Determination of Rheological Parameters from Measurements on a Viscometer with Coaxial Cylinders – Choice of the Reference Radius

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Knowledge about rheological behavior is necessary in engineering calculations for equipment used for processing concentrated suspensions and polymers. Power-law and Bingham models are often used for evaluating the experimental data. This paper proposes the reference radius to which experimental results obtained by measurements on a rotational viscometer with coaxial cylinders should be related.

Keywords: viscometer with coaxial cylinders, power-law fluids, Bingham plastics

1 Introduction

In a recent paper [1], the procedure for determining the rheological parameters from measurements on a viscometer with coaxial cylinders (see Fig. 1) was proposed on the basis of flow analysis. The relations for calculating of consistency coefficient K for power-law fluids and yield stress τ_0 for Bingham plastics were reported. These relations were derived for three reference radii – inner, mean and radius presented by Klein [2].

However, it is possible to find radius R_r at which Newtonian and non-Newtonian shear rates are the same. If the experimental data are related to this radius, K and τ_0 can be obtained directly as the τ-intercept of the measured data $(\tau = f(\dot{\gamma}))$straight line.

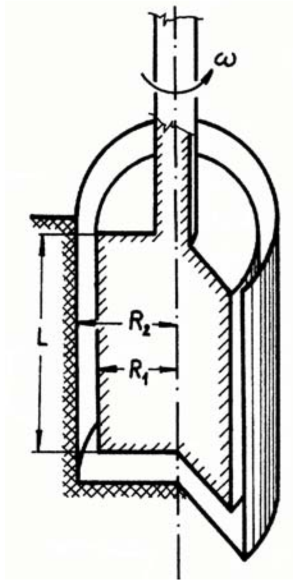

Fig. 1: Viscometer with coaxial cylinders

2 Solution

A) Power-law fluids

The following equation was derived for shear rate (eq.(9) in [1])

$$\dot{\gamma} = -\frac{2\omega}{n(1-\kappa^{2/n})}\left(\frac{R_1}{r}\right)^{2/n}. \tag{1}$$

Inserting $n = 1$, the following equation can be obtained for the shear rate in Newtonian fluids

$$\dot{\gamma} = -\frac{2\omega}{n(1-\kappa^{2})}\left(\frac{R_1}{r}\right)^{2}. \tag{2}$$

The two values are the same at $r = R_r$ and using eqs.(1) and (2) we get

$$\frac{R_1}{R_r} = \left[\frac{n(1-\kappa^{2/n})}{1-\kappa^2}\right]^{n/(2-2n)}. \tag{3}$$

From this equation it can be seen that the R_1/R_r ratio depends on n and κ. The dependence of R_1/R_r on n for selected values of ratio κ is shown in Fig. 2.

Comparison of R_1/R_r with the ratio of R_1 to the mean radius presented by Klein [2]

$$R_K = R_1 R_2\sqrt{\frac{2}{R_1^2 + R_2^2}} \tag{4}$$

for $\kappa = 0.8$ is shown in Fig. 3. This figure shows that R_K represents the mean value of R_r in the presented n interval, and for this reason ratio K/K_K is relatively small, as was shown in [1] (see Figs.12 and 13 in [1]).

B) Bingham plastics

Combining Eqs.(3), (11) and (13) presented in [1], the following equation for shear rate can be obtained

$$\dot{\gamma} = \frac{\tau_0}{\mu_p} + \left(\frac{2\tau_0}{\mu_p(1-\kappa^2)}\ln\kappa - \frac{2\omega}{1-\kappa^2}\right)\left(\frac{R_1}{r}\right)^2. \tag{5}$$

Again we can find radius R_r at which Newtonian and Bingham shear rates are the same by comparing equation (5) with the corresponding relation for a Newtonian fluid (2), and we get

$$\frac{R_1}{R_r} = \sqrt{\frac{1-\kappa^2}{2\ln(1/\kappa)}}. \tag{6}$$

From this equation it can be seen that ratio R_1/R_r depends on κ. The graphical form of this dependence is shown in Fig. 4.

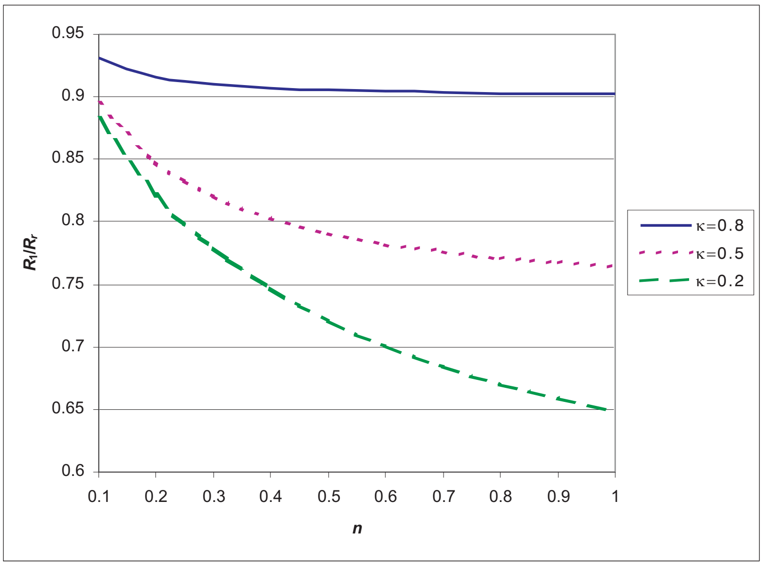

Fig. 2: Dependence of R_1/R_r on n for selected values of ratio κ

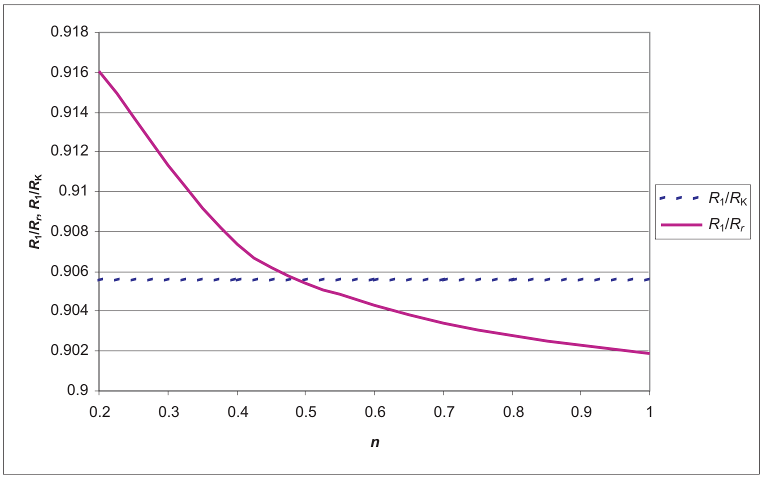

Fig. 3: Dependence of R_1/R_K resp. R_1/R_r on n

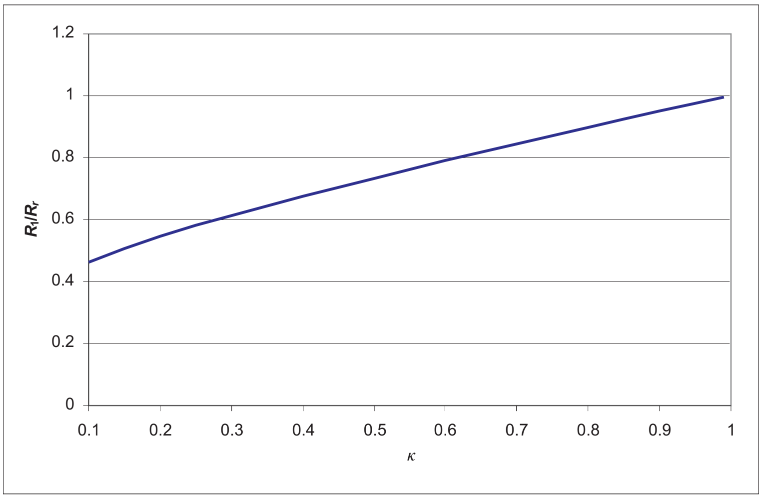

Fig. 4: Dependence of R_1/R_r on κ

3 Conclusion

On the basis of the above paragraph, the following procedure for evaluating the experimental data can be recommended:

1) If the logarithmic plot of shear stress τ_1 on Newtonian shear rate $\dot{\gamma}_{1N}$ is linear (the slope is equal to flow index n) the power-law model can be used and we can calculate R_1/R_r from eq.(3). If the plot of shear stress τ_1 on the Newtonian shear rate $\dot{\gamma}_{1N}$ is linear, the Bingham model can be used and we can calculate R_1/R_r from eq.(6).
2) The shear stresses and shear rates related to radius R_r can be calculated from experimental values τ_1 and $\dot{\gamma}_{1N}$ using the following relations

$$\tau_r = \tau_1 \left(\frac{R_1}{R_r} \right)^2, \tag{7}$$

$$\dot{\gamma}_{rN} = \dot{\gamma}_r = \dot{\gamma}_{1N} \left(\frac{R_1}{R_r} \right)^2 \tag{8}$$

3) If the logarithmic plot of shear stress τ_r on shear rate $\dot{\gamma}_r$ is linear, the consistency coefficient K is τ-intercept and flow index n is the slope of a straight line. If plot of shear stress τ_r on shear rate $\dot{\gamma}_r$ is linear, the yield stress τ_0 is τ-intercept and plastic viscosity μ_p is the slope of a straight line.

4 Nomenclature

K	coefficient of consistency
L	length of cylinder
n	flow index
r	radial coordinate
R_1	inner rotating cylinder radius
R_2	outer stationary cylinder radius
R_K	radius presented by Klein
R_r	radius at which Newtonian and non-Newtonian shear rates are the same
$\dot{\gamma}$	shear rate
κ	R_1/R_2 ratio
μ_p	plastic viscosity
ω	angular velocity
τ	shear stress
τ_0	yield stress

subscripts

1	at radius R_1
r	at radius R_r
N	Newtonian

Reference

[1] Rieger, F.: Determination of Rheological Parameters from Measurements on a Viscometer with Coaxial Cylinders. *Acta Polytechnica*, Vol. **46** (2006), p. 42–51.

[2] Klein, G.: Basic Principles of Rheology and the Application of Rheological Measurement Methods for Evaluating Ceramic Suspensions. In: *Ceramic Forum International Yearbook 2005* (Edited by H. Reh). Baden-Baden: Göller Verlag, 2004, p. 31–42.

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