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MAXWELL-CHERN-SIMONS-HIGGS THEORY

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Abstract.
We consider the three dimensional electrodynamics described by a complex scalar field coupled with the U(1) gauge field in the presence of a Maxwell term, a Chern-Simons term and the Higgs potential. The Chern-Simons term provides a velocity dependent gauge potential and the presence of the Maxwell term makes the U(1) gauge field dynamical. We study the Hamiltonian formulation of this Maxwell-Chern-Simons-Higgs theory under the appropriate gauge fixing conditions.

Keywords: Electrodynamics, Higgs theories, Chern-Simons-Higgs theories, Hamiltonian formulations, gauge-theories.

1. Introduction

We study the Hamiltonian formulation [1] of the three dimensional (3D) electrodynamics [2–22], involving a Maxwell term [20], a Chern-Simons (CS) term [19, 21, 22], and a term that describes a coupling of the U(1) gauge field with a complex scalar field in the presence of a Higgs potential [22]. Such theories in two-space one-time dimension ((2+1)D) can describe particles that satisfy fractional statistics and are referred to as the reletivistic field theoretic models of anyons and of the anyonic superconductivity [21, 22].

A remarkable property of the CS action [21, 22], is that it depends only on the antisymmetric tensor $\epsilon^{\mu\nu\lambda}$ and not on the metric tensor $g^{\mu\nu}$. As a result, the CS action in the flat spacetime and in the curved spacetime remains the same [21, 22]. Hence CS action, in both the Abelian and in the non-Abelian cases represents an example of a topological field theory [21, 22].

The systems in two-space, one-time dimensions (2+1)D (i.e., the planar systems, display a variety of peculiar quantum mechanical phenomena ranging from the massive gauge fields to soluble gravity [19–22]. These are linked to the peculiar structure of the rotation group and the Lorentz and Poincare groups in (2+1)D. The 3D electrodynamics models with a Higgs potential, namely, the Abelian Higgs models involving the vector guage field A^μ with and without the topological CS term in (2+1)D have been of a wide interest [19–22].

When these models are considered without a CS term but only with a Maxwell term accounting for the kinetic energy of the vector gauge field and they represent field-theoretical models which could be considered as effective theories of the Ginsburg-Landau-type [22] for superconductivity. These models in (2+1)D or in (3+1)D are known as the Nielsen-Olesen (vortex) models (NOM) [20]. These models are the relativistic generalizations of the well-known Ginsburg-Landau phenomenological field theory models of superconductivity [2, 20, 22].

The effective theories with excitations, with fractional statistics are supposed to be described by gauge theories with CS terms in (2+1)D and a study of these gauge field theories and the models of quantum electrodynamics involving the CS term represent a broad important area of investigation [21, 22].

The CS term provides a velocity dependent gauge potential [21, 22], and the presence of the Maxwell term in the action makes the gauge field dynamical [20]. We study the Hamiltonian formulation [1] of this Maxwell-Chern-Simons-Higgs theory under the appropriate gauge fixing conditions [20, 22].

The quantization of field theory models with constraints has always been a challenging problem [1]. Infact, any complete physical theory is a quantum theory and the only way of defining a quantum theory is to start with a classical theory and then to quantize it [1]. Theory presently under consideration is also a constrained system. In the present work, we quantize this theory using the Dirac's Hamiltonian formulation [1] in the usual instant-form (IF) of dynamics (on the hyperplanes defined by: $x^0 = t =$ constant) under appropriate gauge-fixing conditions (GFC's) [1, 19–22].

2. Hamiltonian formulation

The Maxwell Chern-Simons Higgs Theory in two space one time is defined by the following action:

$$S = \int \mathcal{L}(\Phi, \Phi^*, A^\mu)\ d^3x, \tag{1}$$

where the Lagrangian density $\mathcal{L}$ (with $\kappa = \frac{\theta}{2\pi^2}$; θ being the CS parameter) is given by:

$$\mathcal{L} = \Big[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + (\widetilde{D}_\mu \Phi^*)(D^\mu \Phi) - V(|\Phi|^2) + \frac{\kappa}{2}\epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \Big] \quad (2)$$

$$\begin{aligned} V(|\Phi|)^2 &= \gamma + \beta|\Phi|^2 + \alpha|\Phi|^4 \\ &= \lambda(|\Phi|^2 - \Phi_0^2)^2 \ ; \quad (\Phi_0 \neq 0). \end{aligned} \quad (3)$$

Where the covariant derivative is defined by:

$$\begin{aligned} D_\mu &= (\partial_\mu + i\ eA_\mu) \\ \widetilde{D}_\mu &= (\partial_\mu - i\ eA_\mu) \\ g_{\mu\nu} &= diag(+1, -1, -1) \\ \epsilon^{012} &= \epsilon_{012} = +1 \\ \mu, \nu &= 0, 1, 2. \end{aligned} \quad (4)$$

In the above Lagrangian density the first term is the kinetic energy term of the U(1) gauge field and second term represents the coupling of U(1) gauge field with the complex scalar field as well as kinetic energy for the complex scalar field. Third term describes Higgs potential and the last term is the CS term.

The model without the CS term describes an Abelian Higgs model and is defined by the Lagrangian density $\mathcal{L} = \mathcal{L}\big(\Phi, \Phi^*, A_\mu\big)$ where $\mathcal{L}$ is a function of a complex scalar field and an Abelian gauge vector field $A_\mu(x)$ defined by the above Lagrangian density. In (2+1)D this theory is called as the Nielsen Olsen (vortex) model (NOM). These models possesses stable, time independent (i.e., static), classical solutions (which could be called 2D solitons). In fact, the model admits the so-called topological solitons of the vortex type [4].

Further, in this model, if we choose the parameters of the Higgs potential to be such that the scalar and vector masses become equal i.e., if we set the Higgs boson and the vector boson (photon) masses to be equal i.e., if we set: $m_{Higgs} = m_{Photon} = e\Phi_0$ then that implies:

$$V(|\Phi|)^2 = \frac{1}{2} e^2 (|\Phi|^2 - \Phi_0^2)^2. \quad (5)$$

The above model then reduces to the so-called Bogomol'nyi model which describes a system on the boundary between type-I and type-II superconductivity [4].

In component form, the above Lagrangian density can be written as:

$$\begin{aligned} \mathcal{L} = &\left(\frac{\kappa}{2}\right)\Big[A_0F_{12} + A_1(\partial_2 A_0) - A_2(\partial_1 A_0)\Big] \\ &+ \left(\frac{\kappa}{2}\right)\Big[A_2(\partial_0 A_1) - A_1(\partial_0 A_2)\Big] - \frac{1}{2}F_{12}^2 \\ &+ \Big[\frac{1}{2}(\partial_1 A_0 - \partial_0 A_1) + \frac{1}{2}(\partial_0 A_2 - \partial_2 A_0)\Big] \\ &+ \Big[\ (\partial_0\Phi^*)(\partial_0\Phi) + i\ e(\partial_0\Phi^*)A_0\Phi \\ &- i\ e(\partial_0\Phi)A_0\Phi^* + e^2A_0^2\Phi^*\Phi\Big] \\ &+ \Big[\ -(\partial_1\Phi^*)(\partial_1\Phi) - i\ e(\partial_1\Phi^*)A_1\Phi \\ &+ i\ e(\partial_1\Phi)A_1\Phi^* - e^2A_1^2\Phi^*\Phi\Big] \\ &+ \Big[\ -(\partial_2\Phi^*)(\partial_2\Phi) - i\ e(\partial_2\Phi^*)A_2\Phi \\ &+ i\ e(\partial_2\Phi)A_2\Phi^* - e^2A_2^2\Phi^*\Phi\Big] \\ &-\ V(|\Phi|^2). \end{aligned} \quad (6)$$

Canonical momenta obtained from the above Lagrangian density are:

$$\begin{aligned} \Pi &= \frac{\partial\mathcal{L}}{\partial(\partial_0\Phi)} = (\partial_0\Phi^* - i\ eA_0\Phi^*) \\ \Pi^* &= \frac{\partial\mathcal{L}}{\partial(\partial_0\Phi^*)} = (\partial_0\Phi + i\ eA_0\Phi) \\ \Pi^0 &= \frac{\partial\mathcal{L}}{\partial(\partial_0 A_0)} = 0 \end{aligned} \quad (7)$$

$$\begin{aligned} E_1(:= \Pi^1) &:= \frac{\partial\mathcal{L}}{\partial(\partial_0 A_1)} \\ &= -(\partial_1 A_0 - \partial_0 A_1) + \frac{\kappa}{2}A_2 \\ E_2(:= \Pi^2) &= \frac{\partial\mathcal{L}}{\partial(\partial_0 A_2)} \\ &= (\partial_0 A_2 - \partial_2 A_0) - \frac{\kappa}{2}A_1. \end{aligned} \quad (8)$$

Here $\Pi, \Pi^*, \Pi^0, E_1, E_2$ are the momenta canonically conjugate respectively to $\Phi, \Phi^*, A_0, A_1, A_2$. The theory is thus seen to possess only one primary constraint (PC):

$$\chi_1 = \Pi^0 \approx 0. \quad (9)$$

The canonical Hamiltonian density of the theory is obtained using the Legendre transformation from the Lagrangian density of the theory in the usual manner. Every term in the Lagrangian density (including the CS term) is equally important. The calculational details are omitted here for the sake of brevity. The total Hamiltonian density of the theory is then obtained from the canonical Hamiltonian density by including in it the primary constraint of the theory with the help of the Lagrange multiplier field $u \equiv u(x^\mu)$ (which is dynamical) as follows:

$$\begin{aligned}\mathcal{H}_T = {} & \Pi^0 u + \Pi\ \Pi^* - ieA_0(\Pi\Phi - \Pi^*\Phi^*) \\ & + \frac{1}{2}({E_1}^2 + {E_2}^2) \\ & + \frac{1}{2}F_{12}^2 + \big[E_1(\partial_1 A_0) + E_2(\partial_2 A_0)\big] \\ & + \frac{1}{2}\left(\frac{\kappa}{2}\right)^2({A_1}^2 + {A_2}^2) \\ & - \left(\frac{\kappa}{2}\right)\big[A_2E_1 - A_1E_2 + A_0F_{12}\big] \\ & + \Big[\ (\partial_1\Phi^*)(\partial_1\Phi) + i\ e(\partial_1\Phi^*)A_1\Phi \\ & - i\ e(\partial_1\Phi)A_1\Phi^* + e^2A_1^2\Phi^*\Phi\Big] \\ & + \Big[\ (\partial_2\Phi^*)(\partial_2\Phi) + i\ e(\partial_2\Phi^*)A_2\Phi \\ & - i\ e(\partial_2\Phi)A_2\Phi^* + e^2A_2^2\Phi^*\Phi\Big], \end{aligned} \tag{10}$$

where

$$H_T = \int \mathcal{H}_T d^2x, \tag{11}$$

with the total Hamiltonian density given by:

$$\mathcal{H}_T = \big[\mathcal{H}_c +\ \Pi^0 u\big]. \tag{12}$$

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It is to be noted here that in the construction of the canonical Hamiltonian density of the theory, all the fields of the theory play an equally important role through the Legendre transformation and through the Lagrangian density of the theory that defines the theory. Also, it is worth mentioning here that the Hamilton's equations of motion of the theory (that are omitted here for the sake of brevity) obtained from the total Hamiltonian density of the theory preserve the constraints of the theory for all time. After preserving the Primary constraint χ_1 in the course of time, one obtains a secondary constraint

$$\begin{aligned}\chi_2 = {} & \Big[ie(\Pi\Phi - \Pi^*\Phi^*) + (\partial_1 E_1 + \partial_2 E_2) \\ & + \frac{\kappa}{2}(\partial_1 A_2 - \partial_2 A_1)\Big] \approx 0. \end{aligned} \tag{13}$$

The matrix of Poisson Brackets (PB's) among the constraints χ_i is a null matrix and thereby theory is a gauge invariant theory and is invariant under the following local vector gauge transformations:

$$\begin{aligned} & \delta\Phi = i\beta\Phi, \quad \delta\Phi^* = -i\beta\Phi^*, \quad \delta\Pi^0 = 0 \\ & \delta A_0 = -\partial_0\beta\ ; \quad \delta A_1 = -\partial_1\beta\ ; \quad \delta A_2 = -\partial_2\beta \\ & \delta\Pi = -i\beta(\partial_0\Phi^*) - e\beta A_0\Phi^* + i(e-1)(\partial_0\beta)\Phi^* \\ & \delta\Pi^* = i\beta(\partial_0\Phi) - e\beta A_0\Phi - i(e-1)(\partial_0\beta)\Phi \\ & \delta E_1 = \frac{-\kappa}{2}\partial_2\beta;\ \delta E_2 = \frac{\kappa}{2}\partial_1\beta;\ \delta u = -\partial_0\partial_0\beta. \end{aligned} \tag{14}$$

Here, β is the gauge parameter $\beta \equiv \beta(x^\mu)$ and the vector gauge current satisfies: $\partial_\mu J^\mu = 0$. The components of J^μ are:

$$\begin{aligned} J^0 = J_0 = {} & (i\beta\Phi)\big[\partial_0\Phi^* - i\ eA_0\Phi^*\big] \\ & - (i\beta\Phi^*)\big[\partial_0\Phi + i\ eA_0\Phi\big] \\ & - (\partial_1\beta)\ F_{01} - (\partial_2\beta)\ F_{02} \\ & - \frac{\kappa}{2}\big[(\partial_1\beta)A_2 - (\partial_2\beta)A_1\ \big] \end{aligned}$$

$$\begin{aligned} J^1 = -J_1 = {} & (i\beta\Phi)\big[\ -\partial_1\Phi^* + i\ eA_1\Phi^*\big] \\ & - (i\beta\Phi^*)\big[\ -\partial_1\Phi - i\ eA_1\Phi\big] \\ & - (\partial_0\beta)\ F_{10} - (\partial_2\beta)\ F_{21} \\ & + \frac{\kappa}{2}\big[(\partial_0\beta)A_2 - (\partial_2\beta)A_0\ \big] \end{aligned}$$

$$\begin{aligned} J^2 = -J_2 = {} & (i\beta\Phi)\big[\ -\partial_2\Phi^* + i\ eA_2\Phi^*\big] \\ & - (i\beta\Phi^*)\big[\ -\partial_2\Phi - i\ eA_2\Phi\big] \\ & - (\partial_0\beta)\ F_{20} - (\partial_1\beta)\ F_{12} \\ & - \frac{\kappa}{2}\big[(\partial_0\beta)A_1 - (\partial_1\beta)A_0\ \big]. \end{aligned} \tag{15}$$

For quantizing the theory using Dirac's procedure we choose the following two gauge-fixing conditions (GFC's):

$$\begin{aligned} \xi_1 &= \Pi \approx 0 \\ \xi_2 &= A_0 \approx 0. \end{aligned} \tag{16}$$

Here the gauge $A_0 \approx 0$ represents the time-axial or temporal gauge and the gauge $\Pi \approx 0$ represents the coulomb gauge. These gauges are acceptable and consistent with our quantization procedure and also physically more interesting. Corresponding to this set of gauge fixing conditions the total set of constraints now becomes:

$$\begin{aligned} \chi_1 &= \Pi^0 \approx 0 \\ \chi_2 &= \big[ie(\Pi\Phi - \Pi^*\Phi^*) + (\partial_1 E_1 + \partial_2 E_2) \\ &\quad + \frac{\kappa}{2}(\partial_1 A_2 - \partial_2 A_1)\big] \approx 0 \\ \chi_3 &= \xi_1\ =\ \Pi \approx 0 \\ \chi_4 &= \xi_2\ =\ A_0 \approx 0. \end{aligned} \tag{17}$$

The non-vanishing matrix elements of the matrix $R_{\alpha\beta}$ $(:= \{\chi_1, \chi_2\}_P)$ of the equal-time Poisson brackets of the above constraints are:

$$\begin{aligned} R_{14} = -R_{41} = {} & -\ \delta(x^1 - y^1)\delta(x^2 - y^2) \\ R_{23} = -R_{32} = {} & ie\Pi\ \delta(x^1 - y^1)\delta(x^2 - y^2). \end{aligned} \tag{18}$$

The above matrix is nonsingular and the set of constraints $\chi_i\ ;\ i = 1, 2, 3, 4$ is now second class and the theory is a gauge non-invariant theory. The non-vanishing matrix elements of the matrix $R^{-1}_{\alpha\beta}$ (which is the inverse of the matrix $R_{\alpha\beta}$) are given by:

$$R^{-1}_{14} = -R^{-1}_{41} = \delta(x^1 - y^1)\delta(x^2 - y^2) \tag{19}$$
$$(e\Pi)R^{-1}_{23} = -(e\Pi)R^{-1}_{32} = \ i\ \delta(x^1 - y^1)\delta(x^2 - y^2).$$

Following the standard Dirac quantisation procedure, the non-vanishing equal time Dirac Brackets (DB's) of the theory are obtained as:

$$(\Pi)\ \{\Pi^*(x^0, x^1, x^2)\ ,\ \Phi(x^0, y^1, y^2)\}_D = (-\Pi^*)\delta(x^1 - y^1)\delta(x^2 - y^2)$$

$$\{\Pi^*(x^0, x^1, x^2)\ ,\ \Phi^*(x^0, y^1, y^2)\}_D = \{\Pi^*(x^0, x^1, x^2)\ ,\ \Phi^*(x^0, y^1, y^2)\}_P = -\delta(x^1 - y^1)\delta(x^2 - y^2)$$

$$(ie\Pi)\ \{E_1(x^0, x^1, x^2)\ ,\ \Phi(x^0, y^1, y^2)\}_D = \left(\frac{\kappa}{2}\right)\ \delta(x^1 - y^1)\ \partial_2\delta(x^2 - y^2)$$

$$\{E_1(x^0, x^1, x^2)\ ,\ A_1(x^0, y^1, y^2)\}_D = \{E_1(x^0, x^1, x^2)\ ,\ A_1(x^0, y^1, y^2)\}_P = -\ \delta(x^1 - y^1)\delta(x^2 - y^2)$$

$$(ie\Pi)\ \{E_2(x^0, x^1, x^2)\ ,\ \Phi(x^0, y^1, y^2)\}_D = -\left(\frac{\kappa}{2}\right)\ \partial_1\delta(x^1 - y^1)\delta(x^2 - y^2)$$

$$\{E_2(x^0, x^1, x^2)\ ,\ A_2(x^0, y^1, y^2)\}_D = \{E_2(x^0, x^1, x^2)\ ,\ A_2(x^0, y^1, y^2)\}_P = -\delta(x^1 - y^1)\delta(x^2 - y^2)$$

$$(\Pi)\ \{\Phi(x^0, x^1, x^2)\ ,\ \Phi^*(x^0, y^1, y^2)\}_D = (-\Phi^*)\delta(x^1 - y^1)\delta(x^2 - y^2)$$

$$(\Pi)\ \{\Phi(x^0, x^1, x^2)\ ,\ A_0(x^0, y^1, y^2)\}_D = (\Phi)\ \delta(x^1 - y^1)\delta(x^2 - y^2)$$

$$(ie\Pi)\ \{\Phi(x^0, x^1, x^2)\ ,\ A_1(x^0, y^1, y^2)\}_D = \partial_1\delta(x^1 - y^1)\delta(x^2 - y^2)$$

$$(ie\Pi)\ \{\Phi(x^0, x^1, x^2)\ ,\ A_2(x^0, y^1, y^2)\}_D = \delta(x^1 - y^1)\ \partial_2\delta(x^2 - y^2). \tag{20}$$

Here one finds that the product of the canonical variables appear in the expressions of the constraints as well as in the expressions of the DB's and therefore for achieving the canonical quantisation of the theory, one encounters the problem of operator ordering while going from DB's to the commutation relations, this problem could however be resolved by demanding that all the fields and the field momenta after quantisation become Hermitian operators and that all the canonical commutation relations need to be consistent with the Hermiticity of these operators. This completes the Hamiltonian formulation of the theory under the choosen gauge fixing conditions.

It may be worthwhile to mention here that our choice of GFC's is by no means unique. In principle, one can choose any set of GFC's that would convert the set of constraints of the theory from first-class into a set of second-class constraints. However, it is better to choose the GFC's that are physically more meaningful and nore relevant like the ones that we have choosen. In our case the gauge $A_0 \approx 0$ represents a time-axial or temporal gauge and the gauge $\Pi \approx 0$ represents a Culomb gauge and both of them are physically important GFC's. Another important point is that one can not choose covariant GFC's here simply because the constraints of the theory are not covariant and therefore it would not work.

In path integral quantization (PIQ) [23], transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory, called the generating functional $Z[J_k]$ of the theory which in the presence of the external sources J_k for the present theory is [23]:

$$Z[J_k] = \int [d\mu] \exp\Bigg[i \int d^3x\ [J_k\Phi^k + \Pi\partial_0\Phi + \Pi^*\partial_0\Phi^* + \Pi^0\partial_0 A_0 + E_1\partial_0 A_1 + E_2\partial_0 A_2 + \Pi_u\partial_0 u - \mathcal{H}_T]\Bigg]. \tag{21}$$

Here $\Phi^k \equiv (\Phi, \Phi^*, A_0, A_1, A_2, u)$ are the phase space variables of the theory with the corresponding respective canonical conjugate momenta: $\Pi_k \equiv (\Pi, \Pi^*, \Pi^0, E_1, E_2, \Pi_u)$. The functional measure $[d\mu]$ of the theory (with the above generating functional $Z[J_k]$) is:

$$[d\mu] = \Bigg[(ie\Pi)\delta(x^1 - y^1)\delta(x^2 - y^2) [d\Phi][d\Phi^*][dA_0][dA_1][dA_2][du][d\Pi] [d\Pi^*][d\Pi^0][dE_1][dE_2][d\Pi_u]\delta[(\Pi^0) \approx 0] \delta[[ie(\Pi\Phi - \Pi^*\Phi^*) + (\partial_1 E_1 + \partial_2 E_2) + \frac{\kappa}{2}(\partial_1 A_2 - \partial_2 A_1)] \approx 0] \delta[\Pi \approx 0]\delta[A_0 \approx 0]\Bigg]. \tag{22}$$

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