*Acta Polytechnica* **62**(1):80–84, 2022
https://doi.org/10.14311/AP.2022.62.0080


# ON THE WESS-ZUMINO MODEL: A SUPERSYMMETRIC FIELD THEORY

Daya Shankar Kulshreshtha[a,*], Usha Kulshreshtha[b]

[a] *University of Delhi, Department of Physics and Astrophysics, Delhi – 110007, India*
[b] *University of Delhi, Kirori Mal College, Department of Physics, Delhi – 110007, India*
[*] corresponding author: dskulsh@gmail.com

Abstract. We consider the free massless Wess-Zumino Model in 4D which describes a supersymmetric field theory that is invariant under the rigid or global supersymmetry transformations where the transformation parameter $\epsilon$ (or $\bar{\epsilon}$) is a constant Grassmann spinor. We quantize the theory using the Hamiltonian and path integral formulations.

Keywords: Wess-Zumino Model, supersymmetric field theories, Hamiltonian and path integral quantization.

## 1. Introduction

Supersymmetry (SUSY) is a symmetry that rotates bosons into fermions and fermions into bosons. It is one of the beautiful symmetries of nature. Also, a field theory (FT) which remains invariant under the rigid or global supersymmetry transformations (where the transformation paremeter is a constant Grassman spinor) and which also satisfies the super Poincare algebra (SPA) is usually referred to as a supersymmetric field theory (SFT). In this article, we consider the free massless Wess-Zumino Model (WZM) in 4D which describes a SFT. It may be important to mention here that the WZM is the first known example of an interacting 4D quantum field theory with linearly realised SUSY, studied by Wess and Zumino using the dynamics of a single chiral superfield (composed of a complex scalar and a spinor fermion). It may be important to mention that the WZM represents a typical SFT which is of central importance in the theory of SUSY, supergravity and superstring theory (SST) and for further details we refere to the work of Refs. [1–8].

The WZM describes an example of a non-manifest supersymmetry [5]. One could of course go to the formalism of superspace and superfields to construct a theory that has a manifest supersymmetry [5]. Taking this theory as an example, it is possible to formulate supersymmertic field theories in different dimensions including in higher dimensions. The WZM also provides a basic framework for the study of Ramond Nievue Schwarz (RNS) SST [8] which is an example of a SST with non-manifest SUSY. Further, starting with the WZM, it is also possible to construct a supergravity theory [1–6, 8].

SPA is a graded Lie algebra that includes anti-commutation relations (ACR's) involving the supercharge $Q_a$ – the generator of the SUSY transformations. WZM is one of the simplest examples of a SFT. In this article, we discuss the supersymmetry of WZM and present some remarks with respect to the rigid or global supersymmetry versus the local supersymmetry (which happens to be a Supergravity theory). Finally we consider the constraint quantization of this theory [7]. It is important to mention that the supersymmetry has profound applications in conformal hadron physics from light-front holography where it even has some observational prospects [9–11].

As mentioned above, the supersymmetry is a symmetry that relates bosonic and fermionic variables (or the bosons and fermions) so that:

$$\delta B = \bar{\epsilon} F \ , \quad \delta F = \epsilon \, \partial B \ ; \quad \partial \equiv \partial_\mu \tag{1}$$

Here, $\delta$ is bosonic, $B$ is bosonic and $F$ is fermionic. The transformation parameter $\epsilon$ (or $\bar{\epsilon}$) is a constant Grassman spinor and is fermionic. Grassman variables are anti-commuting. Supergravity theory on the other hand is a theory that has "local supersymmetry" and it is invariant under local Susy transformations where the transformation parameter depends on the spacetime $x^\mu$. So the transformation parameter for supergravity: $\epsilon(x^\mu)$ or $\bar{\epsilon}(x^\mu)$ depends on $x^\mu$ and hence supergravity is a "gauge theory" of gravity. In contrast to this the WZM is a supersymmetric FT with rigid or global (not local) Supersymmetry.

Let us us consider two consecutive infinitesimal rigid supersymmetry transformations of a bosonic field $B$:

$$\delta_1 \ B = \bar{\epsilon}_1 F \ , \quad \delta_2 F = \epsilon_2 \partial B \tag{2}$$

This then implies that the two internal SUSY transformations lead us to a spacetime translation:

$$\{\delta_1, \delta_2\} B = a^\mu \partial_\mu B \ ; \quad a^\mu = (\bar{\epsilon}_2 \gamma^\mu \epsilon_1) \tag{3}$$

Presence of a spacetime derivative of $B$ on right hand side (RHS) of above equation suggests that the Susy is an extension of the Poincare spacetime symmetry:

$$\{Q_a, \bar{Q}_b\} = 2(\gamma^\mu)_{ab} P_\mu \tag{4}$$

Supercharge $Q_a$ ($a = 1, 2, 3, 4$ in 4D) is the generator of SUSY transformations. It is related to the generator of spacetime translations $P_\mu$ and therefore is not an internal symmetry generator. The SUSY transformation is an extention of the Poincare spacetime symmetry. Supercharge $Q_a$ is a spinor. It is fermionic and anti-commuting. Poincare algebra (PA) after including the supersymmetry becomes the SPA.

## 2. The Wess-Zumino Model

The WZM is defined (on-shell) by the Lagrangian density [5]:

$$\begin{aligned}\mathcal{L} :=\Big[&\frac{1}{2}(\partial_\mu A)\partial^\mu A - \frac{1}{2}m^2A^2 + \frac{1}{2}(\partial_\mu B)\partial^\mu B\\ &- \frac{1}{2}m^2B^2 - mgA(A^2+B^2)\\ &+ \bar{\psi}(i\gamma^\nu\partial_\nu - m)\psi - g\ (\bar{\psi}\psi\ A + i\bar{\psi}\gamma^5\psi B)\\ &- \frac{1}{2}g^2(A^2+B^2)^2\Big]\end{aligned} \tag{5}$$

Here $A$ is a scalar field, $B$ is a pseudoscalar field, $\psi$ is a spin-1/2 Majorana field ($\psi = \psi^C = C\bar{\psi}^T$). $C$ is the charge conjugation matrix and $A = A^\dagger$ and $B = B^\dagger$. All the fields here have the same mass $m$ and they couple with the same strength $g$. This is in contrast to the non-SUSY FT's.

This is due to the fact that states of a particular representation of the super Poincare algebra (SPA) are characterized by the eigenvalue $m^2$ of $P^2$ ($= P_\mu P^\mu$) and different values of spin $s$. Actually, all the fields belong to the same mass multiplet in SPA.

Pauli-Ljubanski polarization vector is defined as:

$$W_\mu := \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma} \tag{6}$$

Here

$$\begin{aligned}P^2 &= P_\mu P^\mu\\ W^2 &= W_\mu W^\mu\end{aligned} \tag{7}$$

are Casimir operators of PA that satisfy:

$$\begin{aligned}\left[P^2, M_{\mu\nu}\right] &= 0\\ \left[P^2, P_\mu\right] &= 0\\ \left[W^2, M_{\mu\nu}\right] &= 0\\ \left[W^2, P_\mu\right] &= 0\end{aligned} \tag{8}$$

We further have:

$$\begin{aligned}P^2 &= m^2 > 0\\ W^2 &= -m^2s(s+1).\end{aligned} \tag{9}$$

Where, $m^2$ and $(-m^2s(s+1))$ are the eigenvalues of $P^2$ and $W^2$. Here $s$ denotes the spin of the representation which assumes discrete values: $s = 0, 1/2, 1, 3/2, \ldots$

This representation is specified in terms of the mass $m$ and spin $s$. Physically a state in a representation $(m, s)$ corresponds to a particle of rest mass $m$ and spin $s$. Also, since the spin projection $S_3$ can take any value from $-s$ to $+s$, (massive particles fall into $(2s + 1)$-dimensional multiplets).

In WZM, all the fields namely, $A$, $B$, $\psi$, $\bar{\psi}$ have the same mass $m$ and they couple with the same strength $g$ (in the unbroken SUSY) – in contrast to the non-supersymmetric field theories. States of a particular representation of SPA are characterized by the eigenvalue $m^2$ of Casimir operator $P^2$ and different values of spin $s$. $W^\mu$ is proportional to $P^\mu$ (generator of the Poincare group):

$$W^\mu = \lambda P^\mu \tag{10}$$

and

$$W_0 = \lambda P_0 = \overrightarrow{P} \cdot \overrightarrow{J} \tag{11}$$

where

$$P^\mu = (P_0\ , \overrightarrow{P}). \tag{12}$$

The constant of proportionality $\lambda$ in $W_\mu = \lambda P_\mu$ is called Helicity and it is defined by:

$$\lambda := \frac{\overrightarrow{P} \cdot \overrightarrow{J}}{P_0} \tag{13}$$

for massless particles with $\lambda := \pm s$ where $s = 0, 1/2, 1, \ldots$ is the spin of representation. $N = 1$ is called as the Minimal Supersymmetry and $N > 1$ is called the Extended Supersymmetry.

For simplicity we set ($g = 0$) yielding the Lagrangian density of the free WZM [5]:

$$\begin{aligned}\mathcal{L} :=\Big[&\frac{1}{2}\partial_\mu A\partial^\mu A - \frac{1}{2}m^2A^2 + \frac{1}{2}\partial_\mu B\partial^\mu B\\ &- \frac{1}{2}m^2B^2 + \bar{\psi}(i\gamma^\nu\partial_\nu - m)\psi\Big]\end{aligned} \tag{14}$$

Theory is seen to be invariant (up to a total derivative) under the rigid SUSY transformation [5]:

$$\begin{aligned}\delta A &= \bar{\epsilon}\ \psi\\ \delta B &= -i\ \bar{\epsilon}\ \gamma^5\ \psi\\ \delta\psi &= -\ (i\gamma^\nu\partial_\nu + m)\ (A - i\gamma^5\ B)\ \epsilon\\ \delta\bar{\psi} &=\ \bar{\epsilon}\ (A - i\gamma^5\ B)\ (i\gamma^\nu\overleftarrow{\partial}_\nu - m)\end{aligned} \tag{15}$$

Here $\epsilon$ is a constant Grasmann variable (which does not depend on spacetime $x \equiv x^\mu$) implying a global or rigid SUSY transformations. However, $\delta\psi$ and $\delta\bar{\psi}$ here, are seen to depend on spacetime derivatives of $A$ and $B$. This implies that this is an extention of Poincare spacetime symmetry (different than an internal symmetry).

Supercurrent $j^\mu$ of the theory could be easily calculated to be [5]:

$$\begin{aligned}j^\mu &= \Big[\frac{i}{2}\bar{\epsilon}(A - i\gamma^5\ B)\ (i\gamma^\nu\overleftarrow{\partial}_\nu - m)\gamma^\mu\ \psi\Big]\\ &\equiv \Big[\frac{1}{\beta}\ \bar{\epsilon}\ k^\mu\Big]\end{aligned} \tag{16}$$

Here $\beta$ is a real constant which could be suitably choosen. The spinor charges $Q_a$ are defined by [5]:

$$Q_a := \int d^3x\ k_a^0$$
$$k_a^0 = \frac{i}{2}\beta\Big[\{(A - i\gamma^5\ B)(i\gamma^\nu \overleftarrow{\partial}_\nu - m)\}\gamma^0\psi\Big]_a \quad (17)$$

Here $k_a^0$ are the spinor charge densities with $a = 1, 2, 3, 4$. Spinor charges and spinor charge densities being fermionic satisfy SPA and the spinor charges are seen to satisfy the anti-commutation relation (ACR) [5]:

$$\{Q_a, \bar{Q}_b\} = 2P_\mu(\gamma^\mu)_{ab} \quad (18)$$

This explicitly shows that the WZM obeys the SPA and it is a supersymmetric FT having a rigid or global SUSY. Also, the supersymmetry of the theory is a non-manifest supersymmetry.

We now set $m = 0$ for making the fields to be massless, so that the free massless WZM is defined by the Lagrangian density:

$$\mathcal{L} := \Big[\frac{1}{2}\partial_\mu A\partial^\mu A + \frac{1}{2}\partial_\mu B\partial^\mu B + \bar{\psi}(i\gamma^\nu\partial_\nu)\psi\Big] \quad (19)$$

This is the simplest example of a supersymmetric FT in 4D with a non-manifest supersymmetry.

We obtain the free WZM by setting $g = 0$ and it is seen to be invariant, up to a total derivative, under the rigid SUSY transformations [5]:

$$\delta A = \bar{\epsilon}\ \psi$$
$$\delta B = -i\ \bar{\epsilon}\ \gamma^5\ \psi$$
$$\delta\psi = -\ (i\gamma^\nu\partial_\nu)\ (A - i\gamma^5\ B)\ \epsilon$$
$$\delta\bar{\psi} = \bar{\epsilon}\ (A - i\gamma^5\ B)\ (i\gamma^\nu \overleftarrow{\partial}_\nu) \quad (20)$$

Here $\epsilon$ is a constant Grasmann variable (which does not depend on spacetime $x \equiv x^\mu$). Here, $\delta\psi$ and $\delta\bar{\psi}$ are seen to depend on spacetime derivatives of $A$ and $B$ which implies that this is an extention or generalization of the Poincare spacetime symmetry. $(\epsilon, \bar{\epsilon})$ being constant, implies that the symmetry is a rigid or global Susy.

It is also possible to consider it as a theory of a single complex scalar field and a fermionic field by combining the fields $A$ and $B$ as follows:

$$\phi(x) := (A + iB)/2$$
$$\phi^\star(x) = (A - iB)/2 \quad (21)$$

implying therefore: $\delta\phi = \bar{\epsilon}\bar{\psi}$ and $\delta\phi^\star = \epsilon\psi$ and

$$\delta\psi_A = 2i(\sigma^\mu\bar{\epsilon})_A\partial_\mu\phi^\star(x)$$
$$\delta\bar{\psi}^{\dot{A}} = -2i(\bar{\sigma}^\mu\epsilon)^{\dot{A}}\partial_\mu\phi(x) \quad (22)$$

Since $A$ is a scalar field and $B$ is a pseudoscalar field, the complex combination $\phi(x)$ transforms under the parity transformation like complex conjugation. Here, $\psi$ and $\bar{\psi}$ are not independent fields as they are the Majorana spinor fields in the Weyl formulation. Hence the transformations of $\delta\psi$ and $\delta\bar{\psi}$ are not independent and one could be obtained from the other. Supercurrent $j^\mu$ of the theory is obtained as:

$$j^\mu = \Big[\frac{i}{2}\bar{\epsilon}(A - i\gamma^5\ B)\ (i\gamma^\nu \overleftarrow{\partial}_\nu)\gamma^\mu\ \psi\Big] \equiv \Big[\frac{1}{\beta}\ \bar{\epsilon}\ k^\mu\Big] \quad (23)$$

Here $\beta$ is a real constant. Spinor charge $Q_a$ are:

$$Q_a := \int d^3x\ k_a^0$$
$$k_a^0 = \frac{i}{2}\beta\Big[\{(A - i\gamma^5\ B)\ (i\gamma^\nu \overleftarrow{\partial}_\nu)\}\gamma^0\ \psi\Big]_a \quad (24)$$

Here $k_a^0$ are the spinor charge densities with $a = 1, 2, 3, 4$. WZM being a supersymmetric FT, spinor charges and the spinor charge densities are seen to satisfy SPA and the spinor charges satisfy the ACR:

$$\{Q_a, \bar{Q}_b\} = 2P_\mu(\gamma^\mu)_{ab} \quad (25)$$

This implies that the WZM obeys SPA and it is a supersymmetric FT with a rigid Susy. SPA reads [5]:

$$[P_\mu, P_\nu] = 0 \quad (26)$$

$$[M_{\mu\nu}, P_\rho] = -i\ (\eta_{\mu\rho}\ P_\nu - \eta_{\nu\rho}\ P_\mu) \quad (27)$$

$$\begin{aligned}[M_{\mu\nu}, M_{\rho\sigma}] =& -i(\eta_{\mu\rho}M_{\nu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}) \\ & + i\ (\eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\rho}M_{\mu\sigma})\end{aligned} \quad (28)$$

$$[P_\mu, Q_a] = 0 \quad (29)$$

$$[M_{\mu\nu}, Q_a] = -(\sigma^4_{\mu\nu})_{ab}\ Q_b \quad (30)$$

$$\sigma^4_{\mu\nu} := \frac{i}{4}[\gamma_\mu, \gamma_\nu]$$
$$\{Q_a, \bar{Q}_b\} = 2(\gamma^\mu)_{ab}\ P_\mu$$
$$\{Q_a, Q_b\} = -2\ (\gamma^\mu\ C)_{ab}\ P_\mu$$
$$\{\bar{Q}_a, \bar{Q}_b\} = 2\ (C^{-1}\gamma^\mu)_{ab}\ P_\mu \quad (31)$$

SPA has 14 generators: 4 generators of Lorentz translations $P_\mu$ , 6 generators of Poincare transformations $M_{\mu\nu}$ and 4 spinor charges $Q_a$ (the Majorana spinors). Here the indices $a$ and $b$ run from 1 to 4 in 4D.

## 3. Free massless WZM

We now set $m = 0$ for making the fields to be massless, so that the free massless WZM is defined by the Lagrangian density:

$$\mathcal{L} := \Big[\frac{1}{2}\partial_\mu A\partial^\mu A + \frac{1}{2}\partial_\mu B\partial^\mu B + \bar{\psi}(i\gamma^\nu\partial_\nu)\psi\Big] \quad (32)$$

We break up the Lagrangian density of the free massless WZM into bosonic and fermionic parts:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_B + \mathcal{L}_F \\ \mathcal{L}_B &= \frac{1}{2}\partial_\mu A \partial^\mu A + \frac{1}{2}\partial_\mu B \partial^\mu B \\ \mathcal{L}_F &= \bar{\psi}(i\gamma^\nu \partial_\nu)\psi \end{aligned} \tag{33}$$

Further, $\mathcal{L}_F$ $(= \mathcal{L}^F)$ could be written in two different looking but conceptually equivalent forms (which differ by a total derivative (t.d.)) as follows:

$$\begin{aligned} \mathcal{L}_1^F &= i\bar{\psi}\gamma^\mu \partial_\mu \psi \\ \mathcal{L}_2^F &= \frac{i}{2}\Big[\bar{\psi}\gamma^\mu(\partial_\mu \psi) - (\partial_\mu \bar{\psi})\gamma^\mu \psi\Big] \end{aligned} \tag{34}$$

$$\begin{aligned} \mathcal{L}_1^F - \mathcal{L}_2^F &= \frac{i}{2}\partial_\mu(\bar{\psi}\gamma^\mu\psi) = \frac{i}{2}\partial_\mu j^\mu \\ j^\mu &= (\bar{\psi}\gamma^\mu\psi) \end{aligned} \tag{35}$$

Theory described by $\mathcal{L}_1^F$ is seen to possess a set of two second class constraints:

$$\begin{aligned} \rho_1 &= (\pi + i\bar{\psi}\gamma^0) \approx 0 \\ \rho_2 &= \bar{\pi} \approx 0 \end{aligned} \tag{36}$$

Here, Fermi fields $\psi$ and $\bar{\psi}$ are to be treated as independent fields. Theory described by $\mathcal{L}_2^F$ is also seen to possess a set of two second class constraints:

$$\begin{aligned} \chi_1 &= (\pi + \frac{i}{2}\bar{\psi}\gamma^0) \approx 0 \\ \chi_2 &= (\bar{\pi} + \frac{i}{2}\gamma^0\psi) \approx 0. \end{aligned} \tag{37}$$

The Fermi fields $\psi$ and $\bar{\psi}$ in this later case are not independent fields. This is consistent with the definition of Majorana spinor fields (we remind ourselves here that in WZM, the fermionic fields are Majorana spinor fields).

We now study the Hamiltonian formulation of the theory [7]. The canonical momenta following from the Lagerangian density of WZM defined by $\mathcal{L} := (\mathcal{L}_B + \mathcal{L}_F)$ with $\mathcal{L}_F = \mathcal{L}_2^F$ (working with the signature $\eta_{\mu\nu} := \mathrm{diag}(+1,-1,-1,-1)$) are:

$$\begin{aligned} \Pi_A &:= \frac{\partial \mathcal{L}}{\partial(\partial_0 A)} = \partial_0 A \\ \Pi_B &:= \frac{\partial \mathcal{L}}{\partial(\partial_0 B)} = \partial_0 B \end{aligned} \tag{38}$$

$$\begin{aligned} \pi &:= \frac{\partial \mathcal{L}}{\partial(\partial_0 \psi)} = -\frac{i}{2}\bar{\psi}\gamma^0 \\ \bar{\pi} &:= \frac{\partial \mathcal{L}}{\partial(\partial_0 \bar{\psi})} = -\frac{i}{2}\gamma^0\psi \end{aligned} \tag{39}$$

Theory thus has 2 primary constraints (PC's):

$$\begin{aligned} \chi_1 &= (\pi + \frac{i}{2}\bar{\psi}\gamma^0) \approx 0 \\ \chi_2 &= (\bar{\pi} + \frac{i}{2}\gamma^0\psi) \approx 0 \end{aligned} \tag{40}$$

In principle, $\chi_1, \chi_2$ represent an infinite number of PC's which could be labeled say by $\alpha, \beta$ (which run from one to infinity). We however, ignore these further labelings in our considerations. The canonical Hamiltonian density of the theory is obtained as:

$$\begin{aligned} \mathcal{H}_c =& (\partial_0 A)\ \Pi_A + (\partial_0 B)\ \Pi_B + (\partial_0 \psi_\alpha)\ \pi_\alpha \\ &+ (\partial_0 \bar{\psi}_\alpha)\ \bar{\pi}_\alpha - \mathcal{L}_B - \mathcal{L}_F \end{aligned} \tag{41}$$

$$\mathcal{H}_c = \frac{1}{2}\Big[\Pi_A^2 + \Pi_B^2 - i\bar{\psi}\gamma_k \partial^k \psi + i(\partial^k \bar{\psi})\gamma_k \psi\Big] \tag{42}$$

The total Hamiltonian density is:

$$\mathcal{H}_T := \mathcal{H}_c + \chi_1\ u + \chi_2\ v \tag{43}$$

Demanding that the constraints $\chi_1$ and $\chi_2$ are preserved in the course of time one does not get any secondary constraints and therefore these are the only 2 constraints that the theory possesses. Non-vanishing matrix elements of the $2 \times 2$ matrix of the PB's of these above constraints among themselves are:

$$R_{12} = -R_{21} = i\gamma^0 \delta(x^1 - y^1)\delta(x^2 - y^2)\delta(x^3 - y^3). \tag{44}$$

The non-vanishing equal-time (ET) commutation relations (CR's) (denoted by a square bracket) and ET anti-commutation relations (ACR's) (denoted by a curly bracket) of the bosonic and ferminic variables of the theory are found to be:

$$[A(x,t), \Pi_A(y,t)] = i\ \delta(x-y) \tag{45}$$

$$[B(x,t), \Pi_B(y,t)] = i\ \delta(x-y) \tag{46}$$

$$\{\psi_\alpha(x,t), \bar{\psi}_\beta(y,t)\} = \gamma^0 \delta_{\alpha\beta}\ \delta(x-y) \tag{47}$$

$$\delta(x-y) := \delta(x^1 - y^1)\delta(x^2 - y^2)\delta(x^3 - y^3) \tag{48}$$

These relations appear to be similar to the usual ones. However, the fermionic spinor field $\psi$ here is not a Dirac spinor but it is a Majorana spinor having real components: $(\psi = \psi^C)$. We need to remember here that the Dirac spinor is a 4-component spinor which has complex elements and it could be expressed in terms of two, 2-component Weyl spinors having complex elements. However, if the elements of these Weyl spinors are taken as real $(\psi = \psi^C)$ then it becomes a Majorana spinor (having real elements).

In path integral quantization (PIQ) [7], transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory, called the generating functional $Z[J_k]$ of the theory which in the presence of the external sources $J_k$ for the present theory is [7]:

$$\begin{aligned} Z[J_k] = \int [d\mu] \exp \Bigg[ i \int & dxdy [J_k \Phi^k + \Pi_A \partial_0 A \\ & + \Pi_B \partial_0 B + \pi \partial_0 \psi + \bar{\pi}\partial_0 \bar{\psi} \\ & + \Pi_u \partial_0 u + \Pi_v \partial_0 v - \mathcal{H}_T] \Bigg] \end{aligned} \tag{49}$$

Here $\Phi^k \equiv (A, B, \psi, \bar{\psi}, u, v)$ are the phase space variables of the theory with the corresponding respective canonical conjugate momenta: $\Pi_k \equiv (\Pi_A, \Pi_B, \pi, \bar{\pi}, \Pi_u, \Pi_v)$. The functional measure $[d\mu]$ of the theory (with the above generating functional $Z[J_k]$) is:

$$[d\mu] = \Bigg[[\delta(x^1 - y^1)\delta(x^2 - y^2)\delta(x^3 - y^3)][dA] \\ [dB][d\psi][d\bar{\psi}][du][dv][d\Pi_A] \\ [d\Pi_B][d\pi][d\bar{\pi}][d\Pi_u][d\Pi_v] \\ \delta[(\pi + \frac{i}{2}\bar{\psi}\gamma^0) \approx 0] \\ \delta[(\bar{\pi} + \frac{i}{2}\gamma^0\psi) \approx 0]\Bigg] \quad (50)$$

## 4. Conclusions and summary

Some important remarks may be helpful. In relativistic quantum mechanics, the Dirac equation (DE) is a single particle relativistic wave equation where $\psi$ represents a wave function. In FT, DE is an Euler-Lagrange field equation which is obtained from the Dirac action or the Dirac Lagrangian by using the variational principle.

WZM is the simplest example of a supersymmetric field theory in 4D. This is also an example of a FT with non-manifest supersymmetry. Taking the example of free massless WZM, one could study many important theories in different dimensions including in higher dimsimensions. The theory also provides a basic framework for the study of Ramond Nievue Schwarz (RNS) superstring theory (SST) which is an example of a SST with non-manifest SUSY. Starting with the WZM, it is possible to construct a supergravity theory by gauging its global (rigid) SUSY into a local SUSY through the Noether's procedure.

Just to summarize in brief, we have studied in this work, the WZM [5], which is a supersymmetric FT that has rigid or global supersymmetry. The theory has a supercharge $Q_a$ ($a = 1, 2, 3, 4$ in 4D) which is a Grassmann spinor having anti-commuting properties. Theory is invariant under rigid supersymmetry transformations where the transformation parameter is a constant Grassmann spinor [5]. Finally, we have also studied the Hamiltonian and path integral quantization of the theory [7].

## Acknowledgements

It is matter of great pleasure to thank the organizers of the International Conference on Analytic and Algebraic Methods in Physics XVIII (AAMP XVIII) – 2021 at Prague, Czechia, Prof. Vít Jakubský, Prof. Vladimir Lotoreichik, Prof. Matej Tusek, Prof. Miloslav Znojil and the entire team for a wonderful orgnization of the conference.

## References

[1] J. Wess, B. Zumino. Supergauge transformations in four dimensions. *Nuclear Physics B* **70**(1):39–50, 1974. https://doi.org/10.1016/0550-3213(74)90355-1.

[2] S. Deser, B. Zumino. Consistent supergravity. *Physics Letters B* **62**(3):335–337, 1976. https://doi.org/10.1016/0370-2693(76)90089-7.

[3] D. Z. Freedman, P. van Nieuwenhuizen, S. Ferrara. Progress toward a theory of supergravity. *Physical Review D* **13**:3214–3218, 1976. https://doi.org/10.1103/PhysRevD.13.3214.

[4] P. van Nieuwenhuizen. Supergravity. *Physics Reports* **68**(4):189–398, 1981. https://doi.org/10.1016/0370-1573(81)90157-5.

[5] H. J. W. Müller-Kirsten, A. Wiedemann. *Introduction to Supersymmetry.* World Scientific, Singapore, 2nd edn., 2010. https://doi.org/10.1142/7594.

[6] J. Wess, J. Bagger. *Supersymmetry and Supergravity.* Princeton University Press, 1992. ISBN 9780691025308.

[7] H. J. W. Müller-Kirsten. *Introduction to Quantum Mechanics: Schrödinger Equation and Path Integral.* World Scientific, Singapore, 2006. ISBN 9789812566911.

[8] K. Becker, M. Becker, J. H. Schwarz. *String Theory and M-Theory: A Modern Introduction.* Cambridge University Press, 2007. ISBN 9780521860697.

[9] S. J. Brodsky. Supersymmetric and conformal features of hadron physics. *Universe* **4**(11):120, 2018. https://doi.org/10.3390/universe4110120.

[10] S. J. Brodsky, G. F. de Teramond, H. G. Dosch. Light-front holography and supersymmetric conformal algebra: A novel approach to hadron spectroscopy, structure, and dynamics, 2020. arXiv:2004.07756.

[11] S. J. Brodsky. Color confinement and supersymmetric properties of hadron physics from light-front holography. *Journal of Physics: Conference Series* **1137**:012027, 2019. https://doi.org/10.1088/1742-6596/1137/1/012027.