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# CRACK ANALYSIS OF RC BEAMS STRENGTHENED WITH CFRP STRIPS UNDER SECONDARY LOADING

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## ABSTRACT

The paper established the calculation formulas on the average crack spacing and the maximum crack width of CFRP（Carbon Fiber Reinforced Polymer）reinforced concrete beam under the secondary loading. Conversion of CFRP plate area into the reinforcement ratio of the reinforced beam, the calculation formula on the average crack spacing of CFRP reinforced concrete beam under the secondary loading was established. Based on calculation formula of the maximum crack width for concrete beam, the calculation formula on the maximum crack width of CFRP reinforced concrete beam under the secondary loading was established. The average crack spacing and the maximum crack width calculated by the formulas in the paper were compared with the test results, which verified that the formula is correct.

## KEYWORDS

Carbon fiber reinforced polymer plates, Reinforced concrete beam, Secondary loading, Average crack spacing, Maximum crack width

## INTRODUCTION

Strengthening with CFRP plate is a kind of new structure strengthening technology with fast research, application and development in the recent years. Compared with traditional reinforced methods, the method has such advantage as high strength, high efficiency, simple construction and excellent durability [1]. Due to the concrete structure has been subjected to load before reinforcement, there is a "stress hysteresis" phenomenon between the reinforcement material attached to the structural surface during the later loading process. Therefore, it is of great significance to the analysis of the structural characteristics of CFRP reinforced concrete beams under secondary loads. When a bridge structure is reinforced, besides bearing its own weight and the constant load of the superstructure, it may also bears vehicle load; for the structures such as beam and plate of a house building, when bearing their own weights, they may bear some loads hard to unload (such as plate decoration layer and fixed equipment, etc.). China Association for Engineering Construction Standardization points out that [2], when the initial bending moment is less than 20% of the bending bearing force of the unreinforced section, the effect of the "stress hysteresis" on the bearing force is negligible; while ACI440 committee and FIB consider the effect of the "stress hysteresis" on the bearing force when calculating bearing force in the relevant specifications [3,4].

As for the destruction theory of the reinforced structure under the secondary loading, the scholars conducted a lot of tests and finite element analyses, researched the stress characteristics of the reinforced structure under the secondary loading and the calculation model on bearing capacity, and the structural safety of the CFRP reinforced beam under secondary loading was guaranteed [5-8]. However, the analysis and calculation of the crack width of the reinforced beam under the secondary loading were seldom studied. Zhuang et al. [9] carried out a research for the calculation method of crack width of CFRP reinforced concrete beam, and on basis of the *Design Code for Concrete Structure* [10], they proposed the calculation formulas on the average crack spacing and crack width that similar to ordinary reinforced concrete beam crack calculation method; Tan et al. [11] deduced the calculation formulas on the average crack spacing and width of CFRP reinforced concrete beam on basis of the bonding-slippage theory, proposed a theoretical analysis method applicable for calculating the crack width of the CFRP reinforced concrete beam. However, the above scholars failed to consider the effect of the “strain hysteresis” in the calculation on the average crack spacing and the crack width of the reinforced beam. The paper will consider the effect of “strain hysteresis” on the reinforced beam, establish a calculation model on the average crack spacing and maximum crack width of CFRP reinforced concrete beam under the secondary loading.

## TEST ON BEARING CAPACITY OF CFRP REINFORCED BEAM

### The loading test of CFRP reinforced beam

In order to research the effect of CFRP plates on structural stress characteristics and crack development under the secondary loading of CFRP reinforced concrete beam. The authors of this paper takes two pieces of 16 m pre-stressed hollow slab beams removed from a highway in Shenyang of China. The two pieces of old beam had served for 20 years, the parameters of the reinforced beams are approximate to the actual bridge beams and the test results are more applicable. The test beam adopts C40 concrete, the longitudinal reinforcement adopts HRB335, with a diameter of 12 mm; the stirrup adopts R235, with a diameter of 8 mm; and the pre-stressed reinforcement of the test beam applies $\phi_j$ 15.24（7$\phi$5）type, the standard strength is 1,860MPa and the tensioning control stress is 1,395Mpa, the elasticity of reinforcements are 200 GPa. The width of the hollow slab was 99 cm and the height was 70 cm. The center hollow section was a circle with a diameter of 50 cm; the thickness of the top plate was 8 cm and the thickness of the floor was 12 cm. The longitudinal design construction length was 1596 cm; the distance from the centerline of bearing to the end was 18 cm.

In order to simulate the load that is hard to unload in bridge, the concrete with 10 cm thick was poured on the test beams and linked with the beam by bonded reinforcements. Two 0.111x200 mm CFRP plates were pasted on the bottom of the test beam to improve the bearing capacity of the test beam, the elasticity of CFRP plates are165 GPa, the test beam was named as “CFRP beam”. In order to contrast the CFRP reinforced effect, one piece of unreinforced test beam, which was named as “non CFRP beam”, was taken as the contrast beam. Bending destruction test was conducted for two pieces of test beam. The test of bending failure was carried out to obtain the stress characteristics and crack development of the two test beams under the secondary load. Reinforcement design of test beams and test loading layout are shown in Figure. 1 and Figure 2.

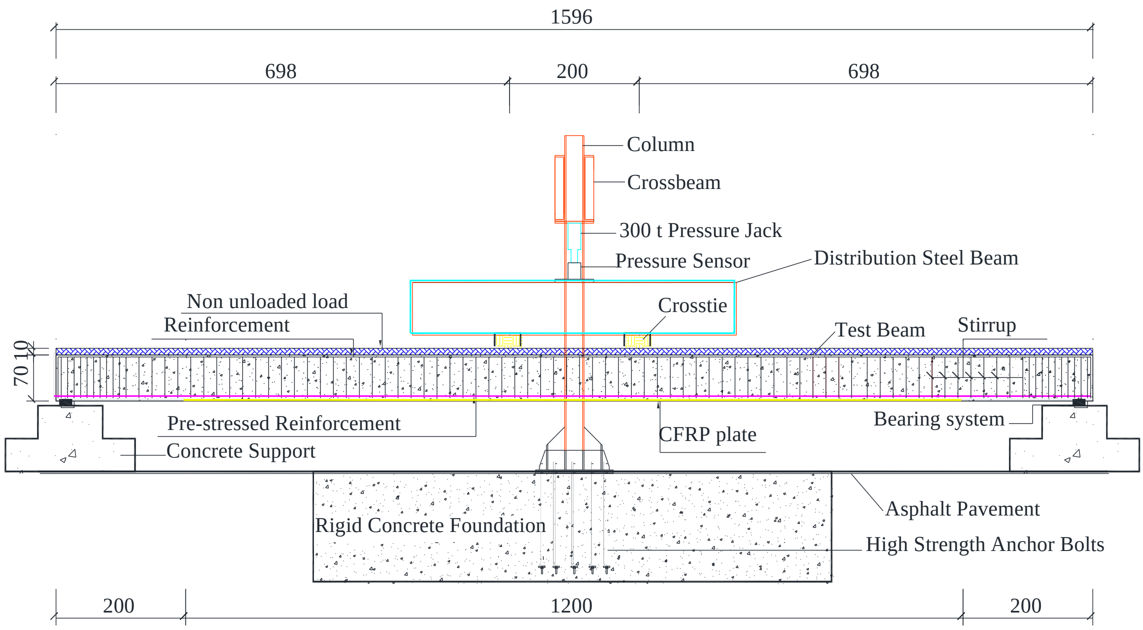


*Fig. 1 - Test set-up of the test beam (unit: cm)*

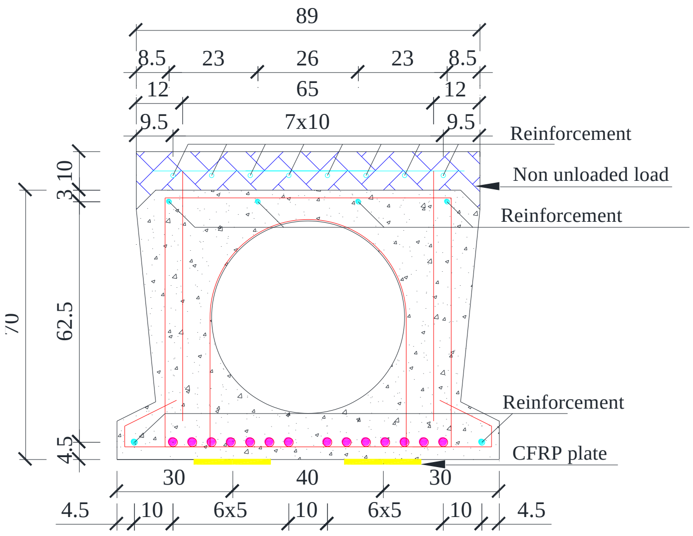


*Fig. 2 - Cross section for the test beam (unit: cm)*

## The results analysis on CFRP reinforced beam under secondary loading test

In order to research the effect of CFRP plates on structural stress characteristics and crack development under the secondary loading of CFRP reinforced concrete beam, a pressure sensor was adopted to record the test load in the loading process of two pieces of beam, the strain of reinforcement and pre-stressed reinforcement at the mid span section was recorded by the reinforcement strain gauge, concrete strain gauges were stuck on the surface of the mid span section to record the strain of concrete, the displacement of the mid span was recorded by the displacement sensor arranged at the mid span section; the surface observation method was adopted to observe the crack development and a crack width gauge was used to measure the maximum crack width of the test beam under all levels of load.

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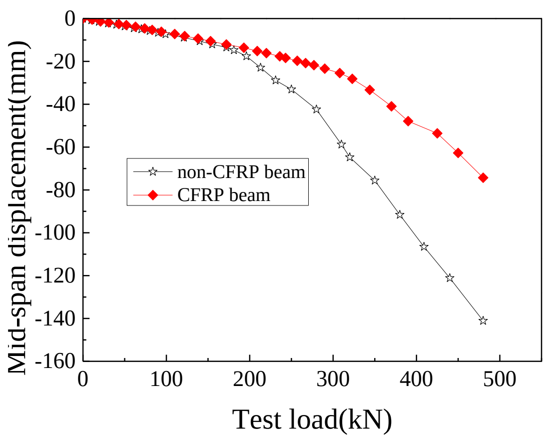
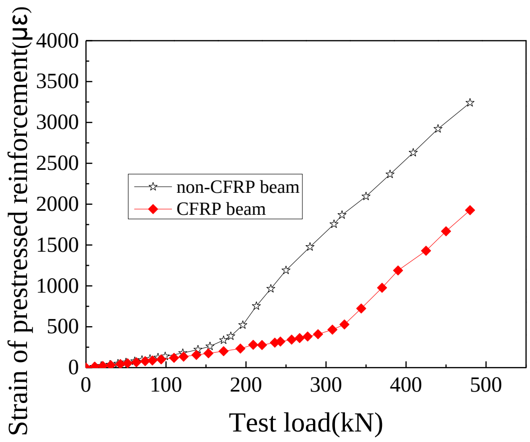
The test beams were loaded for bending test. When the load reached 213 kN, the non CFRP beam cracked, following the continual increase of test load, new crack constantly appeared, and the crack width quickly increased. The concrete strain at the top edge of the mid span section, the strain of reinforcement & pre-stressed reinforcement and displacement changed from linear increase to nonlinear development. For CFRP beam, when the test load reached 308 kN, a crack appeared at the bending position. After a crack appeared, the linear development of the concrete strain at the top edge of the mid span section, the strain of reinforcement & pre-stressed reinforcement and displacement were destroyed, however, the development speed was lower than that of the non CFRP beam. The CFRP plate debonded occurred at the bottom of CFRP beam when the test load reached 480 kN, as shown in Figure 3. After the bond of CFRP plates were damaged, the growth of the crack width and the stress characteristics of structure quickly increased, finally, the destruction loads of the two pieces of beam became basically same. Therefore, the effect of CFRP plates on structural stress characteristics and crack development of the reinforced beam under the secondary loading within the scope of 480 kN will be discussed. Figure 4 gives the contrast of the two pieces of test beams on the stress characteristics and the development of crack width under the secondary load.


*Fig. 3 - Photograph of the test in progress*

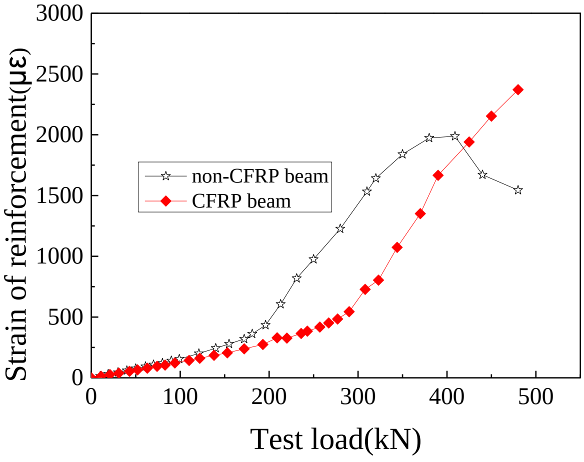



*(a) Mid-span displacement*

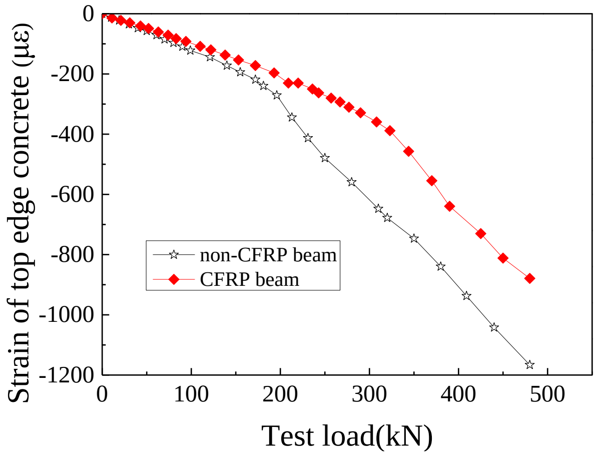



*(b) Strain of pre-stressed reinforcement*

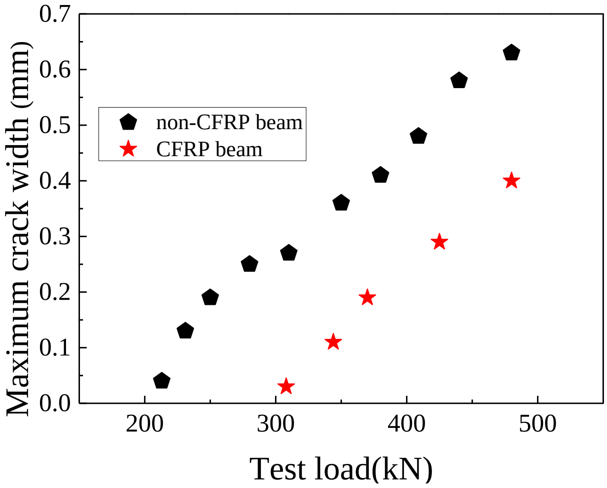
*Fig. 4 - Comparison of non-CFRP and CFRP beam on stress characteristics and width of crack*




*(c) Strain of reinforcement*




*(d) Strain of top edge concrete*




*(e) Maximum crack width*

*Fig. 4 - Comparison of non-CFRP and CFRP beam on stress characteristics and width of crack*

In Figure 4, before the cracking load was reached, the stress characteristics of the test beam developed linearly, and value of the stress characteristics for the non-CFRP and the CFRP beam were same. Following continual increase of load, the non-CFRP beam cracked firstly, this shows that CFRP plate strengthening can increase the cracking load of the reinforced beam. After the test beam cracked, the development speed of the stress characteristics of the non-CFRP beam was apparently higher than that of the CFRP beam, this shows that CFRP plate can effectively increase the stress characteristics of the cracked concrete beam. As for the crack width of the test beam, the development speed of the maximum crack width of CFRP beam was less than the non-CFRP beam after structure cracked, this shows that CFRP plate can restrict the development of the crack width of the reinforced beam. As for the average crack spacing of the test beams, the measured average crack spacing of the non-CFRP beam was 197 mm, while the CFRP beam was 184 mm, this shows that CFRP plate can deduce the average crack spacing and crack width on the surface of the reinforced beam.

Form the results of the test, CFRP plate can reduce the stress characteristics and development of the crack of the reinforced beams under the secondary loading. Therefore, it is significance to research the crack of CFRP reinforced concrete beam under the secondary loading, which can provide experience for the design of CFRP reinforced beams. In order to establish and verify the calculation formula on the crack spacing and width of crack under the secondary loading, the crack data of CFRP reinforced beams under secondary load were collected in this paper [12-14]. The parameters of the CFRP reinforced beams are shown in Table 1.

*Tab. 1 - Parameters of beams*

| Test | Name of Beam | $f_{cu}$ (MPa) | $f_{ct}$ (MPa) | $A_f$ (mm$^2$) | $E_f$ (GPa) | $f_{fu}$ (MPa) |
|---|---|---|---|---|---|---|
| Li | L1-1-1a | 34.4 | 2.20 | 11.1 | 212 | 3500 |
| | L1-1-2a | 34.4 | 2.20 | 11.1 | 212 | 3500 |
| | L1-2-1b | 34.4 | 2.20 | 11.1 | 212 | 3500 |
| | L1-2-2b | 34.4 | 2.20 | 11.1 | 212 | 3500 |
| | L1-2-3b | 34.4 | 2.20 | 11.1 | 212 | 3500 |
| | L2-2-2c | 34.4 | 2.20 | 11.1 | 212 | 3500 |
| | L2-2-3c | 34.4 | 2.20 | 11.1 | 212 | 3500 |
| Huang | LB-2-2 | 36.1 | 2.26 | 33.4 | 220 | 3500 |
| | LB-3-2 | 36.1 | 2.26 | 33.4 | 220 | 3500 |
| Bu | RCFP-2 | 23.4 | 1.78 | 200 | 165 | 2800 |
| | RCFP-3 | 24.4 | 1.82 | 200 | 165 | 2800 |
| | RCFP-4 | 25.4 | 1.86 | 200 | 165 | 2800 |
| This paper | CFRP beam | 26.8 | 1.92 | 400 | 165 | 2800 |

Notes: $f_{cu}$ is the ultimate compressive strength of concrete； $f_{ct}$ is the ultimate tensile strength of concrete； $A_f$ is the area of CFRP section； $E_f$ is the elasticity modulus of CFRP; $f_{fu}$ is the ultimate tensile strength of CFRP.

## ANALYSIS ON AVERAGE CRACK SPACING

The existing research shows that the average crack spacing is only related to the ratio of reinforcement, area of CFRP section and the thickness of concrete protection layer [12]. Therefore, the effect of "stress hysteresis" is not considered when establishing the calculation formula for the crack spacing of CFRP reinforced beams under secondary loading. Suppose that the stress of reinforcement and CFRP plate at the section of the first crack are respectively $\sigma_{s1}$ and $\sigma_{f1}$. The stress of concrete, reinforcement and CFRP plate at the other section where a crack will appear are respectively $f_t$, $\sigma_{s2}$ and $\sigma_{f2}$. The bond stress between reinforcement and concrete, CFRP plate and concrete are respectively $\tau_s$ and $\tau_f$. The second crack will appear within the average bond transfer length $l_{mf}$. Figure 5 shown the calculation model on average crack spacing, and the balance condition between the existing crack and the section where a crack will appear as follows:

$$\sigma_{s1}A_s+\sigma_{f1}A_f=\sigma_{s2}A_s+\sigma_{f2}A_f+f_tA_c. \tag{1}$$

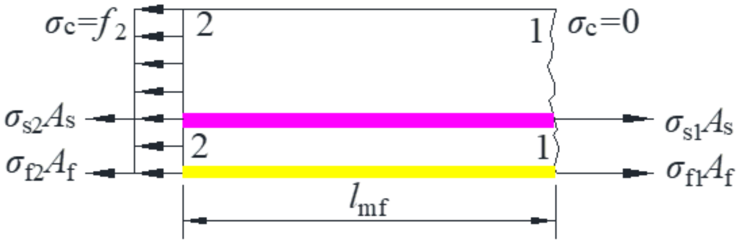
For the reinforcement isolator as shown in Figure 5(b),

$$(\sigma_{s1}-\sigma_{s2})A_s=\tau_s C l_{mf}. \tag{2}$$

For the CFRP isolator as shown in Figure 5(c),

$$(\sigma_{f1}-\sigma_{f2})A_f=\tau_f b_f l_{mf}. \tag{3}$$

In the formula, $C$ is the perimeter of reinforcement; $b_f$ is the sticking width of CFRP plate.




*(a) Stress balance between cracks*


*(b) Stress balance of reinforcement between cracks*


*(c) Stress balance of reinforcement CFRP plate between cracks*

*Figure 5 Calculation model on average crack spacing*

From Formula (1) to (3), it can be gotten that：

$$f_t A_c = 4\tau_s \frac{A_s + A_f}{d} l_{mf} \left[ 1 + \frac{A_f}{A_s + A_f} \frac{\tau_f d/(4t_f) - \tau_s}{\tau_s} \right]. \tag{4}$$

Suppose $l_m = \left[ 1 + \frac{A_f}{A_s + A_f} \frac{\tau_f d/(4t_f) - \tau_s}{\tau_s} \right] l_{mf}$, then,

$$l_m = \frac{f_t A_c}{4\tau_s (A_s + A_f)/d} = \frac{1}{4} \frac{f_t}{\tau_s} \frac{A_c d}{A_s + A_f} = \frac{1}{4} \frac{f_t}{\tau_s} \frac{d}{\rho_f}. \tag{5}$$

In the formula, $\rho_f$ is comprehensive effective reinforcement ratio, $\rho_f = (A_s + A_f)/A_c$ ; $d$ is the diameter of reinforcement; $t_f$ is thickness of CFRP plate.

The expression of the average crack spacing of the reinforced concrete beams in the Formula (5) is same as *Concrete Structure Design Specification GB 50010-2015*, the expression of average crack spacing of the concrete beam is shown as follows:

$$l_m = 1.9c + 0.08 \frac{d}{\rho_{te}}. \tag{6}$$

In the formula, $c$ is the thickness of the protective layer of reinforcement (mm); for reinforced concrete beam, $\rho_{te}$ is the effective reinforcement ratio of the reinforcement.

For CFRP reinforced beam, the effect of CFRP in the tensile zone should be considered. Calculation of effective reinforcement ratio of CFRP plate according to the principle of effective reinforcement ratio of reinforcement. Thus, the calculation formula of $\rho_{te}$ after strengthening is shown as follows:

$$\rho_{te} = \rho_{ste} + \rho_{fte}. \tag{7}$$

$$\rho_{ste} = \frac{A_s}{0.5bh + (b_f - b) h_f}. \tag{8}$$

$$\rho_{fte} = \frac{A_f}{0.5bh + (b_f - b) h_f}. \tag{9}$$

In the formula, $\rho_{\mathrm{ste}}$ is the effective reinforcement ratio of reinforcement in the reinforced beam; $\rho_{\mathrm{fte}}$ is the effective reinforcement ratio of CFRP plate in the reinforced beam; $h$ is the height of section; $b$ is the width of section; $b_{\mathrm{f}}$ is width of the tensile flange; $h_{\mathrm{f}}$ is the height of the tensile flange.

According to the research achievements of the Ref. [15], the bond strength $\tau_{\mathrm{f}}$ of CFRP and concrete is proportional to the tensile strength $f_{\mathrm{t}}$ of concrete, thus the strengthening influence coefficient $\beta$ is introduced:

$$\beta=\frac{A_{\mathrm{f}}}{A_{\mathrm{s}}+A_{\mathrm{f}}}\left(\frac{\tau_{\mathrm{f}} d/\left(4t_{\mathrm{f}}\right)-\tau_{\mathrm{s}}}{\tau_{\mathrm{s}}}\right)=\frac{A_{\mathrm{f}}}{A_{\mathrm{s}}+A_{\mathrm{f}}}\left(k_{\mathrm{f}}\frac{d}{t_{\mathrm{f}}}-1\right). \tag{10}$$

In the formula, $k_{\mathrm{f}}=\tau_{\mathrm{f}}/\left(4\tau_{\mathrm{s}}\right)$ is the coefficient of bonding action between CFRP plate and concrete. By the above analysis, the relationship between the average crack spacing $l_{\mathrm{mf}}$ of the CFRP reinforced concrete beam and the average crack spacing $l_{\mathrm{m}}$ of the unreinforced concrete beam is shown as follows:

$$l_{\mathrm{mf}}=\frac{l_{\mathrm{m}}}{1+\beta}. \tag{11}$$

According to Formula (10), $\beta$ is relevant to the tensile strength of concrete and the area ratio of CFRP plate and reinforcement. By the comparison between the actually measured average crack spacing $l_{\mathrm{mf}}$ of CFRP plate reinforced concrete beam and the calculation value $l_{\mathrm{m}}$ in Formula (6), $\beta$ value can be attained. Then, by utilizing Formula (10), the value of $k_{\mathrm{f}}$ can be solved. The calculation result is shown in Table 2.

*Tab. 2 - Calculation results of average spacing of test beam cracks*

| Name of Test Beams | $l_{\mathrm{mf}}$ (mm) | $c$ (mm) | $d$ (mm) | $\rho_{\mathrm{ste}}$ | $\rho_{\mathrm{fte}}$ | $A_{\mathrm{f}}$ (mm²) | $A_{\mathrm{s}}$ (mm²) | $A_{\mathrm{f}}/A_{\mathrm{s}}$ | $l_{\mathrm{m}}$ (mm) | $k_{\mathrm{f}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| L1-1-1a | 112 | 20 | 12 | 1.21% | 0.06% | 11.1 | 226 | 0.049 | 113.92 | 0.0126 |
| L1-1-2a | 111 | 20 | 12 | 1.21% | 0.06% | 11.1 | 226 | 0.049 | 113.92 | 0.0144 |
| L1-2-1b | 105 | 20 | 12 | 1.21% | 0.06% | 22.2 | 226 | 0.098 | 110.52 | 0.0294 |
| L1-2-2b | 103 | 20 | 12 | 1.21% | 0.06% | 22.2 | 226 | 0.098 | 110.52 | 0.0336 |
| L1-2-3b | 107 | 20 | 12 | 1.21% | 0.06% | 22.2 | 226 | 0.098 | 110.52 | 0.0253 |
| L2-2-2c | 79 | 20 | 12 | 1.21% | 0.06% | 22.2 | 402 | 0.055 | 80.43 | 0.0187 |
| L2-2-3c | 80 | 20 | 12 | 1.21% | 0.06% | 22.2 | 402 | 0.055 | 80.43 | 0.0153 |
| LB-2-2 | 95 | 25 | 12 | 1.70% | 0.17% | 33.4 | 339 | 0.099 | 99.06 | 0.0273 |
| LB-3-2 | 93 | 25 | 12 | 1.70% | 0.17% | 33.4 | 339 | 0.099 | 99.06 | 0.0319 |
| RCFP-2 | 157 | 30 | 22 | 1.22% | 0.27% | 200 | 917 | 0.218 | 175.17 | 0.0748 |
| RCFP-3 | 163 | 30 | 22 | 1.22% | 0.27% | 200 | 917 | 0.218 | 175.17 | 0.0644 |
| RCFP-4 | 161 | 30 | 22 | 1.22% | 0.27% | 200 | 917 | 0.218 | 175.17 | 0.0678 |
| CFRP beam | 184 | 35 | 15.2 | 0.84% | 0.17% | 400 | 1960 | 0.204 | 187.33 | 0.0986 |

Note: $l_{\mathrm{mf}}$ is the average crack spacing of the test beam; $l_{\mathrm{m}}$ is the average crack spacing calculated as Formula (6).

The bonding strength of CFRP plate and concrete is directly proportional to the tensile strength of concrete, therefore, $k_{\mathrm{f}}$ should be a constant. However, there are many factors affecting the bonding strength between CFRP plate and concrete, such as reinforcement and CFRP plate slippage, etc. In addition, $A_f / A_s$ also affect the bonding strength. By the linear fitting for the data of test beams, as shown in Figure 6, the relationship between $k_f$ and $A_f / A_s$ is shown as follows:

$$k_f = 0.337 A_f / A_s . \quad (12)$$

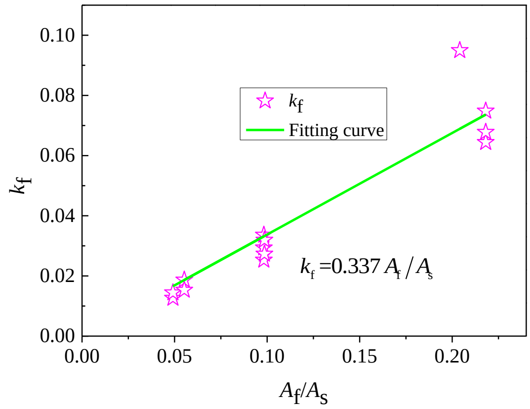



*Fig. 6 - Linear fitting graph of $k_f$ and $A_f / A_s$*

As the relationship between $k_f$ and $A_f / A_s$ is obtained, and the $\beta$ can be calculated by Formula (10), so the average crack spacing of the CFRP plate reinforced concrete beam $l'_{mf}$ could be gotten by Formula (11), which used with the newly calculated $\beta$. The $l'_{mf}$ is compared with the test result, as shown in Tab. 3. In the Tab. 3, the average ratio between the calculation value and the test results is 0.995, the average square error is 0.222, the calculation value is quite in agreement with the test results.

*Tab. 3 - Comparison of average crack spacing between calculation value and test result*

| Test | Name of test beams | $l_{mf}$ (mm) | $l_m$ (mm) | $l'_{mf}$ (mm) | $l'_{mf} / l_{mf}$ |
|---|---|---|---|---|---|
| Li | L1-1-1a | 112 | 113.917 | 110.108 | 0.983 |
| | L1-1-2a | 111 | 113.917 | 110.108 | 0.992 |
| | L1-2-1b | 105 | 110.522 | 103.671 | 0.987 |
| | L1-2-2b | 103 | 110.522 | 103.671 | 1.007 |
| | L1-2-3b | 107 | 110.522 | 103.671 | 0.969 |
| | L2-2-2c | 79 | 80.433 | 79.175 | 1.002 |
| | L2-2-3c | 80 | 80.433 | 79.175 | 0.990 |
| Huang | LB-2-2 | 95 | 99.057 | 92.860 | 0.977 |
| | LB-3-2 | 93 | 99.057 | 92.860 | 0.998 |
| Bu | RCFP-2 | 157 | 175.174 | 158.915 | 1.012 |
| | RCFP-3 | 163 | 175.174 | 158.915 | 0.975 |
| | RCFP-4 | 161 | 175.174 | 158.915 | 0.987 |
| This paper | CFRP beam | 183 | 187.327 | 193.831 | 1.059 |

# ANALYSIS ON CRACK WIDTH

## Deformation relationship of section

Figure 7 shown the deformation relationship of section under the secondary loading. Before strengthening, under the action of the initial bending moment $M_{\mathrm{i}}$, the compressive strain at the compressive edge of section is $\varepsilon_{\mathrm{ci}}$, the strain of the tensile reinforcement is $\varepsilon_{\mathrm{si}}$, the initial strain of the concrete at the tensile edge of section and also the hysteresis strain of the CFRP plate $\varepsilon_{\mathrm{i}}$ are calculated as follows [2]:

$$\varepsilon_{\mathrm{i}} = \frac{h}{h_0}(\varepsilon_{\mathrm{ci}} + \varepsilon_{\mathrm{si}}) - \varepsilon_{\mathrm{ci}}. \tag{13}$$

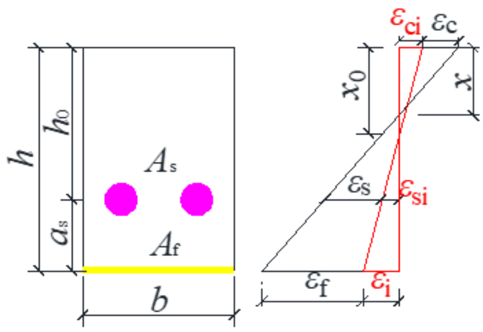
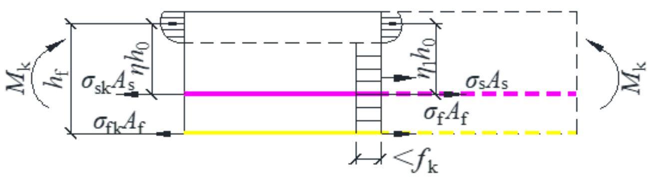



*Fig. 7 - Deformation relationship of section under the secondary loading*

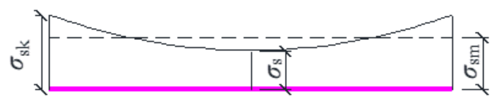
After structure strengthening, under the action of the secondary bending moment $M$, the compressive strain at the compressive edge of section is $\varepsilon_{\mathrm{c}}$, the strain of the tensile reinforcement is $\varepsilon_{\mathrm{s}}$, the tensile strain of the CFRP plate is $\varepsilon_{\mathrm{f}}$.

## Stress of reinforcement at section

Figure 8 presents the stress distribution between cracks. As shown in Figure 8 (a), under the action of the bending moment $M_{\mathrm{k}}$, the reinforcement stress is $\sigma_{\mathrm{sk}}$ and the CFRP plate stress is $\sigma_{\mathrm{fk}}$ at the cracked section. Between two cracks, the reinforcement stress is $\sigma_{\mathrm{s}}$ and the CFRP plate stress is $\sigma_{\mathrm{f}}$. Fig.8 (b), (c) and (d) show the distribution of the stress of reinforcement and the compressive stress of concrete between the cracks of the reinforced beam.

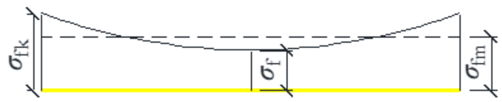



*(a) Section stress*




*(b) Reinforcement stress*

*Fig. 8 - Stress distribution between cracks*




*(c) Stress of CFRP plate*


*(d) Stress of concrete*

*Fig. 8 - Stress distribution between cracks*

Suppose that the mean stress $\sigma_{sm}$ of the reinforcement between the cracks of the reinforced beam is:

$$\sigma_{sm} = \psi_s \sigma_{sk}. \tag{14}$$

In the formula, $\psi_s$ is the nonuniformity coefficient of the strain of the tensile reinforcement.

According to the past test researches [9], the relationship between the mean stress of reinforcement between cracks and the reinforcement stress at the middle section of crack is shown as follows:

$$\sigma_{sm} = 1.1\sigma_s. \tag{15}$$

According to Figure 8(a), the tensile stress of the section is supported by reinforcement, CFRP plate and concrete. If the resistance moment of reinforcement at this section is $M_s = A_s \sigma_s \eta_1 h_0$, the resistance moment of the concrete is $M_{ct}$, the resistance moment of CFRP plate is $M_f = A_f \sigma_f \left(\eta_1 h_0 + a_s\right)$, the nonuniformity coefficient of the strain of the tensile reinforcement is shown as follows:

$$\psi_s = 1.1\frac{\sigma_s}{\sigma_{sk}} = 1.1\left(\frac{M_k - M_{ct} - M_f}{A_s \eta_1 h_0 \sigma_{sk}}\right). \tag{16}$$

By taking $\eta = \eta_1 = 0.87$ [2], after simplified, the formula becomes:

$$\psi_s = 1.1 - \frac{0.65 f_{ct}}{\sigma_{sk}\rho_{ste} + \sigma_{fk}\rho_{fte}} = 1.1 - \frac{0.65 f_{ct}}{\sigma_{sk}\left(\rho_{ste} + \rho_{fte}\frac{E_f}{E_s}\right) + E_f \varepsilon_i \rho_{fte}}. \tag{17}$$

In order to attain the stress of reinforcement at the crack section for the reinforced beam, according to the deformation harmonization relationship between reinforcement and concrete shown in Figure 4, the moment of the sum of the concrete stress in the compressive zone can be attained and shown as follows:

$$M_k = A_s \sigma_{sk} \eta h_0 + A_f \sigma_{fk}\left(\eta h_0 + a_s\right). \tag{18}$$

Formula (13) and Formula (18) can be simplified according to the deformation relationship of the structure under the initial loading and the secondary loading shown in Figure 8, the stress of reinforcement at crack section is shown as follows:

$$\sigma_{sk} = \left(\frac{M_k}{A_s \eta h_0} + E_s \varepsilon_i\right)\left(\frac{1}{1 + \frac{A_f}{A_s}\frac{E_f}{E_s}}\right). \tag{19}$$

## Calculation on the maximum crack of the reinforced beam

According to the calculation formula on the maximum crack width of the concrete beam in *Design Code for Concrete Structure* [10], the calculation formula of the maximum crack width of the CFRP plate reinforced beam can be attained as follows:

$$\omega_{\max}=1.9\psi_s \frac{\sigma_{sk}}{E_s} l_{mf} . \tag{20}$$

In the above formula, $\psi_s$ , $\sigma_{sk}$ and $l_{mf}$ are respectively calculated by Formula (17), (19) and (11), the calculated crack width and the crack width measured in test under the behaviors of different bending moment loads were compared in the Table 4.

*Tab. 4 - Comparison between calculated and test value of maximum crack width*

| Test | Name of Test Beams | $M_k$ ( kN·m ) | $\varepsilon_i$ | $\omega'_{max}$ (mm) | $\omega_{max}$ (mm) | $\omega'_{max} / \omega_{max}$ |
|---|---|---|---|---|---|---|
| Li | L1-1-1a | 57 | 0.000215 | 1.36 | 2.00 | 0.68 |
| Huang | LB-2-2 | 15 | 0.000287 | 0.08 | 0.07 | 1.15 |
| | | 20 | 0.000287 | 0.11 | 0.10 | 1.09 |
| | | 25 | 0.000287 | 0.13 | 0.12 | 1.12 |
| | | 30 | 0.000287 | 0.16 | 0.19 | 0.85 |
| | | 40 | 0.000287 | 0.22 | 0.21 | 1.03 |
| | | 50 | 0.000287 | 0.27 | 0.21 | 1.29 |
| | | 55 | 0.000287 | 0.30 | 0.23 | 1.28 |
| | | 60 | 0.000287 | 0.33 | 0.29 | 1.12 |
| | | 65 | 0.000287 | 0.35 | 0.31 | 1.14 |
| | | 70 | 0.000287 | 0.38 | 0.31 | 1.22 |
| | | 75 | 0.000287 | 0.41 | 0.33 | 1.23 |
| | | 80 | 0.000287 | 0.43 | 0.36 | 1.20 |
| | LB-3-2 | 15 | 0.000338 | 0.09 | 0.07 | 1.26 |
| | | 20 | 0.000338 | 0.12 | 0.10 | 1.19 |
| | | 25 | 0.000338 | 0.15 | 0.12 | 1.25 |
| | | 30 | 0.000338 | 0.18 | 0.19 | 0.95 |
| | | 40 | 0.000338 | 0.24 | 0.26 | 0.93 |
| | | 50 | 0.000338 | 0.30 | 0.26 | 1.17 |
| | | 55 | 0.000338 | 0.33 | 0.27 | 1.24 |
| | | 60 | 0.000338 | 0.36 | 0.29 | 1.25 |
| | | 65 | 0.000338 | 0.40 | 0.32 | 1.23 |
| | | 70 | 0.000338 | 0.43 | 0.34 | 1.25 |
| | | 75 | 0.000338 | 0.46 | 0.37 | 1.23 |
| | | 80 | 0.000338 | 0.49 | 0.39 | 1.25 |
| Bu | RCFP-2 | 117 | 0.000177 | 0.12 | 0.13 | 0.93 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | 135 | 0.000177 | 0.14 | 0.15 | 0.93 |
| | | 150 | 0.000177 | 0.16 | 0.18 | 0.87 |
| | | 165 | 0.000177 | 0.17 | 0.35 | 0.49 |
| | RCFP-3 | 117 | 0.000753 | 0.12 | 0.11 | 1.13 |
| | | 135 | 0.000753 | 0.15 | 0.12 | 1.21 |
| | | 150 | 0.000753 | 0.16 | 0.16 | 1.02 |
| | | 165 | 0.000753 | 0.18 | 0.25 | 0.72 |
| | RCFP-4 | 117 | 0.001615 | 0.13 | 0.10 | 1.25 |
| | | 135 | 0.001615 | 0.14 | 0.11 | 1.31 |
| | | 150 | 0.001615 | 0.16 | 0.14 | 1.15 |
| | | 165 | 0.001615 | 0.18 | 0.16 | 1.10 |
| This test | CFRP beam | 308 | 0.001577 | 0.03 | 0.03 | 1.00 |
| | | 344 | 0.001577 | 0.09 | 0.11 | 0.82 |
| | | 370 | 0.001577 | 0.17 | 0.19 | 0.89 |
| | | 425 | 0.001577 | 0.28 | 0.29 | 0.97 |
| | | 480 | 0.001577 | 0.39 | 0.4 | 0.98 |

By the comparison of the results, the mean ratio of the calculation value and test results was 1.080, the mean square error is 0.186. Therefore, it can be known that the crack calculation formula on the CFRP plate reinforced beam under the secondary loading proposed in the paper is correct.

## CONCLUSION

By carrying out the bending destruction tests on two pieces of test beams in the paper, it was found that CFRP plate can apparently improve the stress characteristics of the reinforced beam under the secondary loading, restrict the development of average crack spacing and crack width. Based on the crack calculation formula of concrete beam, the calculation formula on the average crack spacing and the maximum crack width of CFRP plate reinforced concrete beam under the secondary loading were established, the calculation result is well in agreement with the test results. The calculation formula is simple and could be used for the design on the CFRP plate reinforced concrete beam.

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