of the length of the structure, on the side exposed to the wind. The minimum value is in the middle of the length of the structure, again at periphery angle $\theta = 135°$, measuring this angle from the side exposed to the wind.
- Deformation U_x, which is the deformation of the shell measured perpendicularly to the wind direction. The maximum value is situated in the middle of the length of the structure at periphery angle $\theta = 75°$.
- Circumferential stress $\sigma_{\theta,\max}$, which is the tensional stress in θ direction. The maximum circumferential stress of the shell model was found in the middle of the length of the structure at periphery angle $\theta = 135°$. The minimum circumferential stress $\sigma_{\theta,\min}$ was found in the middle of the length of the structure at periphery angle $\theta = 90°$.
- Meridian stress $\sigma_{X,\min}$, which is the compression stress of the shell. The minimum meridian stress of the shell model is situated in the middle of the length of the structure on the side exposed to the wind.

An example of the calculated parameters of the shell in the model for the middle of the shell length is shown in Fig. 2.

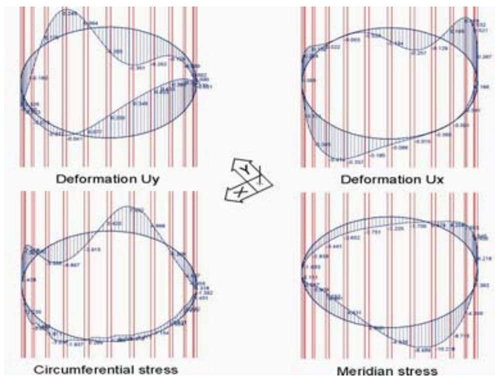


Fig. 2: Calculated deformations and stresses of the shell in the middle of the length; the values are obtained for diameter 400 mm, thickness 0.5 mm and length 2.0 m, and for loading by the wind downstream of axis Y

In the classical linear analysis of buckling, characterised by a bifurcation, the smallest eigenvalue (defined as the critical multiple R_{cr} according to [27] of the given loading) is found when stability is lost.

For comparison, simple beam models were examined at the same time. Numerical models composed from a 1D element were used. These beam models had the same boundary conditions as those used in the shell models. The loading of the beam model was obtained by integrating the loading around the shell. The beam models were used mainly for splitting the beam behaviour of the shell (sometimes called semi-bending behaviour). In all beam models, a linear analysis of the structure (LA) was made, because a geometrically non-linear analysis (GNA) was confirmed to be almost irrelevant for these models. In the beam models, the deformation of the beam and the maximum tension stress was at its highest compared with similar parameters of the shell structures mentioned above.

The dependences between the parameters of the shell (thickness and distance of the stiffeners) and between the parameters described in above (deformations, stresses, and critical buckling resistance) were investigated. The main parameter of the "shell" behaviour was defined as deformation U_y, which was obtained after subtracting the deformation determined by the beam model and the GNA model, respectively. In accordance with practical experience with steel shells, the limit level of deformation was chosen as $D/6000$, where D is the diameter of the shell. For comparison, values corresponding to $D/10000$ were also investigated. At the same time, the circumferential stress was also checked. It was found to be very low, with values not higher than 3 MPa.

In the next step, we investigated the influence of the distance of the ring stiffeners on the thickness of the shell. Linear regression [20] was chosen as the optimum method for this. The relation between the distances of the stiffeners and the thickness of the shell are shown in Fig. 3. Parameters "a" and "b" of Fig. 3 are plotted for different diameters of the shell in Fig. 4. The relation between the distance of the stiffeners L and thickness t is as follows:

$$L = at + b\,, \tag{1}$$

where a, b are parameters of linear regression.

For the allowable (limit) deformation of the shell $D/6000$, formula (1) can be rewritten into formula (2):

$$\begin{aligned} L = {} & (-3.57\times10^{-9}D^3 - 1.82\times10^{-4}D^2 \\ & + 5.21\times10^{-1}D + 2.74\times10^{2})t \\ & + 1.4\times10^{-3}D^2 - 1.7237D + 1348.5\,, \end{aligned} \tag{2}$$

and for $D/10000$ into formula (3):

$$\begin{aligned} L = {} & (9.29\times10^{-8}D^3 - 5.48\times10^{-4}D^2 \\ & + 9.12\times10^{-1}D + 6.23\times10^{1})t \\ & + 1.4\times10^{-3}D^2 - 2.1326D + 1505\,. \end{aligned} \tag{3}$$

In both formulas it is necessary to specify diameter D and thickness t in millimetres. Distance L is also in mm.

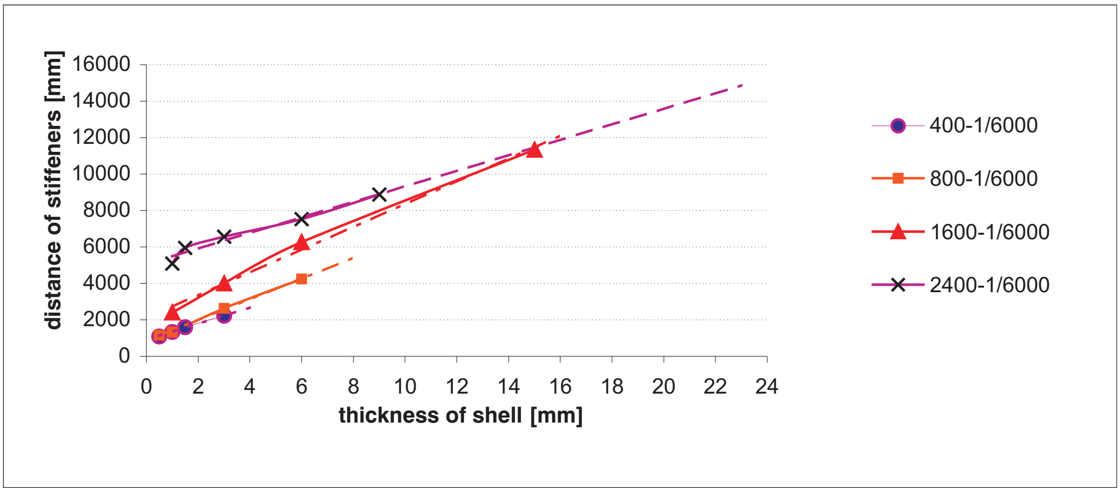
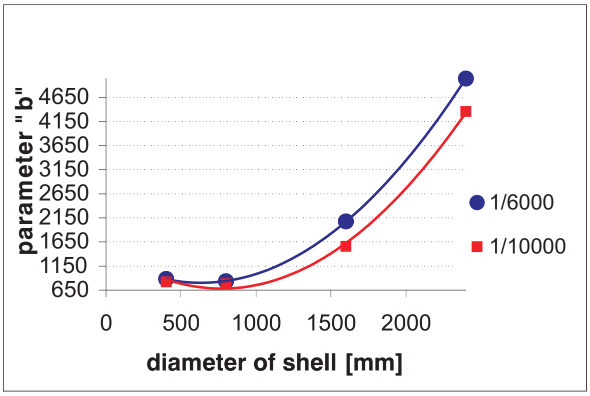


Fig. 3: Dependence of the maximum distance of the stiffeners on the thickness of the shell for diameter 400 mm, 800 mm, 1600 mm and 2400 mm, for limit deformation of the shell $D/6000$



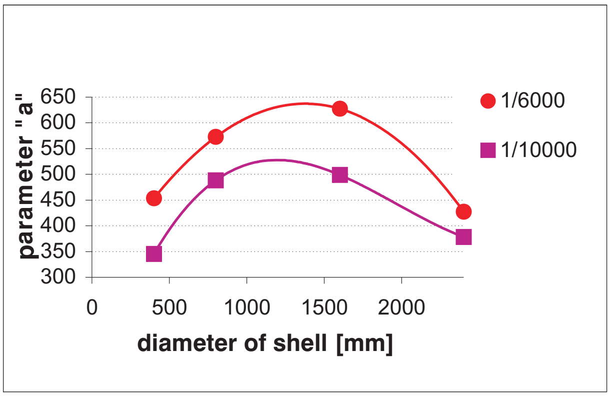


Fig. 4: Dependence of parameters "a" and "b" of the linear regression on the diameter of the shell for limit deformation of the shell $D/6000$ and $D/10000$

We can summarize this section as follows: if the distance between the stiffeners is smaller than L according to (2) or (3) we may apply the beam model for the shell with good results.

4 Assessment of optimum stiffeners

The numerical model of the shell used in the parametric study is shown in Fig. 5 and Fig. 6. For the study, we chose four basic sets of calculations according to the diameter of the shell (400, 800, 1600 and 2400 mm). The varying parameter was the thickness and length of the shell and the stiffness of the ring stiffener in relation to the cross section area of the shell. The boundary conditions in the numerical model simulated the absolutely stiff base of the chimney on the lower end of the shell. The complete numerical model consisted of three shell segments separated by rigid stiffeners. There were two internal stiffeners and one terminal stiffener on the free end of the structure, see Fig. 6. This parametric study investigated the semi-bending component behaviour of the cylindrical shell. LA and classical linear analysis of buckling were performed for all models. For some models, GNA was also performed, in order to verify the original model more thoroughly.

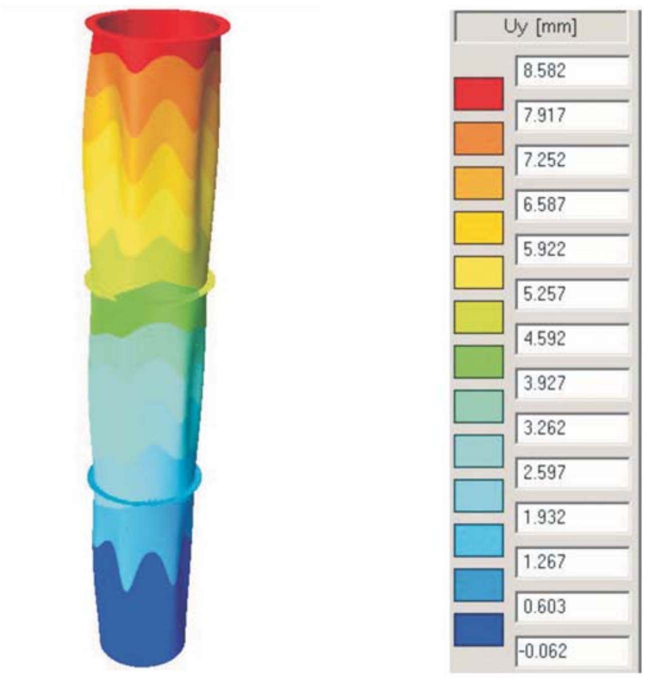


Fig. 5: Deformed numerical model of the shell for diameter 2400 mm and thickness 1.5 mm, with the ring stiffeners modelled with the help of the shell elements, with isozones of the deformation in the wind direction

In the linear analysis, the following parameters were investigated:

- Deformation U_y of the stiffeners, which is the deformation of every stiffener in the wind direction. The maximum deformation of the stiffeners is always situated on the side exposed to the wind at periphery angle $\theta = 0°$. The minimum deformation is situated at periphery angle $\theta = 180°$.
- Maximum values of bending moment M_z, shear force V_y, and normal force N on every stiffener.
- Deformation U_y of the shell in the wind direction. The maximum deformation of the shell is situated in the middle of the length of the structure on the side exposed to the wind.

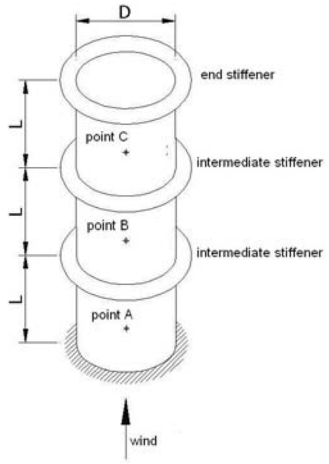


Fig. 6: Numerical model of the shell that was used for determining the optimum distance of the stiffeners. The stiffeners and the other investigated points are identified in the figure

The observed points are marked A, B and C in Fig. 6. An example of the calculated parameters of the ring stiffener is shown in Fig. 7.

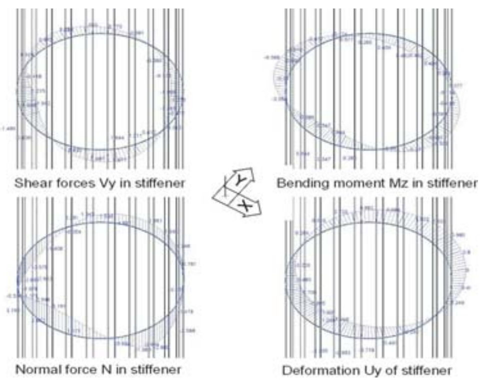


Fig. 7: Parameters of ring stiffener No. 2 for diameter of a cylindrical shell 1600 mm, thickness 3.0 mm and distance between stiffeners 4.0 m. Stiffeners of profile 300/24 (second moment 5.4×10^7 mm^4) and loading by wind downstream of axis Y are investigated.

At the same time, simple beam models for separating the beam and semi-bending behaviour of the shell were also investigated. Fig. 8 and Fig. 9 compare the beam deformation and the semi-bending deformation along the whole length of the structure for one numerical model and a certain stiffness of the stiffener.

The results are as follows: higher stiffness of the internal stiffeners causes higher internal forces in these stiffeners, but an entirely opposite dependence was observed for the end

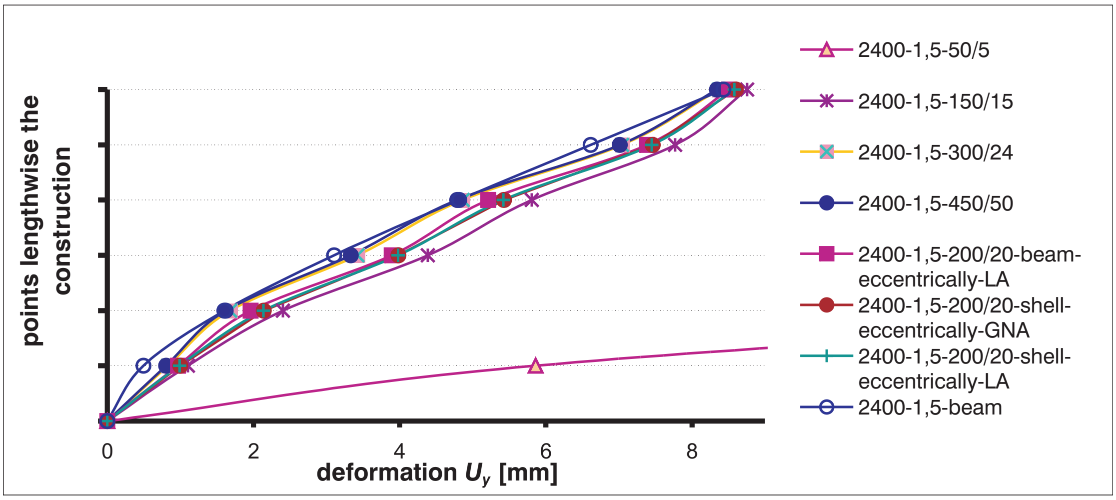


Fig. 8: Comparison of the lengthwise deformation of the shell for one numerical model and a given stiffness of the stiffener on the side exposed to the wind

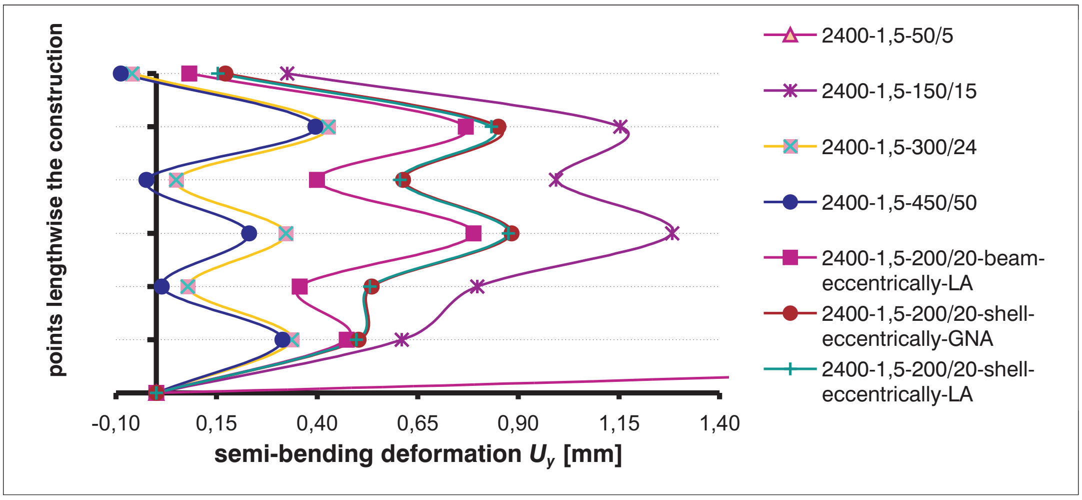


Fig. 9: Comparison of the lengthwise semi-bending deformation of the shell for one numerical model and a given stiffness of the stiffener on the side exposed to the wind

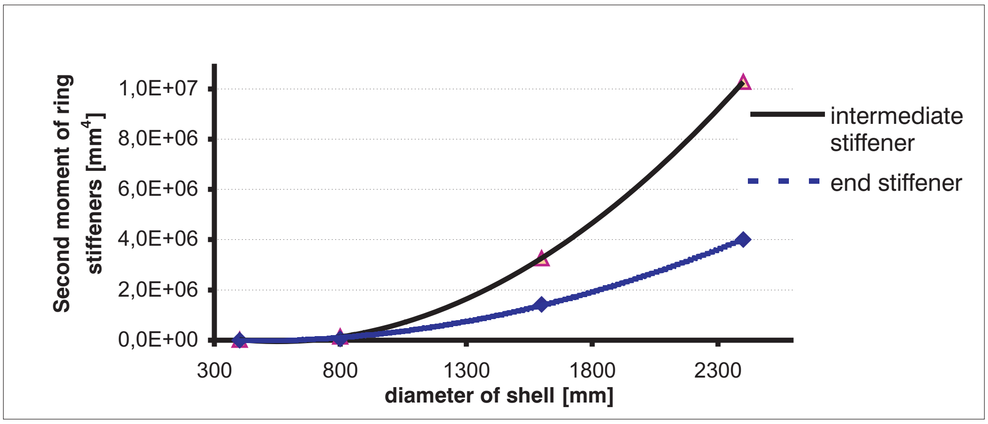


Fig. 10: Dependence of the second moment of the ring stiffeners on the diameter of the shell

stiffener. It seems that the function of the two types of stiffeners is completely different.

For each set of calculations we obtained the relation between the parameters of the shell (thickness, length, and

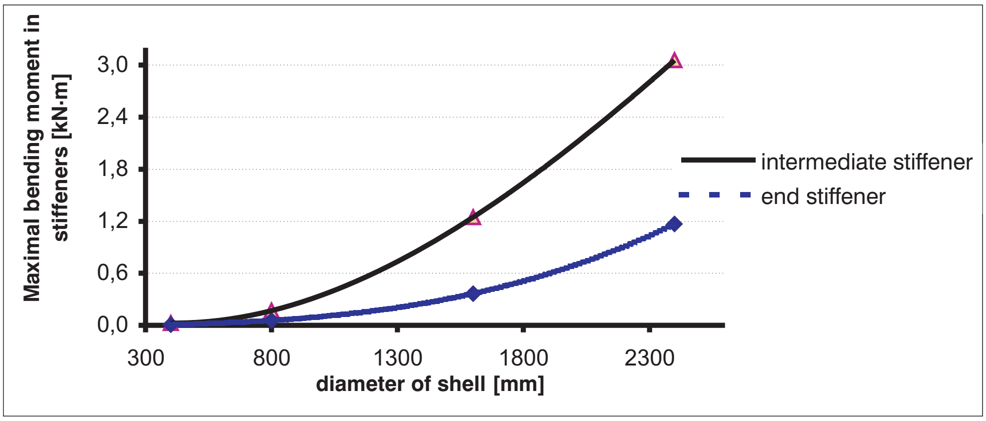


Fig. 11: Dependence of the maximum bending moment of the ring stiffeners on the diameter of the shell

stiffness of the stiffeners) and the most realistic numerical model. The biggest semi-deformation U_y of a stiffener measured in the wind direction was chosen as the master parameter of the shell (for determining the optimum stiffener) was chosen. The deformations were separated for the beam and shell model. All intermediate stiffeners were analysed separately from the end stiffeners.

During the investigation of semi-bending deformations of the stiffeners for varying thickness of shell and the second moment of the stiffeners, we found that if the distance of the stiffeners corresponds with (2), the behaviour of the different stiffeners is almost identical. Therefore in every set of calculations we defined the uniform dependence between the biggest semi-deformation and the second moment of the stiffeners for a given type of stiffener (intermediate, end). With these dependencies, we calculated the optimum stiffness and distance of the stiffeners. The value $D/6000$ was used as the optimum level of semi-bending deformations of the stiffener, the same value as in the first part of the paper.

The next step of the study was to search for the dependencies of the optimum second moment of the stiffeners on the diameter of the shell, and the relationship between the internal forces in the stiffener and the diameter of the shell. By the least squares method, polynomial regressions of the second and third degree were chosen as the best approximation. The relations between the second moment of the stiffeners and the diameter of the shell and between the internal forces in the stiffeners and the diameter are shown in Figs. 10 to 13.

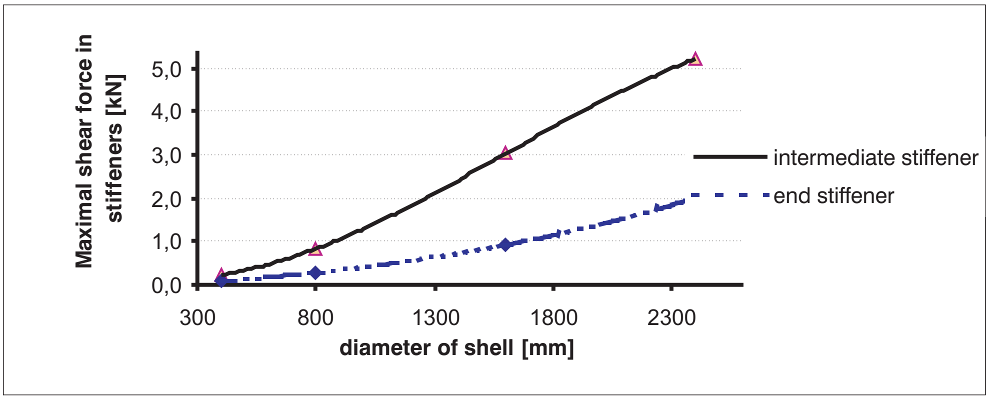


Fig. 12: Dependence of the maximum shear force of the ring stiffeners on the diameter of the shell

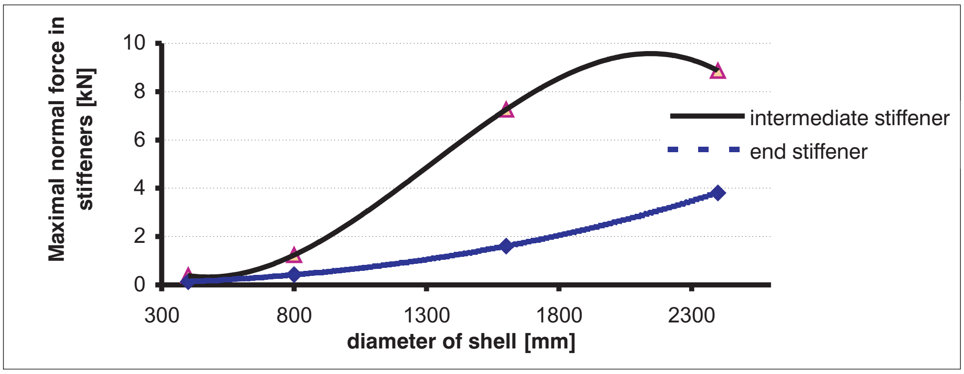


Fig. 13: Dependence of the maximum normal force of the ring stiffeners on the diameter of the shell

The analytical formulas are as follows. It is important to emphasize that these formulas are valid only for distances according to (2).

The optimum second moment of intermediate stiffeners is given by the formula:

$$I = 3.03D^2 - 3350D + 872700\,. \tag{4}$$

The optimum second moment of the end stiffener is given by the formula:

$$I = 1.06D^2 - 935.6D + 178200\,. \tag{5}$$

In both formulas (4) and (5), it is necessary to have the diameter of shell D in millimetres. The second moment of the stiffeners is in mm^4.

In intermediate stiffeners, the internal forces are (maximum values):

$$\begin{aligned} M = {} & -1.244\times10^{-10}D^3 + 1.171\times10^{-06}D^2 \\ & -9.079\times10^{-04}D + 2.097\times10^{-01}, \end{aligned} \tag{6}$$

$$\begin{aligned} V = {} & -5.315\times10^{-10}D^3 + 2.541\times10^{-06}D^2 \\ & -9.710\times10^{-04}D + 2.563\times10^{-01}, \end{aligned} \tag{7}$$

$$\begin{aligned} N = {} & -3.974\times10^{-09}D^3 + 1.562\times10^{-05}D^2 \\ & -1.217\times10^{-02}D + 3.005\,. \end{aligned} \tag{8}$$

In the end stiffener, the internal forces are:

$$\begin{aligned} M = {} & 8.302\times10^{-11}D^3 - 1.194\times10^{-08}D^2 \\ & +4.259\times10^{-05}D - 1.254\times10^{-02}, \end{aligned} \tag{9}$$

$$\begin{aligned} V = {} & 5.252\times10^{-11}D^3 + 9.791\times10^{-08}D^2 \\ & +3.283\times10^{-04}D - 7.084\times10^{-02}, \end{aligned} \tag{10}$$

$$\begin{aligned} N = {} & 8.306\times10^{-11}D^3 + 4.020\times10^{-07}D^2 \\ & +1.441\times10^{-04}D + 2.940\times10^{-03}. \end{aligned} \tag{11}$$

In formulas (6) to (11) it is necessary to have the diameter of shell D in millimetres. The internal forces of the stiffeners are in kN×m (bending moments) and in kN (normal and shear forces).

From all the formulas given above, it is obvious that the second moment of the stiffeners and the maximum internal forces in the stiffeners are dependent only on the diameter of the shell. The effect of the shell thickness was important only for the limit distance of the stiffeners.

5 Conclusion

This paper has investigated the effect of the distance and stiffness of ring stiffeners on the behaviour of a cylindrical steel shell loaded by wind. The aim of work was to determine these characteristics of the shell in a way that would enable these structures to be calculated by beam numerical models and would make the analyses easier for everyday design of structures of this type, for example steel chimneys. All results are presented in analytic formulas that are applicable in practice.

Acknowledgment

This work was carried out at the Department of Steel Structures at the Czech Technical University in Prague. The research was supported by Ministry of Education project 6840770001. This financial support is gratefully acknowledged.

References

[1] Blacker, M. J.: "Buckling of Steel Silos and Wind Action." Proceedings of Silo – conference, University of Karlsruhe, 1988, p. 318–330.

[2] Brendel, B., Ramm, E., Fischer, D. F., Rammerstorfer, F. G.: "Linear and Non-linear Stability Analysis of a Thin Cylindrical Shell under Wind Loads." *Journal Struct. Mech.*, (1981), p. 91–113.

[3] Brown, C. J., Nielsen, J.: *Silos. Fundamentals of Theory, Behaviour and Design.* London and New York, E & FN Spon, 1998.

[4] Derler, P.: "Load-carrying Behaviour of Cylindrical Shells under Wind Load." PhD thesis, Technical University of Graz, 1993 (in German).

[5] Esslinger, M., Poblotzki, G.: "Buckling under Wind Pressure." *Der Stahlbau*, Vol. **61** (1992), No. 1, p. 21–26 (in German).

[6] Koloušek, V., Pirner, M., Fischer, O., Náprstek, J.: *Wind Effect on Civil Engineering Structures*. Academia, Prague and Elsevier, London, 1983.

[7] Lemák, D.: „Vliv obvodových výztuh na chování válcové skořepiny". disertace ČVUT Praha, 2003.

[8] Lemák, D., Studnička, J.: „Vliv obvodových výztuh na působení ocelové válcové skořepiny." *Stavební obzor*, Vol. **13** (2004), No. 4, p. 112–117.

[9] Lemák, D., Studnička, J.: "Behaviour of Steel Cylindrical Shells." Proceedings International Conference VSU 2004, Sofia, May, 2004.

[10] Flügge, W.: *Stresses in Shells*. Berlin, Springer, 1973.

[11] Greiner, R.: "Buckling of Cylindrical Shells with Stepped Wall-thickness under Wind Load." *Der Stahlbau*, Vol. **50** (1981), No. 6, p. 176–179 (in German).

[12] Johns, D. J.: "Wind Induced Static Instability of Cylindrical Shells." *J. Wind Eng. and Ind. Aerodynamics*, Vol. **13**, (1983), p. 261–270.

[13] Kolář, V., Němec, I., Kanický, V.: *FEM Principy a praxe metody konečných prvků*. Computer Press, 1997.

[14] Kolář, V., Němec, I.: "Finite Element Analysis of Structures." United Nations Development Program, Economic Com. for Europe, Workshop on CAD Techniques, Prague – Geneva, Vol. I, June, 1984, 248 p.

[15] Křupka, V., Schneider, P.: *Konstrukce aparátů.* Brno, PC-DIR, 1998.

[16] Křupka, V.: *Výpočet válcových tenkostěnných kovových nádob a potrubí.* Praha, SNTL, 1967.

[17] Kundurpi, P. S., Samevedam, G., Johns, D. J.: "Stability of Cantilever Shells under Wind Loads." Proc. ASCE, 101, EM5, 1975, p. 517–530.

[18] Megson, T., Harrop, J., Miller, M.: "The Stability of Large Diameter and Thin-walled Steel Tanks Subjected

to Wind Loading." Proc. of International Colloquium, University of Ghent, 1987, p. 529–538.

[19] Rammerstorfer, F. G., Auli, W. - Fischer, F.: "Uplifting and Stability of Wind-loaded Vertical Cylindrical Shells." *Engineering Computations*, Vol. **2** (1985), p. 170–180.

[20] Rektorys, K.: *Přehled užité matematiky II*, Prometheus Praha, 2000.

[21] Resinger, F., Greiner, R.: "Buckling of Wind Loaded Cylindrical Shells – Application to Unstiffened and Ring-stiffened Tanks." Proc. State of the Art Colloquium, University of Stuttgart, Germany, 1982, p. 6–7.

[22] Schweizerhof, K., Ramm, E.: "Stability of Cylindrical Shells under Wind Loading with Particular Reference to Follower Load Effect." Proc. Joint US-Australian Workshop on Loading, Analysis and Stability of Thin-Shell Bins, Tanks and Silos, University of Sydney, 1985.

[23] Singer, J., Arbocz, J., Weller, T.: *Buckling Experiments: Experimental Methods in Buckling of Thin-Walled Structures.* New York, Wiley, 2002.

[24] Studnička, J.: *Navrhování tenkostěnných za studena tvarovaných profilů.* Praha, Academia, 1994.

[25] Uematsu, Y., Uchiyama, K.: "Deflection and Buckling Behaviour of Thin, Circular Cylindrical Shells under Wind Loads. *J. Wind Eng. and Ind. Aerodynamics*, Vol. **18** (1985), No. 3, p. 245–262.

[26] Zienkiewicz, O. C.: *The Finite Element Method in Engineering Science.* Mc Graw - Hill, London, 1979 3rd Ed., 787 p., Chapter 18–19 (Nonlinear Problems).

[27] prEN 1993-1-6, "Design of Steel Structures," Part 1–6, General rules – Supplementary Rules for Shells, stage 34, CEN Brussels, 2004.

[28] EN 1991-2-4, "Basis of Design and Actions on Structures," Part 2–4: Actions on Structures – Wind Actions, CEN Brussels, 2003.

[29] prEN 1993-1-1, "Design of Steel Structures," Part 1–1, General Rules and Rules for Buildings, stage 49, CEN Brussels, 2003.

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