Fuzzy Concepts in the Detection of Unexpected Situations

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This paper establishes three essential classes of unexpected situations (UX¹, UX², UX³), and concentrates attention on UX³ detection. The concepts of a special Model of System of Situations (MSS) and a Model of a System of Faults (MSF) are introduced. An original method is proposed for detecting unexpected situations indicating a violation of a proper invariant of MSS (MSF). The presented approach offers a promising application for starting and ending phases of complex processes, for knowledge discoveries on data and knowledge bases developed with incomplete experience, and for modeling communication processes with unknown (disguised) communication subjects. The paper also presents a way to utilize ill-separable situations for UX³ detection. The paper deals with the conceptual background for detecting UX³ situations, recapitulates recent results in this field and opens the ways for further research.

Keywords: unexpected situations, model of system of situations, model of system of faults, invariants, fuzzy variables, degree of unexpectedness, emergence zone, association rules, Hasse Diagram.

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1 Introduction

The approaches to unexpected situations (as a special class of so-called undetectable faults) come from various domains, and they are referenced, e.g., in [1], [2], [3], [4], [5], [6], [7], [8], [9]. (A detailed analysis of these sources is in [17]).

Our approach for UX³ detection is based on the concept of a UX³ type situation, on the concepts of a Model of a System of Situations (MSS) and a Model of a System of Faults (MSF), and on an original method which detects a UX³ type unexpected situation as a violation of a proper structural invariant - constructed on MSS (MSF). The structural invariant is constructed on MSS during the so-called “cognitive phase” of MSS (MSF) development. In this “cognitive phase”, it is considered that some “classes” of situations (faults) have already been established but some of the new situations are processed with large uncertainty. Violation of the structural invariant represents the detection of a UX³ type situation. (A more detailed explanation is given in section 2).

Analysing our approach from the Fault Diagnosis (FDI) point of view, some important issues may be indicated.

The first issue is the concept of MSS (MSF) in the context of the Model-Based approach in FDI, and the assignment of our method to some of the known diagnostic approaches, e.g. to abductive diagnostics. Our approach is model-based, though the development and the use of models (MSS, MSF) are different from the examples introduced, e.g., in [10]. Our models are developed as a result of data and signal analysis (not as a result of preformed knowledge about a diagnosed system, e.g., knowledge about the internal structure or about a mathematical model). As will be explained in the following sections, the phase of fault detection in an abductive diagnostic model (e.g., in [11], [12]) is a rather special case of UX³ detection in our method.

The second issue is the concept of a symptom. Our approach concentrates on processing situation signals (data) of the following types: vectors of outputs from qualitative models, ultrasound signals (representing the internal structure of the material samples), sequences of symbols (signs) and words from monitoring processes, ECG and EEG signals. Special situations (faults) which represent an extraordinary (faulty) behavior of a monitored (diagnosed) system are spread along the run of the signals (e.g., in their morphology) and in the sets of data. It is sometimes hard to speak about symptoms (symptoms of What?).

The presented approach to UX³ detection has been tested by the following three application cases:

- In a supervisory control system for an industrial distillation column, especially in a qualitative model designed for the starting phase of the distillation process (e.g., after maintenance operations), details, in [13], [14].
- In detecting unexpected faults in welds (laser, micro-arc and electron beam welds of thin walled welded structures used in the aerospace industry) in combination with neural fault detectors, [15], [16]
- Within the framework of a special supervisory and monitoring system, e.g., in [17].

2 Unexpected situations – concepts and examples

General features of unexpected situations and three basic classes of unexpected situations will be introduced in this section.

The first class (UX¹) is induced by the relativity of the unexpected situation in respect to the levels of available Data and Knowledge of a reasoning human. (We will denote these situations as UX (UX¹) – emphasizing the intuitive aspect of the detection of such situations.)

Example 2.1: Let us suppose that the extent of values measurable in an instrument (given in the instrument protocol) is \( u_{\text{min}}, u_{\text{max}} \). The situation when we measure by this instrument a value 10 \( u_{\text{max}} \) (without problems) is a UX¹ situation. (One interpretation of such a phenomenon is that wrong information was introduced in the protocol of the instrument.)

Example 2.2: The correct representation of a process by a differential equation depends on the identified type of equation and on the precision of the identified quotients. All cases with unknown noise in the input or in the output variables, unknown drifts in the parameters and cases of so-called hidden parameters, are cases that generate UX¹ situations.
Situations of the second class (UX^2) are generated by models that are a priori insufficient for representing some situations in the modeled process or system. (This means, e.g., that most situations are well represented, but a small number of situations are represented incorrectly.)

**Example 2.3:** Situations generated as an unprovable formula in FOL (First Order Language), situations for which a Turing machine does not stop (or works too long), or situations in complex robotic production lines (which are impossible to simulate wholly), belong to this class.

**Example 2.4:** Situations generated by models of systems with deterministic chaos. For example, systems modeled by Duffing or Lorenz equations belong to this class.

Situations of the third class (UX^3) have causes that differ from those that induce situations of the two previous classes. One principal scenario for UX^3 emergence is shown in Fig. 1.

This brief explanation of essential concepts for the theory of UX^3 situations introduced above will be extended and supplemented by a formal description and a recent application of this theory in section 3.

![Fig. 1: Models, carriers and information](image1)

**Example 2.5:** Approximation of UX^3 type situations by a scheme with standard intentions.

Let us consider the scheme in Fig. 2. The scheme consists of an unavailable (“invisible”) part and a transparent part. The invisible part contains: a process (for which we have no model), process observer variables \((x_1, ..., x_n)\), non-process external variables \((y_1, ..., y_m)\) and switches. The transparent part contains: an observer and a situation recognition block with classes of situations \((S_1, ..., S_q)\) and with the developed MSS.

The situation recognition block (which contains MSS) is developed during a standard operation of the process as a result of the work of the observer. The process is “represented” (for the observer) by the process observer variables \((x_1, ..., x_n)\). Let us consider that after a period of successful function, the structure of the situation recognition block is accepted as stable. Now let us assume that some of the switches are suddenly switched to variables \((y_1, ..., y_m)\) which represent another external reality, but they are formally the same as \((x_1, ..., x_n)\). As a result of this action – the assignment of the new situations coming into classes \((S_1, ..., S_q)\) will work incorrectly. Such a change is undetectable using standard methods, and it requires a special detection approach.

![Fig. 2: Approximation of a UX^3 type situation](image2)

**Example 2.6:** Special UX^3 situations emerge in cases when inappropriate intentions (in Frege’s sense, e.g., in [18]) are used to describe a process. (Usual intentions are propositions, quantities or properties.) Quantities, for example, are sometimes wrongly used for complex processes (or systems with complicated behavior) which are poorly measurable, and representing them by time series or by complicated signals introduces further difficulties in processing and interpretation. Typical examples of such cases are ECG, HRV (Heart Rate Variability) signals. (These facts are known, and they have
been published (e.g., in the Journal of Cardio-Vascular Research from 1996), and have been presented in our research (e.g., in [28]).)

3 Formal models for detecting $UX^3$

The method proposed in this paper for detecting $UX^3$ uses a Model of a System of Situations (MSS) or a Model of a System of Faults (MSF). Both these systems are developed in the cognitive phase (as was mentioned in the Introduction) during operations and experiments with the observed process or with the FDI system. The goal of the “cognitive phase” is to form structural invariants. (A few types of such invariants will be introduced in subsections 3.2–3.4.) In our method, a violation of a structural invariant is a means for detecting a $UX^3$ type situation. (An investigation of the necessary statistical parameters of the “cognitive phase” (e.g., “How many situations need to be analysed and in which classes can they be searched for”, etc.) was made in [26].) In this paper we assume that the “cognitive phase” satisfies the necessary statistical and modelling standards.

Model MSS has in general the following form:

$$MSS = \left\{ S, \left\{ \Gamma_i(S), \ldots, \Gamma_n(S) \right\}, \left\{ Inv(\Gamma_i), \ldots, Inv(\Gamma_p) \right\} \right\},$$

where $S$ represents a basic set of situations, $\left\{ \Gamma_i(S), \ldots, \Gamma_n(S) \right\}$ are structures on $S$ considered as relevant for $UX^3$ detection and $\left\{ Inv(\Gamma_i), \ldots, Inv(\Gamma_p) \right\}$ are invariants on some $\Gamma_i(S), \ldots, \Gamma_n(S)$ for $UX^3$ detection.

Model MSF has in general the following form:

$$MSF = \left\{ \left( S, F \right), \left\{ \Gamma_i(S, F), \ldots, \Gamma_n(S, F) \right\}, \left\{ Inv(\Gamma_i), \ldots, Inv(\Gamma_p) \right\} \right\},$$

where $\left\{ \Gamma_i(S, F), \ldots, \Gamma_n(S, F) \right\}$ are structures on $\left( S, F \right)$ considered as relevant for $UX^3$ detection and $\left\{ Inv(\Gamma_i), \ldots, Inv(\Gamma_p) \right\}$ are invariants on some $\Gamma_i(S, F), \ldots, \Gamma_n(S, F)$ for $UX^3$ detection.

Models for $UX^3$ detection have the forms

$$MD(UX^3) = \left\{ MSS, COND_{\text{Inv}} \right\},$$

or

$$MD(UX^3) = \left\{ MSF, COND_{\text{Inv}} \right\},$$

where $COND_{\text{Inv}}$ represents the conditions of violation of MSS (MSF) invariants. (These conditions are analysed in the process of $UX^3$ detection.) Fig. 3 illustrates the position of the Models for $UX^3$ detection in a block scheme of the FDI system respecting one of many possible structures of the FDI system. The figure expresses only the fact that MSF and MD(UX3) work as a parallel block with a Fault Recognition System (which is usually understood as an ending member of an FDI system).

3.1 A general type of structural invariants

$$Inv(\Gamma_i), \ldots, Inv(\Gamma_p)$$

The general type of invariants $Inv(\Gamma_i), \ldots, Inv(\Gamma_p)$ is connected with the commutation of diagrams (1) and is limited in this paper to the form of morphisms $h_i, g_i$, for $i = 1, \ldots, n$:

$$M_i = g_i \cdot (M_2 \cdot I_{i=1}) \ldots \ldots M_{i=1} \cdot (G_{2} \cdot I_{i=2}) = h_i(M_1(C_1, I_1)).$$

The concrete form of the morphisms depends on the type of models $M_1(C_1, I_1)$ and $M_2(C_2, I_2)$.

The following subsections introduce three examples of MSS and MSF and MD(UX3).

3.2 MD(UX3) with an emergence zone

MSS has (in this case) the form (6)

$$MSS = \left\{ S, \left\{ M(S), N, \mathcal{S} \right\}, \left\{ \forall s \in \mathcal{S}, \mathcal{S}(\rho(s, N) \leq \lambda) \right\} \right\},$$

where $S$ represents a basic set of situations, $M(S)$ is a matroid constructed on this set of situations, $N$ is a Basis on $M(S)$, $3$ represents a Cover on $M(S)$ (a subset of the matroid closure), $\rho(s, N)$ is a metric function and $\lambda$ is a positive real number. In addition to $N$, $3$ there is the so-called emergence zone $\mathcal{S}$. In this zone there are elements that can not be constructed from the elements in $N, 3$, but these elements are relevant to MSS and they are not in zone Outside, see Fig. 4. With regard to the fact that “regular” situations can be assigned in $N$ or in $3$ or they are classified in zone Outside (law of the excluded third alternative), elements of $\mathcal{S}$ are considered as extraordinary situations and they represent (in the context of our paper) $UX^3$ situations.

In this case MD(UX3) has the form (7)

$$MD(UX^3) = \left\{ MSS, (\rho(s^*, N) \in (\lambda, U_p)) \right\}$$

or

$$MD(UX^3) = \left\{ MSS, (s^* \in \mathcal{S}) \right\},$$

where $s^*$ is an unexpected situation and $U_p$ is a real number (Upper bound).

Fig. 3: Position of Models for $UX^3$ detection in a rough block scheme of the FDI system

Fig. 4: Basis, cover and emergence zone
The MSS described above was used in real conditions for UX³ detection in the starting phase of a desisobutanisation process [13, 14].

3.3 MD(UX³) with bipartite graphs

In this case, MSF has the form (8)

$$\text{MSF} = \left\{ \langle S, F \rangle, \{ G \}, \{ \{ G_1, G_2, \ldots, G_m \}, * \} \right\},$$

where $\langle S, F \rangle$ represent basic sets of situations and faults, $G$ is a bipartite graph (Situation $\rightarrow$ Faults). $\{ \{ G_1, G_2, \ldots, G_m \}, * \}$ is a special Dulmage-Mendelsohn decomposition, [19], [21]. ($G_1, G_2, \ldots, G_m$ are irreducible sub-graphs, “*” is a tree ordering on the set of the sub-graphs.) The detection of UX³ has been represented by a violation of ordering “*” and has been indicated by the following conditions discovered for a situation $s^*$.

In this case, MD(UX³) has the form (9)

$$\text{MD(UX³)} = \left\{ \text{MSF}, (\langle \text{ACC}(G, s^*) \rangle \text{AND} \text{not ACC}(G_j, s^*) \rangle), \right.$$

$$\text{for some } j(G_j, G_i) \right\},$$

where $s^*$ is an UX³ related to $\{ G_1, G_2, \ldots, G_m \}$.

Note 3.1: Expression ACC(G, s) denotes “situation s is ACCepted by bipartite graph G”.

This MD(UX³) model has been used, e.g., in [15] and [16]. (However, taking into account the well-studied concept of DM-irreducibility (e.g., in [20], page 63, 64) we are aware of the limited applicability of DMD.)

3.4 MD(UX³) with the association rules

The background for MD(UX³) model presented in this section continues in the line started in [22],[23] and nowadays utilises formulations, e.g., from [24], [25].

In this case, MSF has the form (10)

$$\text{MSF} = \left\{ \langle S, F \rangle, \{ M_G, HD \}, \{ ER \} \right\},$$

where $\langle S, F \rangle$ represent basic sets of situations and faults, $M_G$ is a qualitative matrix with data acquired from the cognitive phase of FDI system operation (the concept “cognitive phase” was introduced and sectioned in section 1.), HD is a Hasse Diagram [25] derived from $M_G$ and ER is a set of Evaluated Association Rules extracted from HD (each rule evaluated by quantities of Supp (Rule Support) and Conf (Rule Confidence), [24].

Note 3.2: The Hasse Diagram facilitates the process of extracting the rules from matrix $M_G$, but its use is not obligatory for the formation of ER.

MD(UX³) has (in this case) the form (11)

$$\text{MD(UX³)} = \left\{ \text{MSF}, (\exists r \in \text{ER}, \text{not ACC}(r, s^*) \rangle \right\},$$

where $s^*$ is a UX³ situation.

Note 3.3: The expression ACC(r, s*) denotes “situation s* is ACCepted by rule r”.

3.5 Additional important circumstances

A. The basic purpose of MSS (MSF) and of MD(UX³) is in the formation of rules that enable to distinguish between “ill-separable situations” and “UX³ situations”. Such decision rules are simple:

IF situation s satisfies the invariant \( \Rightarrow \) (THEN s it is an ill separable situation).

IF situation s does not satisfy the invariant \( \Rightarrow \) (THEN s is UX³ situations).

B. When detecting a UX³ situation, we should like to know how important the discovered UX³ situation is in comparison with other possible UX³ situations (which could be detected by the considered invariant). For this reason a numerical function $D(\langle \text{INV}(\Gamma_1), \ldots, \text{INV}(\Gamma_p) \rangle, \text{UX³})$ has been suggested. It is called the Degree of UX³ (the Degree of Unexpectedness) with respect to invariants $\langle \text{INV}(\Gamma_1), \ldots, \text{INV}(\Gamma_p) \rangle$ and depends on the following two factors:

- the complexity (the constructibility) of the applied invariant (invariants),
- the sensitivity of the invariant (invariants) to the measure of the violation.

4 Degree of UX³

The following form has been introduced for the computation of $D(\text{INV}(\Gamma_p), \text{UX³})$ (with one invariant INV(Γₚ))

$$D(\text{INV}(\Gamma_p), \text{UX³}) = \frac{1}{Q_{cpmx}} \left( \sum_{x_i \in \text{ELM(INV}(\Gamma_p))} \left( \sum_{x_j \in \text{ELM(INV}(\Gamma_p))} \left( \frac{1}{x_i} \right)^{\alpha_i} \left( \frac{1}{x_j} \right)^{\alpha_j} \right)^{\gamma} \right)^{1/\gamma},$$

where $Q_{cpmx}$ is a quotient of the invariant complexity, $x_i$ are elements of the invariant structure (from the ground set ELM(Inv(Γₚ))), $\alpha_i$ are quotients of importance for elements $x_i$. Quantities $\mu(x_i)$ are quotients of deployment of elements $x_i$ before invariant violation and $\gamma(x_j)$ are quotients of deport-

Table 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Name of Structure</th>
<th>$Q_{cpmx}$ [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Klein group of the 4th order</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Permutation group of the 6th order (4x4)</td>
<td>0.9455</td>
</tr>
<tr>
<td>3</td>
<td>Group of linear transformations of the $\infty$th order</td>
<td>0.7432</td>
</tr>
<tr>
<td>4</td>
<td>Semi-group of binary equivalences</td>
<td>0.5571</td>
</tr>
<tr>
<td>5</td>
<td>Semi-group of binary relations</td>
<td>0.1979</td>
</tr>
<tr>
<td>6</td>
<td>Matroid of the 2nd order</td>
<td>0.1750</td>
</tr>
<tr>
<td>7</td>
<td>Linear regular grammar</td>
<td>0.1345</td>
</tr>
<tr>
<td>8</td>
<td>Non context grammar</td>
<td>0.1047</td>
</tr>
<tr>
<td>9</td>
<td>Structure of association rules</td>
<td>0.0883</td>
</tr>
</tbody>
</table>
ment for elements $x_i$ after violation of the invariant by means of $x_i$. (Number $a = 2$ (usually) but not necessarily).

Note 4.1: If an element is not violated, it holds: $\mu(x) = \mu(x_i)$.

Quotient of structure complexity $Q_{cpxs}$ expresses the difficulty to form a model of such a structure. The complexity of the structures is compared in [26]. Some illustrative examples of $Q_{cpxs}$ quantities for structures with one composition operation are introduced in Table 1.

Note 4.2: The specification of structures 1–5, 7, 8 from Table 1 is known from the literature. Structure 6 is a matroid with 10 independent sets and with the cardinality of its basis equal to 2.

The quantity of $D(\text{Inv}(\Gamma_p), \text{UX}^3)$ gives a qualitative evaluation of the difficulty of the operations that follow after $\text{UX}^3$ detection. Usual such operations (e.g., in the FDI field) are: localisation, isolation, identification and interpretation a $\text{UX}^3$ situation. The higher the $D(\text{Inv}(\Gamma_p), \text{UX}^3)$ quantity is, the more difficult these operations are to execute.

**Example 4.1:** Let us suppose Klein group $B = \langle (I, N, R, C), o \rangle$ as an invariant $\text{Inv}(\Gamma_p)$, and let us suppose the violation of elements N and R given by the following quantities of quotients:

$$\omega_1 = \omega_2 = \omega_3 = \omega_4 = 1.2, \omega_5 = 1.5, \omega_6 = 0.9, \omega_7 = 0.8, \omega_8 = 0.5, \omega_9 = 1.2, \omega_{10} = 1.0, \omega_{11} = 0.7, \omega_{12} = 0.6, \omega_{13} = 0.5$$

$$\mu(x_1) = \mu(x_2) = \mu(x_3) = \mu(x_4) = \mu(x_5) = \mu(x_6) = \mu(x_7) = \mu(x_8) = \mu(x_9) = \mu(x_{10}) = \mu(x_{11}) = \mu(x_{12}) = \mu(x_{13}) = 0.8$$

$$a = 2$$

$$\text{D}(B, \text{UX}^3) = (0.08/4)^{1/2} = 0.1414.$$  

**Example 4.2:** Let us suppose a set of rules ER (with 11 rules) as an invariant $\text{Inv}(\Gamma_p)$, and let us suppose violation of rules $r_3$ and $r_5$ given by the following conditions:

$$\omega_1 = 0.7, \omega_2 = 0.7, \omega_3 = 1.4, \omega_4 = 0.9, \omega_5 = 1.0, \omega_6 = 1.2, \omega_7 = 0.5, \omega_8 = 0.8, \omega_9 = 0.5, \omega_{10} = 1.2, \omega_{11} = 0.7, \omega_{12} = 0.6, \omega_{13} = 0.5$$

$$\mu(x_1) = \mu(x_2) = \mu(x_3) = \mu(x_4) = \mu(x_5) = \mu(x_6) = \mu(x_7) = \mu(x_8) = \mu(x_9) = \mu(x_{10}) = \mu(x_{11}) = \mu(x_{12}) = \mu(x_{13}) = 0.8$$

$$a = 2$$

$$\text{D}(\text{ER}, \text{UX}^3) = \frac{1}{Q_{cpxs}} \left( \frac{\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 + \omega_5^2 + \omega_6^2 + \omega_7^2 + \omega_8^2 + \omega_9^2 + \omega_{10}^2 + \omega_{11}^2 + \omega_{12}^2 + \omega_{13}^2}{12} \right) = 0.565$$

The examples introduced here illustrate the $D(\text{Inv}(\Gamma_p), \text{UX}^3)$ quantities for violation of two very different invariant structures. The results correspond to an intuitive understanding of variable $D(\text{Inv}(\Gamma_p), \text{UX}^3)$. The more complicated the structure $\text{Inv}(\Gamma_p)$ is and the higher its violation is, the higher is the potential quantity of $D(\text{Inv}(\Gamma_p), \text{UX}^3)$. (For illustration: $D(B, \text{UX}^3) \in [0, 1], D(\text{ER}, \text{UX}^3) \in [0, 11.325].$)

5 Conclusions

This paper demonstrates the use of a fuzzy approach for modeling very complex problems. Fuzzy concepts are contained in all essential conceptual constructs as $\text{UX}^3$, $\text{MS}, \text{MSF}$, violation of an invariant, emergence zone, Evaluated Association Rules, Hasse Diagram (and in the fuzzy values and variables).

The paper has introduced methods for $\text{UX}^3$ detection. It has introduced a general approach for the development of $\text{MSS}, \text{MSF}$ and $\text{MD}(\text{UX}^3)$ models, and three variants of these models have been described.

Examples of $\text{MD}(\text{UX}^3)$ with an emergence zone, with Bipartite Graphs and with Evaluated Associations rules in the role of structural invariants of $\text{MSS}$ and $\text{MSF}$ have been presented, e.g., in [13]–[17].

In [17] we introduced an illustrative application of $\text{MD}(\text{UX}^3)$ with Evaluated Association Rules for the conditions of an industrial monitoring system (developed with data support from “Ventilation system of the Mrazovka road tunnel in Prague” in Czech Republic). The proposed method may be applied for processes with a similar formal description, e.g., in transport systems, power supply systems, and in the chemical industry.

The approach is also well applicable in special signal based cases (with no available model of the observed and detected system), where the signals are acquired from special sensors and especially when neural networks or fuzzy systems are used for processing them. (Such applications fields are described, e.g., in [16], [27], [28].)

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References


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