

# INFLUENCE ANALYSIS OF NATURAL VIBRATION CHARACTERISTICS OF STEEL BOX GIRDER WITH SINGLE CABLE PLANE AND LARGE CANTILEVERS

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## ABSTRACT

In order to study the effect of cable damage on the self-vibration characteristics of steel box girder cable-stayed bridges with a single-cable plane and large cantilevers, a finite element model of the bridge was elaborated using the finite element software Midas/Civil to find out self-vibration characteristics of the cable-stayed bridge. The self-vibration characteristics of the bridge were also analysed for different damages to the cables and for different degrees of the damages to the cables. The structural health of a bridge can be diagnosed and assessed based on self-vibration characteristics of the studied structure.

## KEYWORDS

Cable-stayed bridge, Natural vibration characteristics, Vibration mode, Steel box girder

## INTRODUCTION

Steel box girder cable-stayed bridges with a single-cable plane and large cantilevers are composed of a composite girder, of a single tower and of stay cables arranged in one plane [1,2]. In order to increase the torsional and flexural capacity of the main girder and tower, main girder and pier are connected together [3-5]. The main girder is composed of steel box, cantilevers and the concrete deck, and the driving lanes are located on cantilevers. The main bridge design adopts a harp (parallel) system of stay cables with distinctive characteristics of the times [6-8]. The analysis of dynamic characteristics is the basis for the study of its dynamic behaviour – natural vibrations as well as dynamic response characteristics. Through the analysis of the self-vibration characteristics of the studied structure, a theoretical basis is provided for the future design of such structure. [9-12]. 1st natural-vibration characteristics determine its dynamic response characteristics. Through the analysis of the natural-vibration characteristics of the studied structure, a theoretical basis is provided for a future design of steel box girder cable-stayed bridges with a single stay cable plane and large cantilevers.

## BACKGROUND

The cable stayed bridge is located in Fuyuan City, Heilongjiang Province. Its superstructure is 280 m long, with two spans of 140 m + 140 m and a width of 26.5 m. A longitudinal section of the bridge is shown in Figure 1 and a cross section is shown in Figure 2. The bridge deck is composed of a steel box girder, steel cantilevers and a concrete bridge deck. Carriageways are located on cantilevers on both sides of the structure. The steel box adopts single box and double chamber structure, and the steel cantilevers on the both sides are made as variable height I-girders. The cantilevers are made of Q370qE steel, the bridge deck is made of C50 concrete. The bridge tower

is a single column type with a height of 117 m. The part of the tower above the deck is made of C55 concrete and the part below the deck is made of C50 concrete. Stay cables are arranged in a harp shaped (parallel) system and are made of high-strength and low relaxation steel strands. Their characteristic tensile strength is 1860 MPa and the modulus of elasticity is  $1.95 \times 10^5$  MPa. The cable spacing on the main girder is 9 m, and the anchorage spacing on the tower is 5.533 m - 5.606 m from the top to the bottom. There are 52 stay cables over the whole bridge, the shortest stay cable is 20.575 m long, and the longest stay cable is 147.651m long. The tower is made of C55 concrete. The section effect of the main girder is shown in Figure 3.

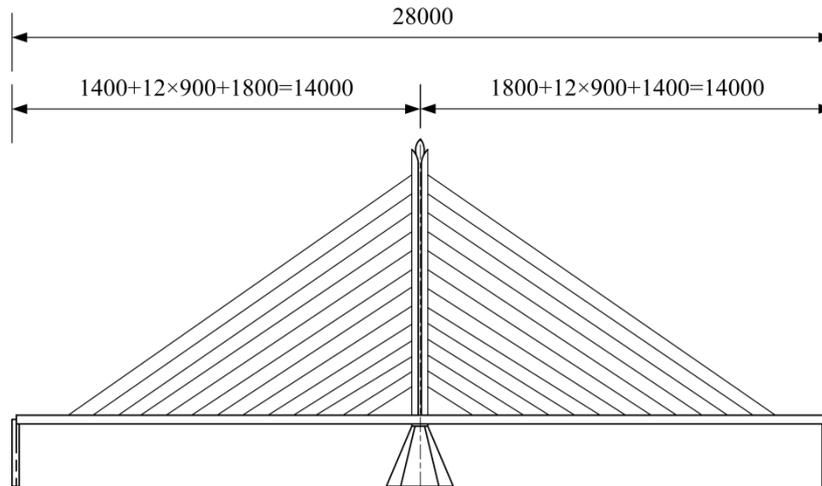


Fig. 1 – Elevation of the bridge

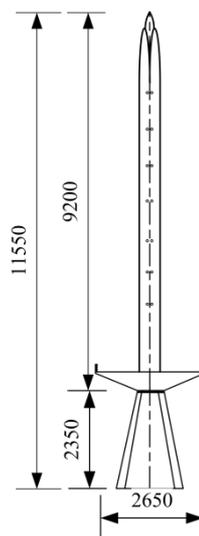


Fig. 2 – Cross section of the bridge

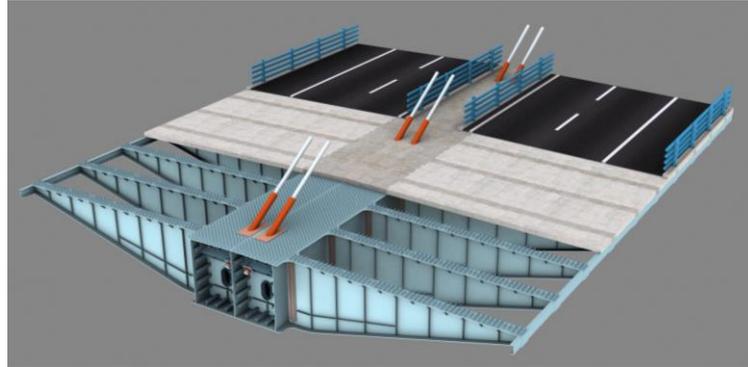


Fig. 3 – Section effect diagram of the main girder

To analyze dynamic characteristics of the bridge, a three-dimensional finite element model of the bridge was elaborated using the finite element analysis software Midas / Civil 2019. The main tower and main girder were simulated using beam units, and the tension cables were simulated using truss units. Totally, 798 beam units and 52 truss units were used. A rigid connection between the deck and the top part of the tower is simulated, stay cables are rigidly connected to the deck, as well as to the bridge tower. An overall view of the bridge finite element model is shown in Figure 4.

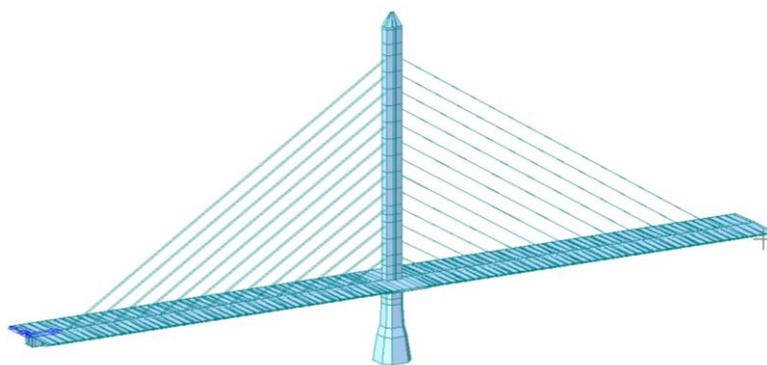
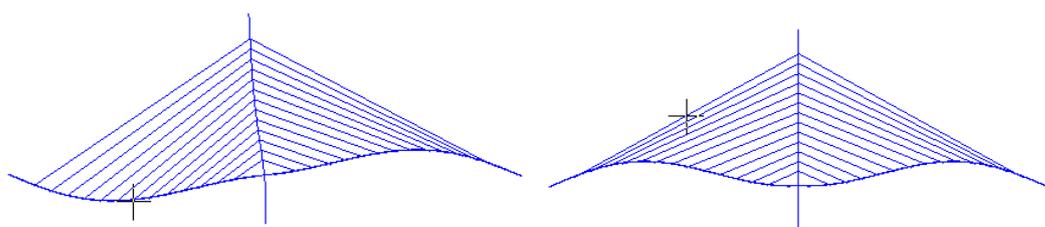


Fig. 4 – Bridge finite element model

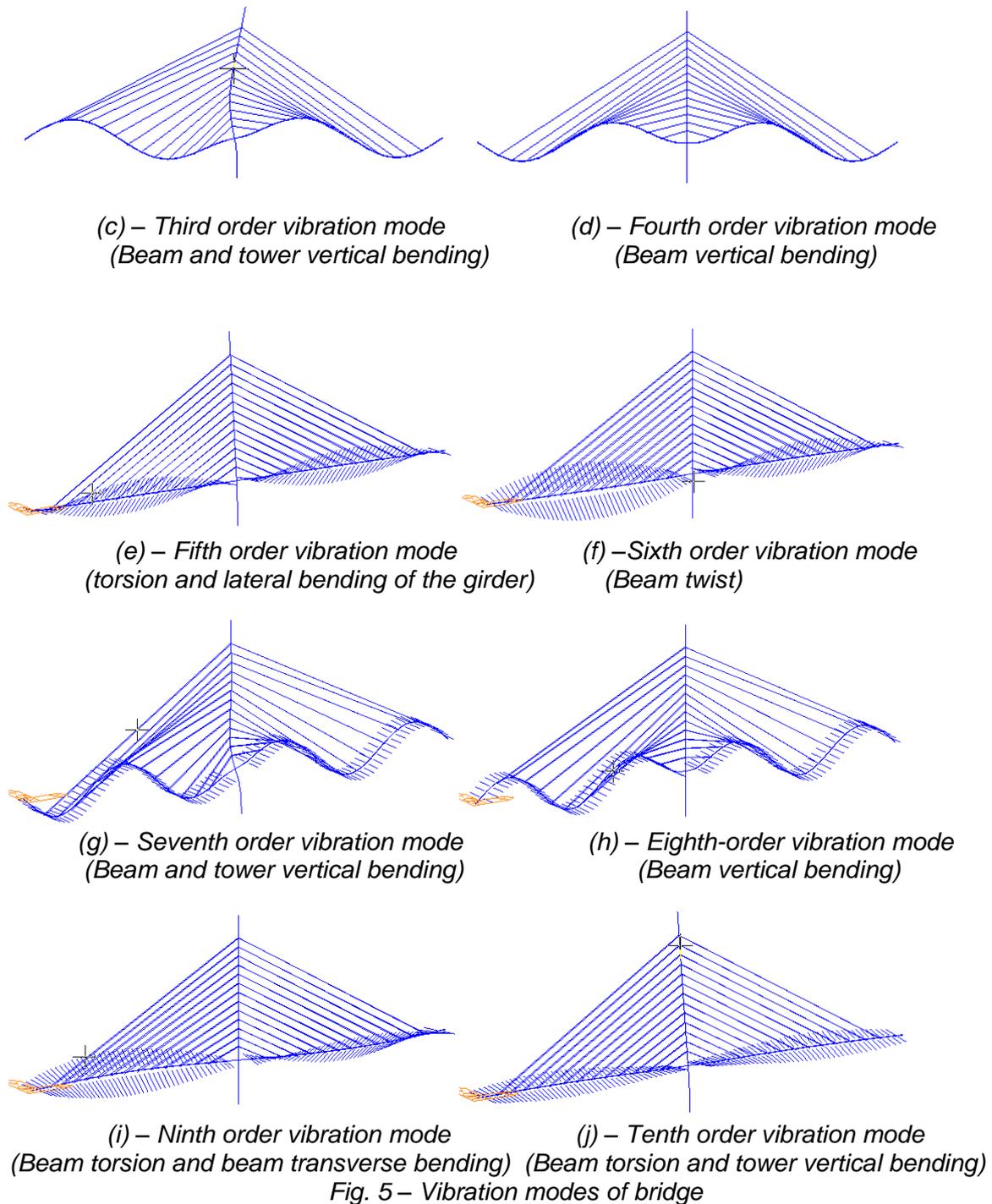
### ANALYSIS OF BRIDGE NATURAL VIBRATION CHARACTERISTICS

Modal analysis is an analysis method, which describes behaviour of the structure according to the inherent characteristics of the structure, mainly including frequency, damping and modal shape [13-15]. For this, it is necessary to determine natural frequencies and modes of the structure, because any structure has its own natural frequencies and corresponding modes of vibration, which are principal properties of the structure. The first 10 natural vibration modes of the studied structure are shown in Figure 5.



(a) – First order vibration mode  
(Beam vertical bending)

(b) – Second order vibration mode  
(Beam vertical bending)



The first 10 natural periods and modes of the studied structure are shown in Table 1.

According to the calculation results in Table 1, the basic natural vibration period of the bridge is 1.85s (i.e. the corresponding frequency is 0,54 Hz). Generally, the basic natural vibration period of ordinary cable-stayed bridges is above 2s, while that of beam bridges is about 1s. It shows that the basic natural period of the studied structure is between the value of a continuous girder (rigid frame) and an ordinary cable-stayed bridge [16-17].

From the first 10 natural vibration modes of the model, it can be seen, that the vibration modes of the studied structure are dominated by vertical bending vibrations of the deck, which corresponds to the structural characteristics of a rigid deck and flexible stay cables. The vibration

behaviour of the studied structure is more similar to that of a continuous girder (rigid structure) and other girder bridges, unlike general cable-stayed bridges where the torsion of towers and cables is the main vibration pattern [18].

Tab.1 - Period and vibration pattern at each order of mode

Order	Period/s	Original Natural Frequency/Hz	Vibration patterns
1st	1.85	0.54	Vertical bending of deck
2nd	1.12	0.89	Vertical bending of deck
3rd	0.64	1.56	Vertical bending of deck + <i>longitudinal</i> bending of tower
4th	0.57	1.75	Vertical bending of deck
5th	0.36	2.73	Torsion and transverse bending of deck
6th	0.35	2.80	Torsion of deck
7th	0.34	2.93	Vertical bending of deck + <i>longitudinal</i> bending of tower
8th	0.31	3.17	Vertical bending of deck
9th	0.31	3.22	Torsion and transverse bending of deck
10th	0.25	3.88	Torsion of deck + lateral bending of tower

## EFFECT OF DAMAGE TO ONE SIDE OF THE CABLE ON THE DYNAMIC CHARACTERISTICS

### Effect of complete damage to the cables

In order to further understand the dynamic characteristics of the structure, it is necessary to consider the analysis of the factors influencing the dynamic characteristics of the bridge. The influence of a cable damage on natural vibration frequency of the bridge is studied. For a simulation of a cable damage, its modulus of elasticity was considered by the value of  $1 \times 10^{-5}$  MPa. The frequency change of the stay cable after a damage was calculated, while the frequency variation was = (frequency value after damage - frequency value without damage)/frequency value without damage. Some valuable conclusions can be drawn by comparing and analyzing the variation of natural vibration frequencies under various conditions.

There are 13 pairs of stay cables at each side of the tower of the bridge. The cables at each side of tower are numbered from short cables to long cables, S01 to S13 respectively. With S02 and S04 representing short cords, S06 and S08 representing medium cords, and S10 and S12 representing long cords, the frequency values and frequency variation values after damage are shown in Tables 2 and 3. The relationship between the order of different cables and the frequency variation is shown in Figure 6. It can be seen from Table 3 and Figure 5: when one side of a tension cable is completely damaged, the cable damage has a greater effect on the 2nd and 4th order frequency variation of the studied structure, in which the vertical deformation of the main beam occurs both in the 2nd and 4th order. A complete damage of a tension cable has a small effect on the stiffness of the main tower, so the frequency change of the main tower *longitudinal* bending modes is not significant and can be ignored. The frequency change curves of the second and fourth order of the main beam are shown in Figure 7 and Figure 8. The second order frequency curve of the main beam is a parabola shape with an upward opening. The minimum natural frequency is 0.86 Hz. The fourth order frequency curve of the main beam is W - shaped. The minimum natural vibration frequency is 1.72 Hz. The second order vertical bending frequency of the main deck decreases first and then rises, and the fourth order vertical bending frequency of

the main deck varies irregularly, while the frequencies of other modes almost do not change. Therefore, the cable damage has a certain effect on the vertical bending mode frequency of the deck.

*Tab.2 - Frequency variations after damage of stay cables*

Order	Original Natural Frequency/Hz	Frequency Variation after Cable Damage /Hz					
		S02	S04	S06	S08	S10	S12
1st	0.54	0.54	0.54	0.54	0.54	0.54	0.54
2nd	0.89	0.88	0.88	0.87	0.87	0.87	0.88
3rd	1.56	1.55	1.55	1.55	1.55	1.55	1.55
4th	1.75	1.73	1.73	1.74	1.75	1.74	1.74
5th	2.73	2.73	2.73	2.73	2.73	2.73	2.73
6th	2.80	2.80	2.80	2.80	2.80	2.80	2.80
7th	2.93	2.93	2.93	2.92	2.92	2.92	2.93
8th	3.17	3.17	3.17	3.17	3.17	3.17	3.17
9th	3.22	3.22	3.22	3.22	3.22	3.22	3.22
10th	3.88	3.88	3.88	3.88	3.88	3.88	3.88

*Tab.3 - Proportional change of frequency after damage of stay cables*

Order	Proportional change of frequency/%					
	S02	S04	S06	S08	S10	S12
1st	-0.09	-0.4	-0.67	-0.49	-0.03	-0.53
2nd	-0.21	-1.09	-2.2	-2.64	-1.99	-0.72
3rd	-0.35	-0.55	-0.1	-0.07	-0.35	-0.12
4th	-0.62	-1.19	-0.56	0	-0.32	-0.36
5th	0	0	0	0	0	0
6th	0	0	0	0	0	0
7th	-0.16	0	-0.34	-0.32	0	-0.06
8th	-0.46	-1.09	-0.05	-0.16	0	-0.12
9th	0	0	0	0	0	0
10th	0	0	0	0	0	0

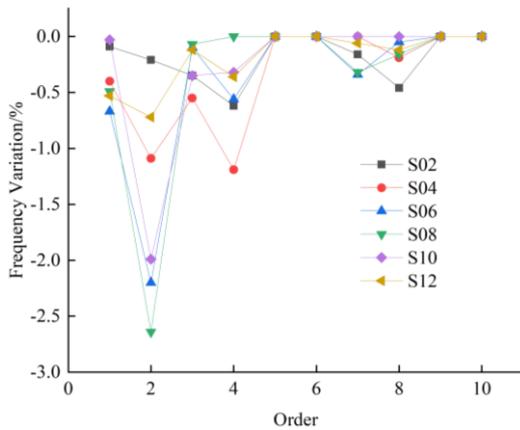


Fig.6 - Relationship between the order variation

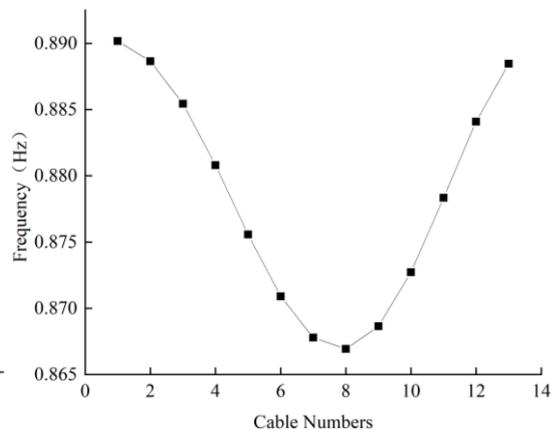


Fig.7 - Natural frequency of deck and frequency with a damaged stay cable – 2nd order

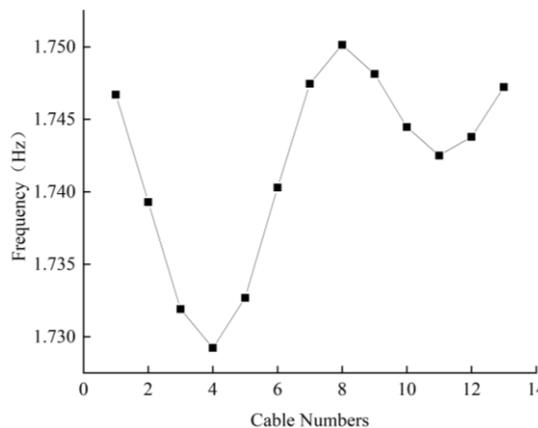


Fig.8 - Natural frequency of deck with a damaged stay cable – 4th order

### Influence of different elastic modulus of cable

In order to investigate the effect of different degrees of damage to stay cables at one side of the tower on natural frequencies of the bridge, three different lengths of stay cables are analyzed. Three S03, S04 and S05 are taken as representatives of short cables, three S07, S08 and S09 are taken as representatives of medium and long cables, and three S11, S12 and S13 are taken as representatives of long cables. In order to further study the influence of the elastic modulus of the cable on natural vibration frequencies of the cable-stayed bridge, calculations were made with an elastic modulus ratio of 0.25-1 for each order, so the elastic modulus values were  $1.95 \times 10^5$ ,  $1.4625 \times 10^5$ ,  $0.975 \times 10^5$ , and  $1 \times 10^5$  MPa, respectively.

Figure 9, Figure 10 and Figure 11 show the relationship between frequency and order for a damage to a short cable, middle cable and long cable. See Figure 12, Figure 13 and Figure 14 for frequency variation curves of S04, S08 and S12 cables. Figures 9 and 12 show that the frequency proportional change of the main beam increases with the increase of a cable damage. When the damage of cable S04 is from 25% to 100%, the change of a vertical bending frequency of the beam increases by 0.55%, which is mainly reflected in the third order. The frequency variation of vertical bending increases by 1.19%, which is mainly reflected in the first, second, fourth and eighth orders. The second and fourth order frequency changes are more significant than the first, third and eighth order frequency changes. Therefore, a cable damage has a great influence on the frequency of the vertical bending mode of the main beam.

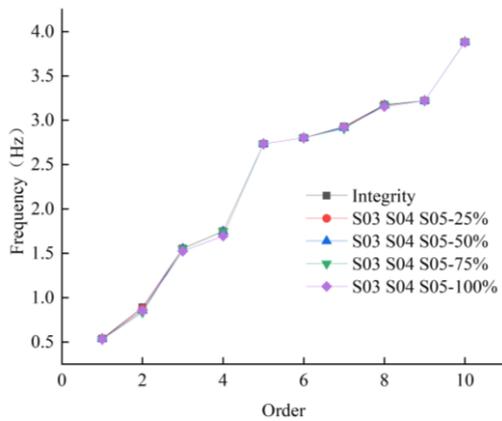


Fig.9 - Relationship between frequency and order for damaged cables S03, S04, S05

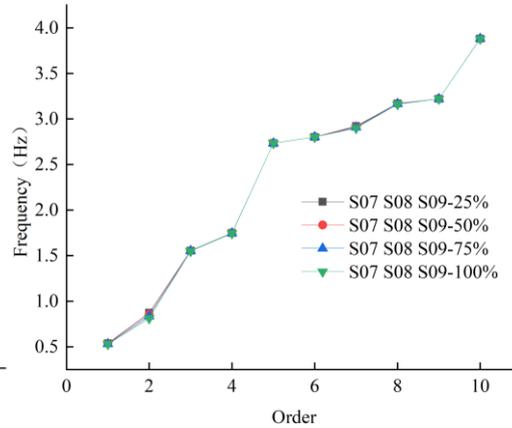


Fig.10 - Relationship between frequency and order for damaged cables S07, S08, S09

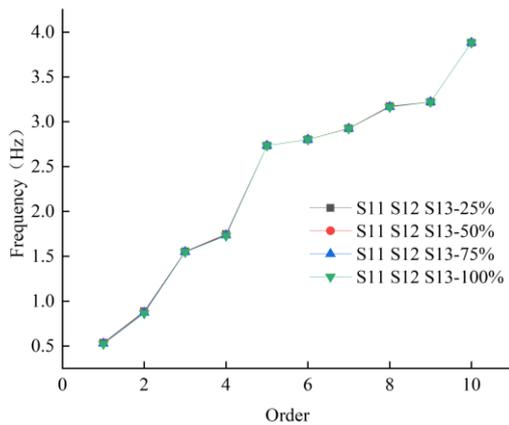


Fig.11 - Relationship between frequency and order for damaged cables S11, S12, S13

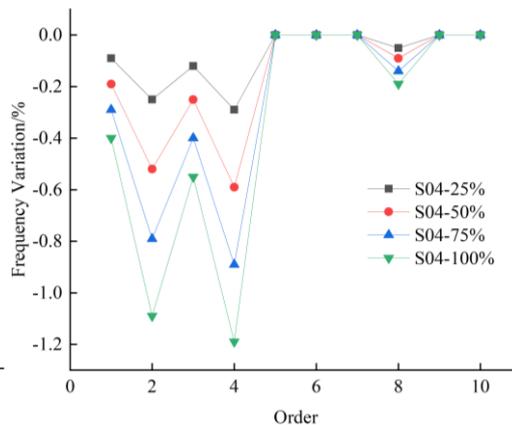


Fig.12 - Curve of frequency change for damaged cable S04

According to Figure 10 and 13, when a damage of S08 cable is from 25% to 100%, the variation of the deck vertical bending frequency increases by 2.64%. The effect is concentrated in the main girder 2nd order vertical bending vibration type. The increase of a cable damage has little influence on the stiffness of the tower, so frequencies of the vertical bending modes of the tower do not change much and can be ignored.

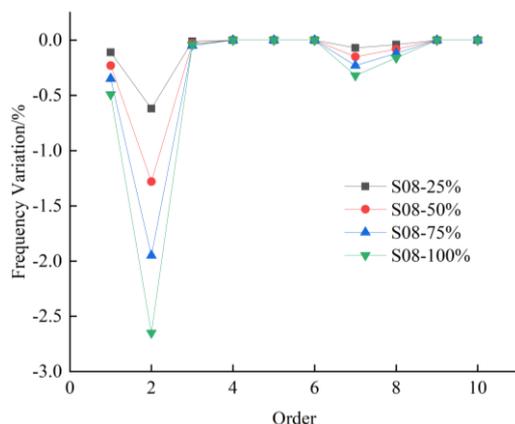


Fig.13 - Curve of frequency change for damaged cable S08

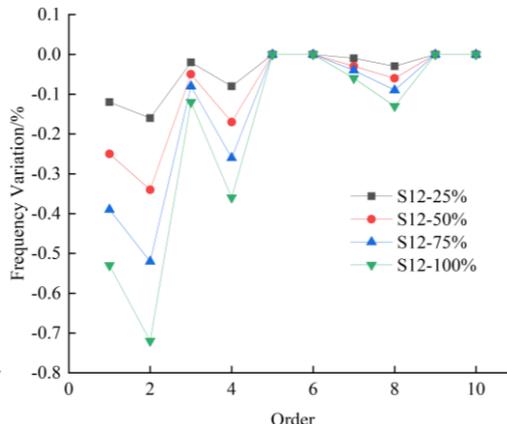


Fig.14 - Curve of frequency change for damaged cable S12

From Figure 11 and Figure 14 can be seen: when a damage of S12 cable is from 25% to 100%, the damage is concentrated in the main beam 1, 2, 4 and 8 stages of vertical bending vibration, for beam transverse bending vibration and tower transverse bending vibration almost no effect. With the increase of cable damage ratio, the first order proportional change of frequency of cable S12 increased by 0.53%, the second order proportional change of frequency increased by 0.72%, the fourth order proportional change of frequency increased by 0.36%, and the eighth order proportional change of frequency increased by 0.12%. Therefore, the cable damage has a certain effect on the frequency of the vertical bending mode of deck.

## ANALYSIS OF THE EFFECT OF DAMAGE TO THE TENSION CABLES ON BOTH SIDES OF THE TOWER ON NATURAL -VIBRATION CHARACTERISTICS OF THE CABLE-STAYED BRIDGE

### Influence of complete failure of a cable

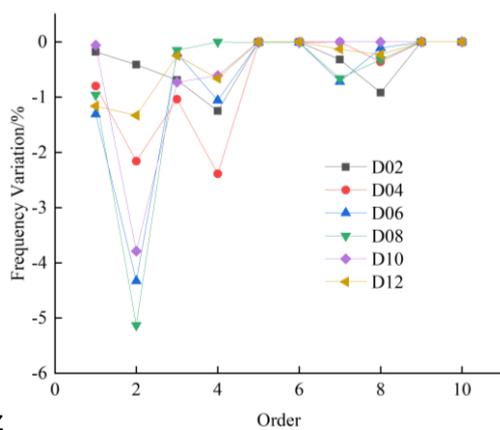
There are 26 stay cables on both sides of the tower. The cables on both sides of the tower are numbered from short cables to long cables, D01 ~ D13. D03, D04, D05 for short cables, D07, D08, D09 for medium cables, D11, D12, D13 for long cables. For damaged stay cables, modulus of elasticity considered is  $1 \times 10^5$  MPa. The frequency values and frequency variation values after the damage are shown in Table 4 and Table 5. The relationship between frequency proportional change and order for a damaged cable is shown in Figure 15. It can be seen from Table 5 and Figure 15 that when the cables on the both sides are completely damaged, the cable damage has a great influence on the frequency amplitude of the order 1, 2 and 4 of the studied structure, and the vertical deformation of the main beam occurs at the order 1, 2 and 4. The first, second and fourth order frequency variation curves of the main beam are shown in Figure 16, 17 and 18. The first order frequency curve of the main beam changes irregularly. The minimum frequency of natural vibration occurs under a damage of the long cable D13, which is 0.51 Hz. The second order frequency curve of the main beam is a parabola shape with an upward opening. The minimum value of natural vibration frequency appears in the middle cable D08, which is 0.84Hz. The fourth order frequency curve of the main beam is W - shaped. The minimum natural frequency appears on the short cable D04, which is 1.70 Hz. The change rate of cable D08 is 5.13%. The second order frequency of the main beam decreases first and then rises, and the vertical bending mode frequency of the main beam is significantly affected by the cable damage, so the cable damage has a certain influence on the vertical bending mode frequency of the deck.

Tab.4 - Frequency under cable damage

Order	Original Natural Frequency/Hz	Frequency Variation after Cable Damage /Hz					
		D02	D04	D06	D08	D10	D12
1st	0.54	0.54	0.53	0.53	0.53	0.54	0.53
2nd	0.89	0.89	0.87	0.85	0.84	0.86	0.88
3rd	1.56	1.54	1.54	1.55	1.55	1.54	1.55
4th	1.75	1.73	1.71	1.73	1.75	1.74	1.74
5th	2.73	2.73	2.73	2.73	2.73	2.73	2.73
6th	2.80	2.80	2.80	2.80	2.80	2.80	2.80
7th	2.93	2.92	2.93	2.91	2.91	2.93	2.92
8th	3.17	3.15	3.16	3.17	3.16	3.17	3.17
9th	3.22	3.22	3.22	3.22	3.22	3.22	3.22
10th	3.88	3.88	3.88	3.88	3.88	3.88	3.88

Tab.5 - Frequency variation

Order	Frequency variation /%					
	D02	D04	D06	D08	D10	D12
1st	-0.18	-0.8	-1.31	-0.96	-0.06	-1.16
2nd	-0.41	-2.16	-4.33	-5.13	-3.79	-1.33
3rd	-0.69	-1.04	-0.21	-0.15	-0.73	-0.25
4th	-1.25	-2.39	-1.06	0	-0.61	-0.66
5th	0	-0.01	-0.01	-0.01	0	0
6th	0	-0.01	-0.01	-0.01	0	0
7th	-0.32	0	-0.72	-0.66	0	-0.13
8th	-0.92	-0.36	-0.11	-0.32	0	-0.23
9th	0	0	0	0	0	0
10th	0	0	0	0	0	0



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Fig.15 - The relation between frequency variation and order

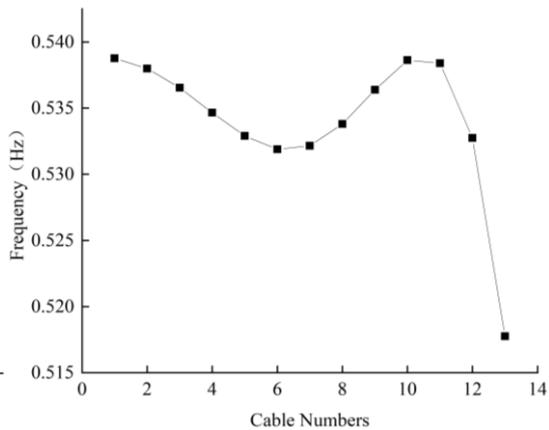


Fig.16 - First order frequency curve of main deck for damaged cable

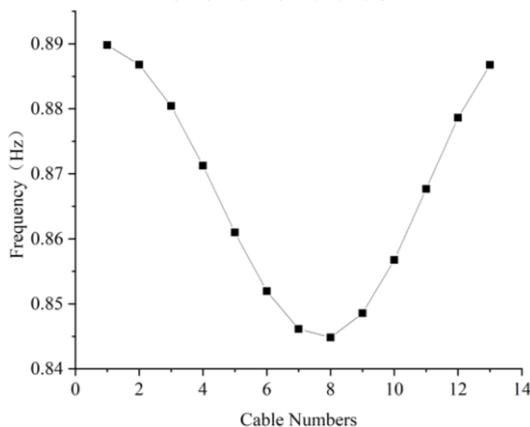


Fig.17 - Second order frequency curve of main deck for damaged cable

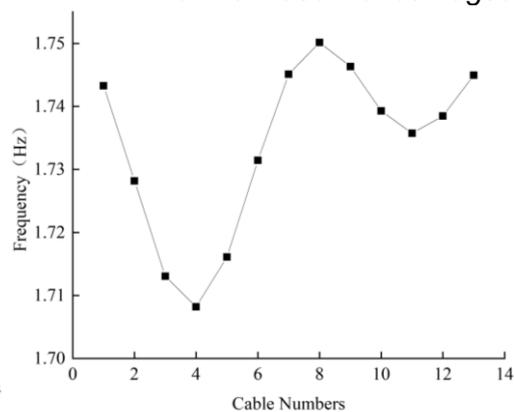


Fig.18 - Fourth order frequency curve of main deck for damaged cable

### Influence of different elastic modulus of a cable

In order to explore the influence of different damage degrees of stay cables on both sides of the tower on the natural vibration frequency of the bridge, the damage of three different lengths of

stay cables are analyzed respectively. Three D03, D04 and D05 are taken as representatives of short cables, three D07, D08 and D09 are taken as representatives of medium and long cables, and three D11, D12 and D13 are taken as representatives of long cables. The elastic modulus of the cables was  $1.95 \times 10^5$ ,  $1.4625 \times 10^5$ ,  $0.975 \times 10^5$  and  $1 \times 10^5$  MPa, respectively.

Figure 19, Figure 20 and Figure 21 show the frequency values for damaged short cables, middle cables and long cables under different damage degrees. The proportional frequency change curves for damaged cables D04, D08 and D12 cables are shown in Figure 22, 23 and 24. It can be seen from Figure 19 and Figure 22 that the damage of short cables is mainly reflected in the beam vertical bending mode (the 1st, 2nd, 4th and 8th modes) and beam tower *longitudinal* bending mode (the 3rd mode), in which the 2nd and 4th frequencies are more obvious than the 1st, 3rd and 8th frequencies.

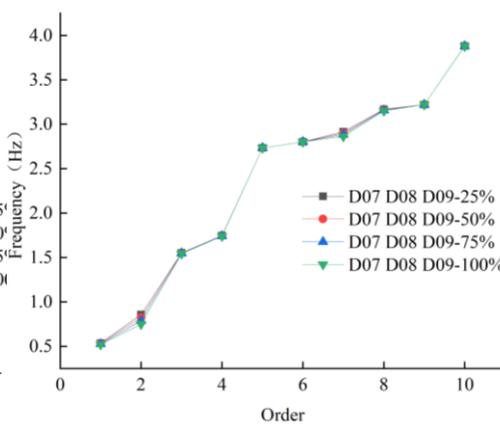
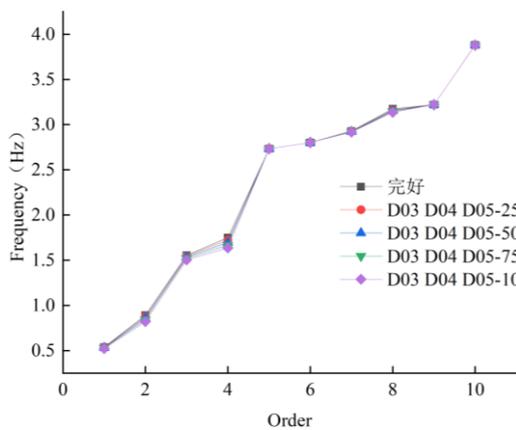


Fig.19 - Relationship between frequency and order for damaged cables D03, D04, D05

Fig.20 - Relationship between frequency and order for damaged cables D07, D08, D09

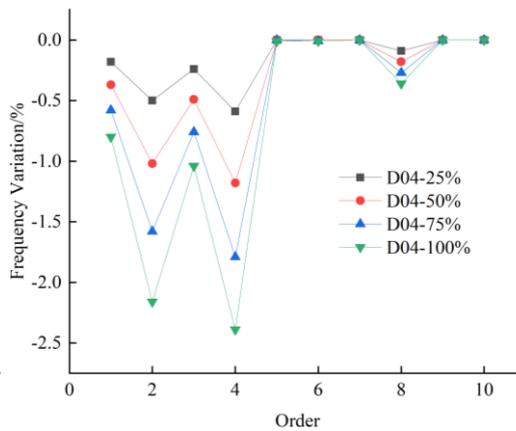
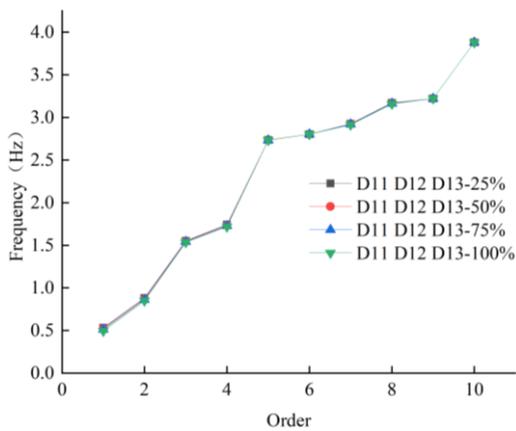


Fig. 21 - Relationship between frequency and order for damaged cables D03, D04, D05

Fig. 22 - Curve of frequency proportional change of for damaged cable D04

From Figure 20 and Figure 23, it can be seen that: the frequency values of different degrees of damage in the cable show regular changes, and the damage is concentrated in the beam vertical bending vibration (1st and 2nd order vibration) and beam vertical bending and tower vertical bending vibration (7th order vibration), where the 2nd order frequency variation is more significant than the 1st and 7th order frequency variation, and with the increase of the degree of damage, the frequency variation is more and more significant. There is almost no effect on the beam and tower transverse bending vibration.

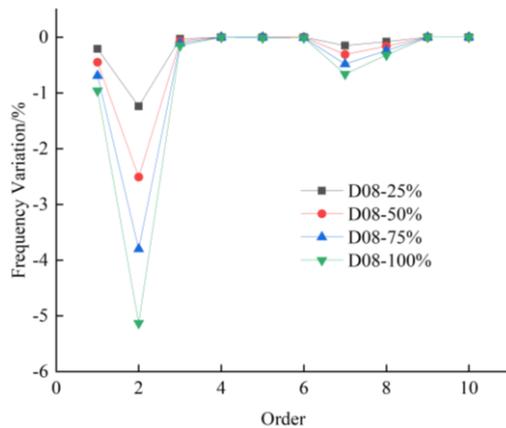


Fig.23 - Curve of frequency proportional change for damaged cable D08

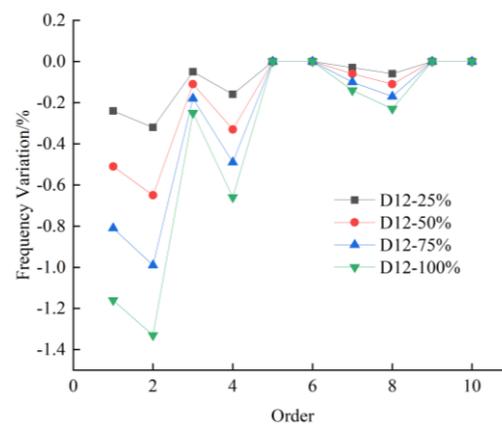


Fig.24 - Curve of frequency proportional change for damaged cable D12

From Figure 21 and Figure 24, it can be seen that the frequency values with different degrees of damage of long cables show regular changes, and the damage is concentrated in the beam vertical bending vibration (1st, 2nd, 4th and 8th order vibration) and the beam vertical bending and tower vertical bending vibration (3rd and 7th order vibration), where the 1st and 2nd order frequency variation is more obvious than the 3rd, 4th, 7th and 8th order frequency variation, and the frequency variation is increasing with the increase of the degree of damage. The proportional changes of the 1st and 2nd order frequencies are more obvious than the 3rd, 4th, 7th and 8th order frequencies.

## CONCLUSION

This paper analyses natural vibration characteristics of a steel box girder cable-stayed bridge with a single-cable plane and large cantilevers, and studies the influence of various stay cable damages on natural vibration characteristics of the bridge, including the effects of different degree of such damages at one side as well as on the both sides of the tower, which can diagnose and evaluate the health condition of such bridges, which was shown by the results of this research.

- (1) The bridge has no main girder torsional vibration type in the first 4- orders, because the main girder is a steel box combined with large cantilevers, the steel box girder size is relatively small, and the stay cables are arranged in the central cable plane, which causes, that the main girder of the studied structure has large torsional stiffness and good resistance to torsional deformation.
- (2) The tower side and both sides of the damage cable with the increase in number, the main beam 2nd order vertical bending and main beam 4th order vertical bending frequency variation effect is larger. Main beam 2nd order vertical bending self-oscillation frequency is open upward parabolic change, S08 and D08 cable damage, main beam 2nd order vertical bending self-oscillation frequency is the smallest; Main beam 4th order vertical bending self-oscillation frequency is irregular change, S04 and D08 order main beam vertical bending frequency is the smallest. The second order vertical bending frequency of the main deck decreases first and then rises, and the fourth order vertical bending frequency of the main deck varies irregularly, while the frequencies of other modes almost do not change. Therefore, the cable damage has a certain effect on the vertical bending mode frequency of the deck.
- (3) For a short cable at one side as well as at both sides of the tower with different degrees of damage, the damage is concentrated in 1st, 2nd, 4th and 8th order, where the vertical bending modes of the main deck are generated. For beam transverse bending and torsional vibrations almost no effect. With the increase of cable damage ratio, the first order proportional change of frequency of cable S12 increased by 0.53%, the second order proportional change of frequency

increased by 0.72%, the fourth order proportional change of frequency increased by 0.36%, and the eighth order proportional change of frequency increased by 0.12%. Therefore, the cable damage has a certain effect on the frequency of the vertical bending mode of deck.

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