BENDING ANALYSIS OF BEAMS AFFECTED BY FIRES

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Abstract

The fire resistance of reinforced concrete elements can be determined by simple isotherm methods or detailed analyses using a full finite element model. Many design offices do not have sufficient resources to make use of finite element methods and currently must rely on the crude isotherm method. A new methodology of intermediate complexity is presented that determines rapidly the strength of RC beams affected by fire. It is considerably more accurate than existing simple performance based methods, yet is implemented in spreadsheet software that is available to all engineers. Any cross-sectional temperature field can be considered and the method accounts for non-linear and temperature dependent material behaviour in both steel and concrete. It can handle concrete sections of arbitrary cross-section. Results can be numeric or displayed graphically.

Keywords: concrete, fire, analysis, excel, isotherm, spreadsheet

INTRODUCTION

The fire resistance of reinforced concrete elements is normally determined by one of three main procedures which, in increasing order of complexity, are: (i) Using tabulated data from Standard Fire tests which for basic analyses this may be sufficient. (ii) Simple performance based designs using the isotherm method [1]. (iii) More detailed analyses using full finite element models.

A problem with these methods is the enormous gap in complexity between the simplest performance based method (ii) and a full finite element model (iii). Finite element software is expensive to purchase and requires skilled users to produce useful results. Many design offices do not have sufficient funds or expertise to make its use worthwhile for fire design. Consequently they must rely on crude options for assessing the fire resistance of RC structures.

This paper presents a methodology that provides a usable tool to perform calculations to determine the strength of RC beams affected by a fire. It is considerably more accurate than existing simple performance based methods, yet can be implemented in spreadsheet software that is available to all engineers. Any cross-sectional temperature field can be considered and the method accounts for non-linear and temperature dependent material behaviour in both steel and concrete. It can handle concrete sections of arbitrary cross-section. Results can be numeric or displayed graphically.

1 OUTLINE OF METHOD

On heating, the key material parameters of a concrete section that affect its capacity to resist load - ultimate stress, ultimate strain change. Since in a typical fire cross-sectional temperatures are non-uniform, the ultimate stress and strain within a concrete section will vary continuously. This has previously been handled very crudely in the “isotherm” method presented in Eurocode 2 [1] where concrete properties are assumed to either remain as at ambient temperature, or be completely removed due to fire. A temperature of 500°C is normally taken as the transition between “strong” and “weak” concrete. This assumption is clearly very crude so to provide a better approach to estimating the strength of concrete members in fire, the method in this paper adopts the following more sophisticated method

1. Determine the temperature field in the cross-section by discretising the section in to (i,j) cells.
2. For each cell, determine the appropriate ultimate strain for its temperature.
3. Determine the cross-section strain distribution assuming i) no cell can exceed its ultimate strain and ii) there is no overall force in the cross-section.
4. For this strain distribution determine the force in each cell based on temperature dependent stress-strain data.
5. Calculate the overall capacity of the section.

The method will be discussed in detail below considering, for simplicity, a rectangular cross-section.

2 DETAIL OF METHOD

2.1 Heat Transfer

The method requires an estimate of the temperature field in the cross-section being considered in the form of temperatures at each “cell” of a discretization scheme. Cells with typical dimensions of 5 mm are appropriate. The temperature field can be derived by any means and is not, as for the isotherm method, limited to using results from Standard Fire exposure. For this paper, temperatures were derived from an Excel-based 2-d finite difference heat transfer model with an assumption of a Standard fire.

2.2 Constitutive Models

Inputs into the method include material constitutive models for both reinforcing steel and concrete. These include temperature dependency of ultimate stress and strain and, if desired, material softening. None of these phenomena are included in the isotherm method. In this paper, data from EC2 has been used but any similar material data would be appropriate.

2.3 Strain Distribution

The method presented here modifies the compression block approach typically used at ambient temperature for a different procedure to determine a more accurate shape of the compression zone. Once the temperature field in a section has been obtained, the allowable compressive strain in each cell in the cross-section has to be determined. At ambient temperature a value of 0.003 is widely used in concrete design but this value is not appropriate for elevated temperatures due to the temperature dependency of ultimate strain. Here, a more general value is assumed, the strain at the peak stress ε_{c1} amplified by a constant β ($\varepsilon_{Allowable} = \beta\varepsilon_{c1}$). This value for traditional ambient temperature design takes the value $\beta_{ambient} = \varepsilon_{design}/\varepsilon_{c1} = 0.003/0.0025 = 1.2$. Thus, for a given temperature matrix and β, an allowable strain matrix can be determined as indicated for a typical case in Fig. 1.

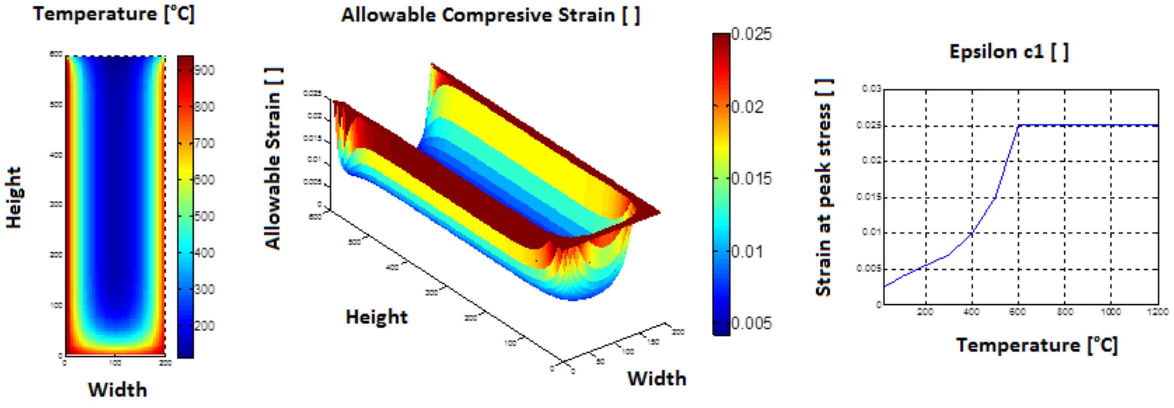

Fig. 1. Temperature profile (Left), Allowable strain $\beta*\varepsilon c1$ (Centre) and ε_{c1} according EC2 (Right).

Assuming that plane sections remain plane after bending, two conditions have to be satisfied in order to calculate the bending resistance of the section: i) The mechanical strain field has to be tangent to the allowable strain field.

$$\max\left(\varepsilon^{ij}_{mechanical} - \varepsilon^{ij}_{allowable}\right) = 0 \tag{2}$$

Where $\varepsilon^{ij}_{mechanical}$ and $\varepsilon^{ij}_{allowable} = \beta \cdot \varepsilon^{ij}_{c1,T}$ ($\varepsilon^{ij}_{c1,T}$ is the concrete strain at peak stress [] and β constant). All parameters are the complete matrices of strains and the difference is taken cell to cell. The point at which this condition is satisfied is the point at which crushing first occurs in the section and corresponds to compression failure of the concrete. The second condition ii) is that there must be no overall axial force in the section, ie:

$$C = T$$

Where *C* the total compressive force in the concrete and *T* the tensile force in the steel. The strain distribution that satisfies both conditions is determined rotating the mechanical strain plane iteratively. The resulting situation is shown graphically in Fig.s 2 and 3 for hogging and sagging moment respectively.

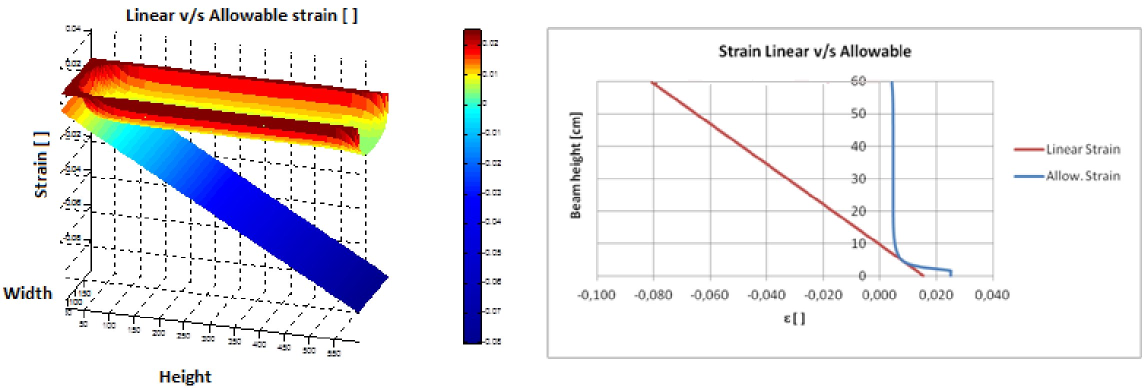

Fig. 2. Interaction between allowable strain and the mechanical strain for hogging moment (β=1).

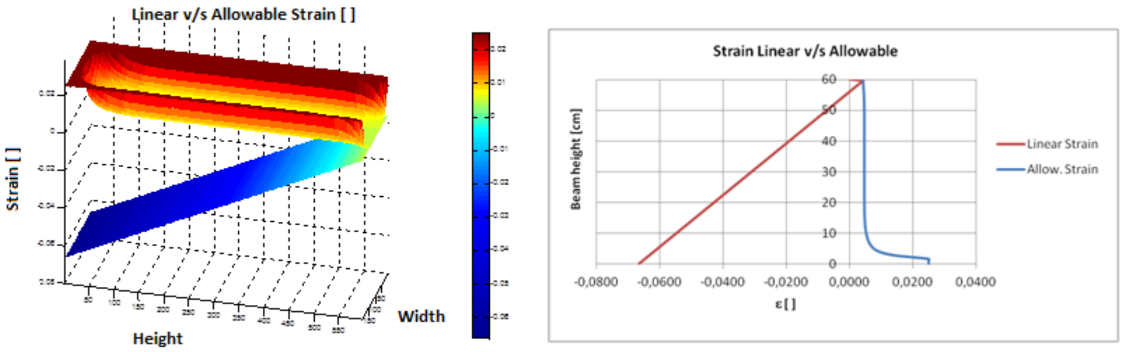

Fig. 3. Interaction between allowable strain and the mechanical strain for sagging moment (β=1).

2.4 Stress Distribution and Capacity.

With the temperature and strain in each cell determined, it is possible to determine the stress in each cell based on the temperature dependent material data. This leads to a good estimate of high temperature stress block, as indicated in Fig. 4

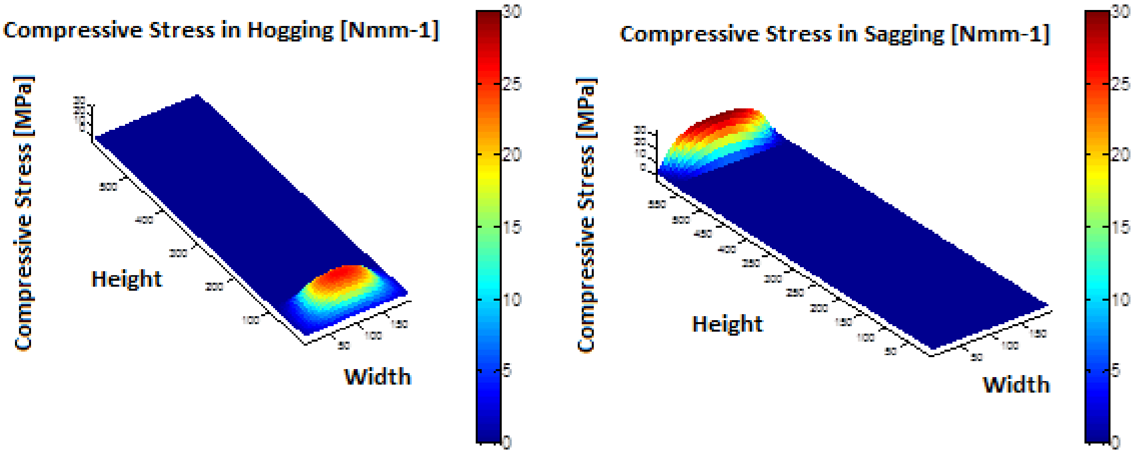

Fig. 4. Stress diagram for hogging (Left) and sagging (Right) moments.

As the coordinates of each cell are known and the value of the stress is also known after the rotation of the mechanical strain plane, the centroid of the compression (C.C) is obtained from.

$$C.C = \frac{\Sigma f_{cT}^{ij} \cdot y_j}{\Sigma f_{cT}^{ij}} \tag{3}$$

Where f_{cT}^{ij} is the compressive stress in the concrete cell (i,j) at temperature T [Nmm^{-1}] and y_j the vertical position of the cell. So finally the moment resistance can be determined, as usual.

$$M - (C \text{ or } T) \cdot (d - C.C) \tag{4}$$

3 CASE OF STUDY

For this purpose two beams will be compared for two different reinforcement ratios: a) 30 and b) 75% of the maximum reinforcement ratio. For four exposure conditions: a) Ambient T°, b) Standard fire during 60 minutes, c) Standard fire during 90 minutes, d) Standard fire during 120 minutes. And finally, for Sagging and Hogging Moment, totalling 32 cases.
For a beam designed using steel with a yield stress 400 [MPa] and concrete compressive resistance 30 [MPa], ρ_{min}=0.0033 and ρ_{max}=0.023, being ρ = As/bd (ACI 2005, [3]).

3.1 Validation of the method at ambient temperature.

The calculation performed for the four cases was done varying the value of β and studying the flexural resistance. Normalizing the resistance obtained by the resistance obtained at β=1 a curve of the enhancement due to β can be obtained.

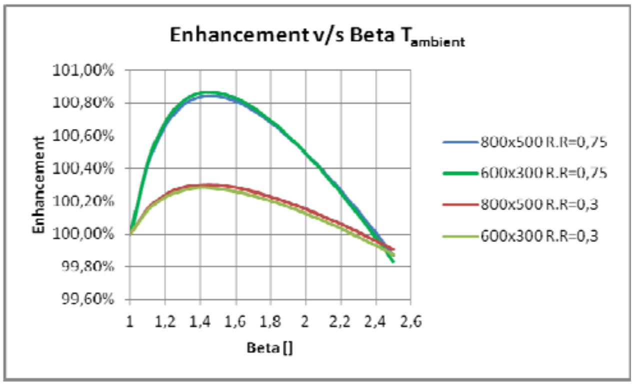

Fig. 5. Enhancement v/s β.

The method captures the fact that the enhancement at ambient temperature is constant for different beam sizes and for different reinforcement ratios. For the cases studied the optimal value of β is 1.4 and the maximum enhancement found was 0.86%. In particular, was found for all cases that when a β=1.2 is used, the utilization of the maximum enhancement is 80%.

3.2 Behaviour of the method under fire conditions.

A summary of the results obtained is presented graphically for the 32 cases in terms of the enhancement achieved and the optimal β at that point.

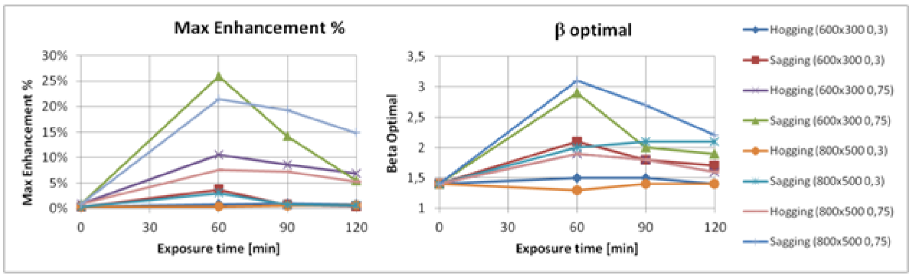

Fig. 6. Maximum Enhancement [%] (Left) and Optimal Beta for max. enhancement (Right).

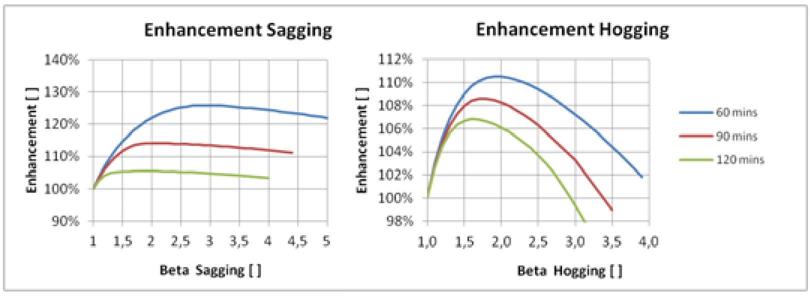

Fig. 7. Enhancement v/s Beta, beam 600x300 with high reinforcement ratio (particular case).

The results for this beam are presented now in detail. The values for resistance obtained by the isotherm method are compared with the resistance calculated from the proposed method for β=1 and for the optimal β in each case. The over resistance in the last column correspond to the percentage in resistance obtained from the isotherm method respect to the value at the optimal β.

Tab 1. Summary of the comparison between methods.

Beam	Reinfor. Arrangement.	Moment Type.	Exposure time [min]	Isotherm Resistance [kNm]	Calculated Resistance. (β=1)	Optimal β	Max. Resistance (Opt. β) [kNm]	Over Resistance (Isotherm Method)
600 x 300	3+3 ϕ 16	Sagg.	60	186,97	171,40	2,1	177,69	5%
	3+3 ϕ 16	Hogg.	60	247,08	228,38	1,5	230,25	7%
	3+3 ϕ 16	Sagg.	90	115,08	117,03	1,8	117,86	-2%
	3+3 ϕ 16	Hogg.	90	211,46	196,40	1,5	198,28	7%
	3+3 ϕ 16	Sagg.	120	86,94	84,41	1,7	84,75	3%
	3+3 ϕ 16	Hogg.	120	179,39	168,68	1,4	170,08	5%
	3+3 ϕ 25	Sagg.	60	471,45	323,58	2,9	407,38	16%
	3+3 ϕ 25	Hogg.	60	596,42	437,74	1,9	483,85	23%
	3+3 ϕ 25	Sagg.	90	318,27	252,47	2	288,09	10%
	3+3 ϕ 25	Hogg.	90	525,63	394,90	1,8	428,77	23%
	3+3 ϕ 25	Sagg.	120	226,62	199,63	1,9	210,59	8%
	3+3 ϕ 25	Hogg.	120	448,75	348,04	1,6	371,81	21%

Note that the highest errors are found in the hogging analysis for high reinforcement ratios and that because no correction due to the high temperature in the corners of the compression block was made.

4 CONCLUSIONS

The proposed method is a useful tool to be used in any engineering office. The results obtained show perfect agreement for ambient temperature using currently recommended allowable strains. Additionally, the traditional method is slightly conservative respect to the method proposed.

A comparison between the isotherm method and the one proposed shows the necessity to correct the isotherm method by considering a compression block with round corners, which reduces its simplicity. In general the results obtained via the isotherm method are slightly higher than the ones obtained with the proposed method, this shows that the isotherm method, while simple, is not as conservative as engineers might think.

Using an approach like the one presented it is possible to study the effect of different assumptions about the allowable compressive strains in concrete. The value of β that produces a maximum resistance varies from 1.4 (In perfect agreement with the value recommended for ambient temperature) to 2.9. In general a good utilization of the enhancement is reached using a value of $\beta=1.4$ for hogging and $\beta=2.0$ for sagging moments.

The enhancement achieved is different for different exposures and different fire curves, in this particular case a peak enhancement of 25% was found, which shows the value of studying the maximum allowable strain for design.

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