STUDY ON OPTIMAL COMBINATION SETTLEMENT PREDICTION MODEL BASED ON LOGISTIC CURVE AND GOMPERTZ CURVE

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ABSTRACT

The Logistic and Gompertz embankment settlement prediction models have poor prediction accuracy for the late settlement of high-filled soil. This study proposes a combination of the two models based on their common characteristics and individuality, and their respective advantages and specific limitations. The minimum logarithmic error square sum of the combined model was used as the objective function to solve the optimal weighting coefficient. The optimal weighted geometric mean combination prediction model was deduced, to improve the confidence of the prediction accuracy of the settlement of high-filled soil. By fitting and analysing the measured settlement data of the engineered high-filled soil with each prediction model, the feasibility of the proposed optimal combination prediction model in the settlement prediction of high-filled soil was tested. It was found that the proposed optimal combination forecasting model was more accurate and adaptable compared to any single model, and was more reliable. Therefore, the proposed combination forecasting model could be used as an effective method to predict the settlement of high-filled soil in the later stages of settlement.

KEYWORDS

High-filled soil, Optimal combination model, Settlement prediction

INTRODUCTION

The central and western regions of China are occupied with cities in mountainous terrain. The continuous development of these urbanized areas has resulted in large cities expanding into surrounding areas, resulting in new urban districts, airports and railway and (highway) roads being built on land formed using high-filled soil. Uneven settlement and deformation of high-filled soil in hilly and gully terrain is of concern to geotechnical experts. Many structures built on high-filled soil have been destroyed or condemned due to the settlement of the foundations. The settlement of high-filled soil results in the deformation of the filling soil and the settlement of the foundation. Reported data indicate compression of the soil caused by the weight of the fill often accounts for 80%-90% of the total settlement [1-2]. After completion of the construction of the fill soil, uneven settlement often occurs over a long period to varying degrees. The settlement is primarily influenced by the filling speed, filling height, filling properties and local climatic conditions. This multi-dimensional problem based on time and space cannot be solved using empirical formulas [3]. The final settlement calculated using the finite element method based on the consolidation theory is often significantly different to the measured value. Therefore, it is necessary to understand the dynamic process of settlement of high-filled soil to accurately predict trends in settlement using data during the early period, to ensure safety and quality [4]. To date, there are three main methods used globally for predicting settlement of high-filled soil: the empirical formula deduction method, numerical simulation and the curve fitting method. The most commonly used curve fitting

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methods include hyperbolic, modified hyperbolic, grey prediction, grey Theory-Markov, power function, exponential equations and Asaoka methods [5-14]. In recent years, the Logistic and Gompertz curves have been favoured as they reflect more accurately the relationship between settlement and time [15-18], however, the prediction accuracy is often limited by the amount of monitoring data, geographical conditions and construction technology. It is difficult to use a single prediction model to make an accurate prediction of settlement, resulting in low confidence of prediction precision.

The common characteristics and individuality of Logistic and Gompertz prediction models, together with their advantages and specific limitations are used in this study which combines the two models to calculate the optimal weighting coefficients. The sum of the minimum logarithmic error squares of the combined model was used as the objective function which derived the optimal weighted geometric mean of the combination prediction model. The prediction model was used to improve the prediction accuracy of the curve fitting method. The combined prediction model could be used as an effective method for predicting the late settlement of high-filled soil.

THE TWO SETTLEMENT PREDICTION MODELS

The Logistic model

In 1938, Pierre-Francois Verhulst proposed the Logistic model for predicting world population growth (Verhulst-Pearl model). Due to the development of inter-discipline fields, this model is widely used in ecology, medicine, economics and management. The model can be used for settlement prediction and its differential form can be rewritten as：

$$\frac{dy}{dt} = ky(1 - \frac{y}{A}) \tag{1}$$

where: y =settlement; t =time; k =immediate settlement rate; A =final settlement.

The integral of equation (1) is:

$$\int (\frac{1}{y} + \frac{1}{A-y})dy = \int k dt \tag{2}$$

Solution:

$$y = \frac{A}{1 + Be^{-kt}} \tag{3}$$

where: B =parameter to be sought; e =Euler number.

The Gompertz model

The Gompertz model was put forward by the British statistician Benjamin Gompertz. It was originally used to describe the law of the growth and change of nature. Since its birth, the Gompertz model has played an important role not only in the field of biology, but also in market economy forecasting and traffic forecasting. The model can be used for settlement prediction and its differential form can be rewritten as:

$$\frac{dy}{dt} = -k \ln \frac{y}{A} \tag{4}$$

where: y =settlement; t =time; k =immediate settlement rate; A =final settlement.

The integral of equation (4) is:

$$\int \frac{d(\ln y - \ln A)}{\ln y - \ln A} = -\int k \, dt \tag{5}$$

Solution:

$$y = Ae^{-Be^{-kt}} \tag{6}$$

where: B =parameter to be sought; e =Euler number.

Then, the first derivative of equations (3) and (6) can be obtained:

$$\left. \begin{array}{l} \dfrac{dy}{dt} = \dfrac{ABke^{-kt}}{(1+Be^{-kt})^2} > 0 \\[4mm] \dfrac{dy}{dt} = ABke^{-Be^{-kt}} \cdot e^{-kt} > 0 \end{array} \right\} \tag{7}$$

The second derivative of equations (3) and (6) are:

$$\left. \begin{array}{l} \dfrac{d^2 y}{dt^2} = \dfrac{ABke^{-kt}(Be^{-kt}-1)}{(1+Be^{-kt})^3} \\[4mm] \dfrac{d^2 y}{dt^2} = ABk^2 e^{-Be^{-kt}} \cdot e^{-kt} \cdot (Be^{-kt}-1) \end{array} \right\} \tag{8}$$

The limits of equations (3) and (6) are:

$$\left. \begin{array}{l} \lim\limits_{t\to\infty} \dfrac{A}{1+Be^{-kt}} = A \\[4mm] \lim\limits_{t\to\infty} Ae^{-Be^{-kt}} = A \end{array} \right\} \tag{9}$$

Using the definition of consolidation degree U, equations (3) and (6) can be written:

$$\left. \begin{array}{l} U = \dfrac{y_t - y_0}{y_\infty - y_0} = \dfrac{(A/1+Be^{-kt}) - (A/1+B)}{A - A/1+B} \\[4mm] U = \dfrac{y_t - y_0}{y_\infty - y_0} = \dfrac{Ae^{-Be^{-kt}} - Ae^{-B}}{A - Ae^{-B}} \end{array} \right\} \tag{10}$$

Equation (7) demonstrates that the Logistic and Gompertz curves increase monotonously, Equation (8) demonstrates that when $Be^{-kt}=1$ (that is $t = \ln B/k$) there is an inflection point in the curve, and Equation (9) demonstrates that when time is infinite, the maxima in both curves equals A. According to the properties of the two curves, the shape of the Logistic curve is an "S" shape, and the Gompertz curve has the characteristics of an "anti-S" shape distribution. This is in agreement with the proposal by Mei et al. that it is theoretically demonstrated that the settlement-time curve is an "S" shape when the load is linear or approximately linear [19]. Equation (10) demonstrates that when time equals zero, U=0, and when time approaches infinity, U= 1. Therefore, the Logistic and Gompertz models can be used to predict the variation in settlement of high-filled soil over time.

SOLUTION OF THE MODEL

Using the Logistic and Gompertz models, settlement can be predicted for any time when the values of the parameters A, B and k are known. In order to improve the fit, nonlinear Gompertz and Logistic curves were converted into modified exponential curves using the basic principles of linear regression. The transformation of the modified exponential equations is shown in Table 1.

Tab. 1 - Transformation of the modified exponential equation

Model type	Model equation	Transformation process	Transformation model
Logistic	$y = \dfrac{A}{1 + Be^{-kt}}$	if $Y = 1/y$, $A' = 1/A$, $B' = B/A$, $k' = e^{-k}$	$Y = A' + B'k'^t$
Gompertz	$y = Ae^{-Be^{-kt}}$	$\ln y = \ln A - Be^{-kt}$, if $Y = \ln y$, $A'' = \ln A$, $B'' = -B$, $k' = e^{-k}$	$Y = A'' + B''k'^t$

This study proposes the use of the Bryant method to solve the parameters of the curve. The three parameters in the model require that the measured y~t in the settlement data be of equal time interval; the measured data were generally non-isometric. Therefore, before using the two curves to fit the data, measured y~t data must be converted into an isochronous y~ sequence that meets the requirements. In this study, the Lagranre interpolation method was used to perform the isochronous transformation of non-isochronous settlement time series data. The equations of each parameter were calculated using the Bryant method as follows:

$$k = \frac{(n-1)\sum\limits_{t=1}^{n-1} Y_t Y_{t+1} - \sum\limits_{t=1}^{n-1} Y_t \sum\limits_{t=1}^{n-1} Y_{t+1}}{(n-1)\sum\limits_{t=1}^{n-1} Y_t^2 - (\sum\limits_{t=1}^{n-1} Y_t)^2} \tag{11}$$

$$B = \frac{n\sum\limits_{t=1}^{n} k^t Y_t - \sum\limits_{t=1}^{n-1} k^t \sum\limits_{t=1}^{n} Y_t}{n\sum\limits_{t=1}^{n-1} k^{2t} - (\sum\limits_{t=1}^{n-1} k^t)^2} \tag{12}$$

$$A = \frac{(\sum\limits_{t=1}^{n} Y_t - B\sum\limits_{t=1}^{n} k^t)}{n} \tag{13}$$

OPTIMAL WEIGHTED AVERAGE GEOMETRIC PREDICTION MODEL

If there were m kinds of settlement prediction models for a problem, $\hat{y}_{1t}$, $\hat{y}_{2t}, \cdots \hat{y}_{mt}$, $t = 1,2,\ldots N$, then at the same time, $\hat{y}_{it}$ represents the i type of the prediction model and the predicted value of the t phase, $\hat{y}_t$ represents the weighted geometric average combination prediction model of the above m types of prediction models, and also represents the prediction value of the t phase of the model.

$$\hat{y}_t = \prod_{i=1}^{m} \hat{y}_{it}^{w_i} \tag{14}$$

$$\text{if } W = (w_1, w_1, \ldots, w_m)' \in R^m, \text{ and } \sum_{i=1}^{m} w_i = 1, w_i \geq 0, i = 1,2,\ldots,m \tag{15}$$

If y_t is used to represent the measured values of the actual settlement in the t period, $s_{it} = \ln \hat{y}_{it} - \ln y_t$ and $S_t = \ln \hat{y}_t - \ln y_t$ are used to denote the logarithmic errors of $\hat{y}_{it}$ and $\hat{y}_t$ respectively, then:

$$S_t^2 = (\ln \hat{y}_t - \ln y_t)^2 = (\ln \prod_{i=1}^{m} \hat{y}_{it}^{w_i} - \ln y_t)^2 =$$

$$(\sum_{i=1}^{m} w_i \ln \hat{y}_t - \ln y_t)^2 = \left[\sum_{i=1}^{m} w_i (\ln y_{it} - \ln y_t) \right]^2 =$$

$$(w_1, w_2, \ldots w_m) \begin{vmatrix} \ln \hat{y}_{1t} - \ln y_t \\ \ln \hat{y}_{2t} - \ln y_t \\ \vdots \\ \ln \hat{y}_{mt} - \ln y_t \end{vmatrix} (\ln \ln \hat{y}_{1t} - \ln y_t,$$

$$\ln \hat{y}_{2t} - \ln y_t, \ldots, \ln \hat{y}_{mt} - \ln y_t) \begin{vmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{vmatrix} = W'(s_{it} s_{jt})W \tag{16}$$

where: $W = (w_1, w_2, \cdots, w_m)'; (s_{it} s_{jt})$ is an m order square matrix.

Therefore, the sum of the square of logarithmic errors is:

$$S^2 = \sum_{t=1}^{N} S_t^2 = \sum_{t=1}^{N} W'(s_{it} s_{jt})W = W' \left[\sum_{t=1}^{N} (s_{it} s_{jt}) \right] W \tag{17}$$

where matrix $A = \sum_{t=1}^{N} (s_{it} s_{jt})$ is generally a positive definite matrix. Due to

(1) $\left[\sum_{t=1}^{N} (s_{it} s_{jt}) \right]' = \sum_{t=1}^{N} (s_{it} s_{jt})' = \sum_{t=1}^{N} (s_{it} s_{jt})$, matrix A is a symmetric matrix;

(2) For $X = (x_1, x_2, \cdots, x_m)' \in R^m - \{0\}$, there are:

$$X' \left[\sum_{t=1}^{N} (s_{it} s_{jt}) \right] X = \sum_{t=1}^{N} X'(s_{it} s_{jt}) X = \sum_{t=1}^{N} (\sum_{k=1}^{m} x_k s_{kt}) \geq 0 \tag{18}$$

If $X'AX$ is the constant zero, $\sum\limits_{k=1}^{m} x_k s_{kt} = 0, t = 1,2,\cdots,N$. That is, for any nonzero m dimensional vector X , it is the solution of the equations in (19).

$$\left.\begin{aligned}
s_{11}x_1 + s_{21}x_2 + \cdots + s_{m1}x_m = 0, \\
s_{12}x_1 + s_{22}x_2 + \cdots + s_{m2}x_m = 0, \\
\vdots \\
s_{1N}x_1 + s_{2N}x_2 + \cdots + s_{mN}x_m = 0,
\end{aligned}\right\} \tag{19}$$

If $x_k = 1, x_i = 0, i \neq k, i$ 、 $k = 1,2,\cdots,m$, there is $s_{kt} = 0, k = 1,2,\cdots,m, t = 1,2,\cdots,N,$

$s_{kt} = \ln \hat{y}_{kt} - \ln y$ is sometimes not zero, therefore:

$$X'AX \geq 0 \tag{20}$$

From equations (1) and (2) the matrix A is a positive definite matrix, so that A is invertible and for any nonzero vector W , $S^2 > 0$. $Q = (1,1,\cdots,1)' \in R^m$, so the optimal weight coefficient W can be determined by minimizing the sum of the logarithmic error square $S^2 = W'AW$ of the combined prediction model under the constraint conditions of $W'Q = 1$, $W \geq 0$.

Using the result from [20], the solution is:

$$W = \left\{\left[\sum_{t=1}^{N}(s_{it}s_{jt})\right]^{-1}Q\right\} \Big/ \left\{Q^{-1}\left[\sum_{t=1}^{N}(s_{it}s_{jt})\right]^{-1}Q\right\} \tag{21}$$

For the weighted geometric combination prediction model composed of Logistic and Gompertz models, the optimal weight coefficients are:

$$\left.\begin{aligned}
w_1 = (\sum_{t=1}^{N}s_{2t}^2 - \sum_{t=1}^{N}s_{1t}s_{2t}) \Big/ \left[\sum_{t=1}^{N}(s_{1t}^2 + s_{2t}^2 - 2s_{1t}s_{2t})\right] \\
w_2 = \sum_{t=1}^{N}(s_{1t}^2 - s_{1t}s_{2t}) \Big/ \left[\sum_{t=1}^{N}(s_{1t}^2 + s_{2t}^2 - 2s_{1t}s_{2t})\right]
\end{aligned}\right\} \tag{22}$$

Due to $w_1 + w_2 = 1$, only one of the w_1, w_2 was solved.

ACCURACY OF THE COMBINED PREDICTION MODEL

According to the square sum of absolute errors (SSE), square sum of relative errors (SSRE), standard error (SE), relative standard error (RSE) and mean absolute percentage error (MAPE), the accuracy of the optimal combination prediction model proposed in this study was tested using five parameters. The formulas of each parameter are as follows:

$$
\left.\begin{array}{l}
SSE = \sum_{t=1}^{N} (\hat{y}_t - y_t)^2, SSPE = \sum_{t=1}^{N} (\frac{\hat{y}_t - y_t}{y_t})^2 \\[4mm]
SE = (\frac{SSE}{N})^{0.5}, SPE = (\frac{SSPE}{N})^{0.5} \\[4mm]
MAPE = \frac{1}{n} \sum_{t=1}^{N} (\frac{\left| \hat{y}_t - y_t \right|}{y_t}) \times 100\%
\end{array}\right\} \qquad (23)
$$

According to the value MAPE, the prediction accuracy can be divided, as shown in Table 2.

Tab. 2 - MAPE classification of prediction accuracy

MAPE %	Prediction accuracy	MAPE %	Prediction accuracy
<10	High accuracy	20~50	Feasible
10~20	Good	>50	Infeasible

ENGINEERING EXAMPLE ANALYSIS

High-filled soil engineering in Wuhan TianHe airport [21]

Located in the Wuhan City, Hubei Province, China, the Wuhan Tianhe Airport Second Channel Expressway has a total length of approximately 15.5 km. Most of the landforms are low mountains and hills with little structural erosion. The topography is undulating and there are more high-filled sections across the whole line. After the completion of the construction, the settlement deformation of the high-filled section of the whole line was monitored in real time. This study focuses on the S2 section, the pile number of the section is K4+200~K5+325 and total length is 1.215 km. Section K4+860 is a special subgrade section and the settlement deformation of this section was analysed and predicted. The embankment filling height of this section was 8.2 m and measurement period was from June 12 to December 15, 2016. Based on the measured settlement data across 153 days, using the Bryant method, logarithmic transformation and linearization equations listed in Table 1, the Logistic and Gompertz models were determined, respectively:

$$
\hat{y}_{1t} = \frac{132.2949}{1 + 4.2697 \times e^{-0.0577t}} \text{ and } \hat{y}_{2t} = 133.8612 \times e^{-1.9656e^{-0.0427t}}
$$

Therefore:

$$
\sum_{t=1}^{20} s_{1t}^{2} = 0.0548, \sum_{t=1}^{20} s_{2t}^{2} = 0.0581 , \sum_{t=1}^{14} s_{1t}s_{2t} = 0.0539
$$

The Optimal weight coefficient $w_1 = 0.8204, w_2 = 0.1796$. Therefore, the optimal weighted geometric average combination prediction model was:

$$
\hat{y}_t = \hat{y}_{1t}^{\,0.8204} \cdot \hat{y}_{2t}^{\,0.1796}
$$

Using the Logistic model, Gompertz model and the proposed optimal weighted geometric average combination prediction model, the predicted settlement values and precision index values

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were calculated (Tables 3 and 4). The relationship curve of settlement and time of section K4+860 is shown in Figure 1. The curve of relationship between measurement time and relative error of section K4+860 and a diagram of measurement time and residual scatter plot are shown in Figure 2 and Figure 3, respectively.

Tab. 3 - Settlement datas of K4+860 by measured and predicted

Time/d	Measured data /mm	Predicted data/mm		
		Logistic model	Gompertz model	The combined model
10	36.707	38.936	37.126	38.604
13	45.111	43.855	43.316	43.758
18	59.202	52.681	53.808	52.888
31	66.11	77.194	79.334	72.574
35	79.321	84.443	86.125	78.742
38	96.485	89.594	90.815	94.812
45	103.898	100.357	100.390	102.363
47	105.427	103.069	102.783	104.018
52	110.353	109.110	108.138	109.935
56	112.551	113.199	111.822	112.950
57	113.509	114.122	112.666	113.860
68	122.903	121.997	120.186	122.670
88	126.203	128.864	127.859	126.683
94	127.358	129.850	129.191	127.731
99	129.063	130.454	130.076	130.086
115	130.757	131.557	131.936	130.625
123	131.109	131.829	132.490	131.248
130	133.617	131.984	132.843	133.138
145	133.983	132.164	133.324	134.071
153	134.203	132.212	133.479	134.339

Tab. 4 - Comparison of precision index values of each prediction model

Model type	SSE	SSRE	SE	RSE	MAPE%
Logistic model	293.234	0.057	3.829	0.054	3.461
Gompertz model	330.611	0.064	4.066	0.057	3.364
The combined model	96.776	0.025	2.200	0.036	1.866
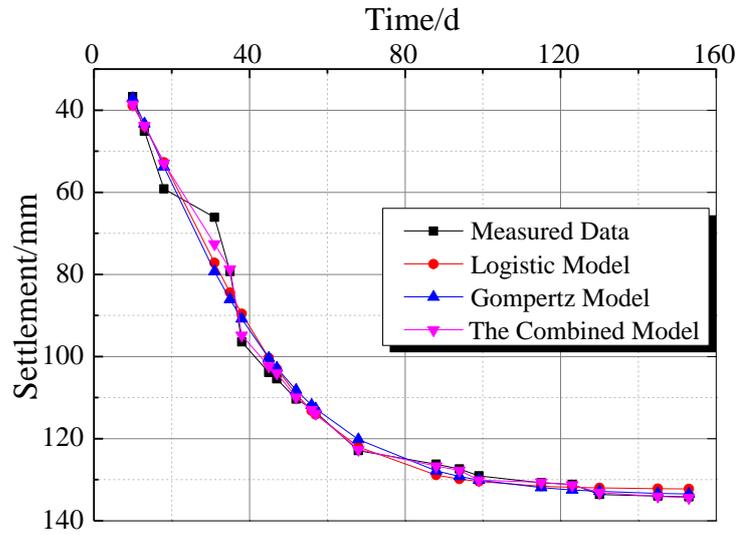
Fig. 1 - Measured settlement and predicted settlement curves
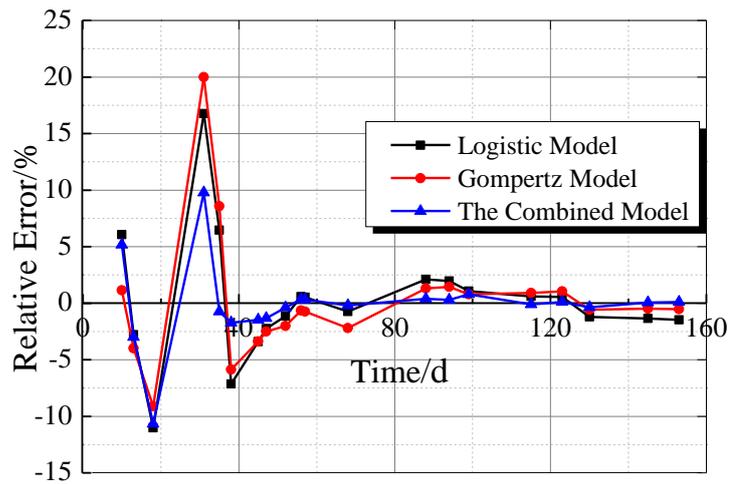
Fig. 2 - Measurement time and relative error curves
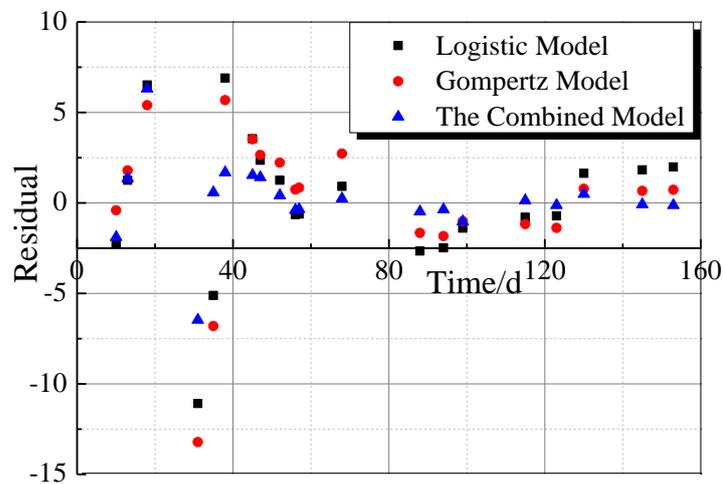
Fig. 3 - Measurement time and residual scatter plots

The high-filled embankment project of Wuhan Tianhe Airport, the regularity of SSE and SE values of the accuracy index of the prediction model, and SSRE values and RSE values of the stability reliability index of the prediction model were similar to that of the high-filled embankment project of Yan Jiafeng Section (Table 8). However, in this study the MAPE values of the Logistic, Gompertz and proposed combined prediction models were 3.461%, 3.364% and 1.866%, respectively. All indicate a high precision prediction and the MAPE values in this study were smaller, suggesting greater accuracy. When the monitoring time was 38 days, the relative error was large, and the variation range was -11.02%-20.00% (Figure 5). However, when the monitoring time was longer than 38 days, the variation range of the relative error was - 3.41%-0.07%. The residual diagram was consistent with the randomness and unpredictability of probability theory, and the points described by the combined prediction model were scattered randomly about a line with residuals equal to 0 (Figures 3 and 6), suggesting that the proposed prediction model was effective.

CONCLUSION

(1)   To address the deficiencies in single settlement prediction models in predicting the settlement of high-fill soil, this study combined the Logistic and Gompertz prediction models to solve optimal weighting coefficients. This was achieved taking the sum of the minimum logarithmic error squares of the combined model as the objective function, and then deriving the optimal weighted geometric mean combination prediction model.

(2)   Selecting the engineer measured data for the fitting analysis, the comparative analysis indicated that the accuracy (SSE, SE and MAPE) and reliability (SSRE, RSE) of the optimal combination predicting model was better than any single model, and the adaptability was greater. When the monitoring time exceeded the curve inflection point, it was similar to the measured value, indicating the optimal combination predicting model was good for settlement prediction.

(3)   The proposed combination prediction model could be used to optimize the combination of multiple models and buffer the advantages and disadvantages of each single model. When there is no negative weight ratio, the settlement prediction accuracy was improved significantly.

In this study, linear loading or approximate linear loading only was used for the analysis. The application of a non-linear growth model to settlement prediction of step loading and more complex loading processes requires further study and analysis.

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REFERENCES

[1]   Akira Asaoka., 1978. Observational procedure of settlement prediction. Soils & Foundations, 18(4): 87-101.
[2]   Zheng Jiangguo, Cao Jie, Zhang Jiwen. et al.,2018. Analysis of influencing factors of loess high fill settlement based on centrifuge model test. Journal of Rock Mechanics and Engineering. Vol. 38, 1-13.
[3]   Zhuo Wenze, Zhou Ren.,2015. Study on settlement Prediction of Deep soft Foundation of Henei-Haifang Expressway. Journal of Hebei University of Engineering (Natural Science Edition), Vol. 32, No. 3, 19–24.
[4]   Xue Jie.,2018. Study and Analysis on Prediction of subgrade settlement of Expressway. Traffic North, No. 8, 53–55.

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[5]   Feng Huiping, Geng Huiling, Han Bowen.,2017. Prediction Model for subgrade settlement of High Speed Railway in unsaturated soil region. Journal of Geotechnical Engineering, Vol. 39, No. 6, 1089–1095.

[6]   Siew-Ann Tan.,1994. Hyperbolic method for settlement in clays with verticaldrains. Can. Geotech.J, Vol. 31, No. 1, 125–131.

[7]   AKIRAASAOKA.,1978. Observational procedure of settlement predictions[J]. Soils and Foundations, Vol. 18, No. 4, 87–101.

[8]   Siew-Ann Tan, Soon-Hoe Chew.,1996. Comparison of the hyperbolic and Asaoka observational method of monitoring consolidation with vertical drains. Soils and foundation, Vol. 36, No. 3, 31–42.

[9]   Lei Xuewen, Bai Shiwei, Meng Qingshan.,2000. Application of Grey Prediction in settlement Analysis of soft soil Foundation. Rock and Soil Mechanics, Vol. 21, No. 2, 145–147.

[10]   Song Yanhui, Nie Dexin.,2003. Verhuslst model for prediction of foundation settlement. Rock and Soil Mechanics, Vol. 24, No. 1, 123–126.

[11]   Zhao Minghua, Zeng Guangxian, Xiang Zhenfeng.,2006. Grey Optimum-Markov Prediction Model for settlement of High fill Embankment. Road Traffic Technology, Vol. 23, No. 11, 19–22.

[12]   Akira Asaoka, Koh Misumi.,1988. Analysis and prediction of consolidation settlement of normally consolidated clay foundation under embankment loading. Soils and foundations, 28(3):35-49.

[13]   Dalgic, Suleyman, Simsek. et al., 2002. Settlement predictions in the Anatolian motorway, Turkey. Engineering Geology, 67(1): 185-199.

[14]   Bergado, Dennes T. Daria, Peter M, Sampaco. et al., 1991. Prediction of embankment settlements by in-situ tests. Geotechnical Testing Journal, 14(4): 425-439.

[15]   Gompertz B., 1825. On the Function Expressive of the Law of Human Mortality, and on a new Method of Determining the Value of Life Contingencies: Philosophical Transactions of the Royal Society, 513-585.

[16]   Zhang Yuting, Sun Hao, Wang Jinxiang.,2009. Application of Gompertz Curve Model to settlement Prediction of soft soil Foundation of Sea Embankment. Port of Waterways, Vol. 30, No. 4, 257–262.

[17]   Wu Qixing, Hu Hui.,2006. Analysis of settlement of soft soil under vacuum preloading based on Gompertz growth curve. Journal of Rock Mechanics and Engineering, Vol. 25, No. 2, 3600–3606.

[18]   Xu Hongzhong, Shi Bin, Li Xuehong.,2005. Study on Logistic growth model and its applicability for prediction of total process settlement Rock and Soil Mechanics, Vol. 26, No. 3, 387–391.

[19]   Mei Guoxiong, Zai Jinmin, Yin Jinze.,2005. The proof of the "S" shape of the settlement-time curve and its application from the constitutive relation of the soil. Rock and Soil Mechanics, Vol. 26, 21–24.

[20]   Tang Xiaowo.,1992. Optimal combination prediction method and its application. Statistics and Management, Vol. 11, No. 1, 31–35.

[21]   Wei Daokai, Kou Hailei.,2018. Study on data fitting and Prediction of settlement and deformation of High- fill subgrade on Expressway. Road Engineering, Vol. 43, No. 1, 251–256.