

# FILTRATION VELOCITY OF WATER IN COARSE PLASTIC PARTICLES

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## ABSTRACT

Experimental investigation of the effect of particle geometry on the values of hydraulic conductivity using 5 plastic particle fractions of different shapes and sizes is presented in this paper. Conducted experiments serve as a background for ongoing research focused on sediment transport of erodible bed in tilting flume. When a flowing mixture of water and particles evolves above a stationary bed, the determination of flow rate of water filtered through stationary bed is very important in order to estimate the inaccuracy of total flow rate through the flume. In order to evaluate the amount of infiltrate water, experiments were carried out in both laminar and turbulent flow regimes. The transition area was identified, and the results show only small effect of particle geometry on the transition between regimes. Experimentally identified values of hydraulic conductivity were used to calculate the amount of infiltrate water in the tilting flume for different inclination angles. Experimentally identified values of hydraulic conductivity are, for example, useful in designing industrial water filters, as the materials used to conduct experiments and those in filters are similar.

## KEYWORDS

Hydraulic conductivity, Filtration, Plastic particles

## INTRODUCTION

The necessity to determine the amount of infiltrate water flowing through the homogenous porous media occurred when working on a research project focused on intense sediment transport in open channel flow [1]. Porous materials can be very roughly characterized by the value of hydraulic conductivity, which is a commonly used parameter in various branches of civil engineering (for example in water or environmental management). The hydraulic conductivity is most frequently used to calculate the flow rate through the porous materials. The computations are realized by the generally known Darcy's law equation [2,3].

## METHODS AND EXPERIMENTAL SET UP

Our experimental evaluation of the hydraulic conductivity was carried out in an experimental cell situated in the Water Engineering Laboratory at Faculty of Civil Engineering of Czech Technical University in Prague. Experimental cell was made of transparent plexiglass. Height of cell was 1.05 m and dimension of rectangular cross-section was 0.22 m x 0.20 m (Fig. 1). The direction of the water flow through the cell was from the bottom to the top. The inflow rate was adjusted using a control valve on the inlet. The total flow rate was measured by mass method on the outlet. Tested porous material was placed between two steel sieves in order to mitigate the effects of buoyancy and hydrodynamic pressure.



Fig. 1 – Photo of experimental cell

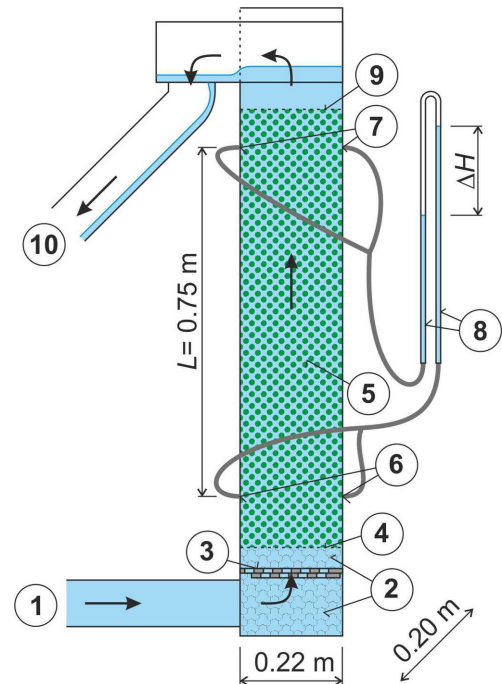


Fig. 2 – Schematic of experimental cell – 1 - inflow, 2 – geotextile, 3 – system of meshes, 4 and 9 - steel sieve, 5 – tested material, 6 and 7 – connection of U-tube, 8 – U-tube, 10 – outflow

## Materials

Five fractions of plastic particles of different sizes and shapes were used to conduct the experiments, and as this work is connected to our earlier experiments in tilting flume [1], tested materials are identical. Properties of the particle fractions are given in Tab. 1 and photos of the particles can be seen in Figures 3 – 7.

Tab. 1 - Properties of used materials –  $d_1$  and  $d_2$  - diameters,  $h$  – height,  $d_{esp}$  - equivalent spherical diameter,  $\rho_s$  – density,  $P$  – porosity

Quantities	units	F30	F60	HSF3	TLT25	TLT50
$d_1$	mm	4.05	6.43	3.32	4.99	4.75
$d_2$	mm		5.78	2.54		
$h$	mm	2.24	5.14	3.89	2.08	4.9
$d_{esp}$	mm	3.65	6.42	3.18	4.23	5.41
$\rho_s$	kg/m <sup>3</sup>	1368	1419	1358	1381	1307
$P$	-	0.38	0.38	0.40	0.40	0.40

Where  $d_1$  is the major axis length of the particle ellipse cross section and  $d_2$  is the minor axis length,  $h$  is the averaged height of a particle (maximum dimension in case of tri-axial ellipsoid HSHF3, and height of the remaining particle fractions F30, F60, TLT25 and TLT50), value of porosity,  $P$ , was measured using a graduated cylinder,  $d_{esp}$  is the diameter of the equivalent sphere derived from the volume of the averaged particle of each fraction.

Regarding shape of the particles, F30 and TLT25 can be described as contact lenses, F60 and TLT50 as cylinder with concave and convex face and the HSF3 a tri-axial ellipsoid. [1]



Fig. 3 – F30



Fig. 4 – F60



Fig. 5 – HSF3



Fig. 6 – TLT25



Fig. 7 – TLT50

## Procedures

All materials were put in the cell filled with water and simultaneously mixed to attainment a random position of particles as in the experiment of a sediment transport in the tilting flume.

The air on the particles was removed from the experimental cell by repeated mixing using a steel rod during the cell filling procedure. In order to achieve similar experimental conditions to those of our earlier experiments, focused on sediment transport of erodible bed in tilting flume, no additional stress was applied to the surface of the particle bed (which would achieve higher degree of particle consolidation). The temperature of flowing water was in ranging from 14 to 24 °C.

Various experimental measurements covering wide range of flow rates were performed in order to determine possible range of hydraulic conductivity values in both laminar and turbulent flow regimes. Experimental procedure consisted of four steps. First step was to set the flow rate. Second step was to wait for the flow and pressure conditions to stabilize. After the stabilization period, the pressure heights difference  $\Delta H$  was measured between lower and upper pressure sampling using a U-tube and hydraulic gradient  $i$  was calculated as a ratio of  $\Delta H$  and the distance between lower and upper pressure sampling  $L$ .

The third step was the determination of flow rate by equation  $Q = V/t$ . First, a water sample was taken in the given time  $t$ , then weight of the water sample  $m$  was measured. The value of the density  $\rho$ , used in the calculations, was derived from the temperature of the water sample. Finally, the volume of the sample was given by equation  $V = m/\rho$ .

The last step was to calculate the value of the hydraulic conductivity  $k$  using a standard Equation 1 [2]

$$k = \frac{L}{\Delta H} \cdot \frac{Q}{A} \quad (1)$$

Where  $A$  ( $0.22 \times 0.20 = 0.044 \text{ m}^2$ ) is the area of cross section and  $Q$  is the flow rate of water.

## RESULTS

### Hydraulic conductivity

Calculated values of hydraulic conductivity are plotted in Fig. 8 against the flow rate. Results show, that the values of hydraulic conductivity are affected by the size and shape of pores between particles, which are mostly affected by the shape and the size of the particles. Value of the hydraulic conductivity is affected by the volume of the pore, which is directly proportional to the volume of the particle, and thus a simple trend is expected: The larger the particle, the higher value of hydraulic conductivity.

This applies to all particles except the TLT25 and F30 fractions. The F30 particles are smaller than TLT25, yet calculated values of hydraulic conductivity for TLT25 are smaller than those calculated values for F30.

However, the difference between hydraulic conductivity values is small, and could be caused by the difference in the shapes of the particle fractions. F30 particles (contact lenses) are more curved than TLT25 (contact lenses).

The values of hydraulic conductivity decrease with an increase of the flow rate, which indicates a turbulent flow regime (see Fig. 8.), the region of more or less constant value of hydraulic conductivity in the Figure 8 indicates a laminar flow regime.

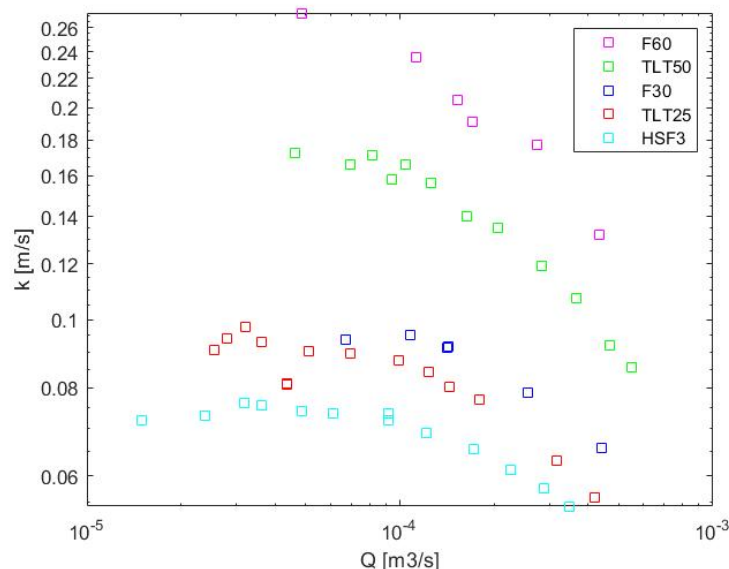


Fig. 8 – Calculated values of hydraulic conductivity (logarithmic scale)

### Comparison of Reynolds number critical values and the transition between laminar and turbulent flow regimes

Hydraulic conductivity is given not only by properties of the particles but also by flow rate of water. The determination of hydraulic conductivity is based on Darcy's law (linear relationship  $v_f = k \cdot i$ , where  $i$  is the hydraulic gradient and  $k$  the hydraulic conductivity). It should be noted that the constant value of hydraulic conductivity is applicable only in laminar flow regime. In turbulent flow

regime the relationship between  $v_f$  and  $i$  is non-linear and the value of  $k$  must be modified when the Darcy's law is used in the turbulent flow regime. [2,3]

A standard procedure for evaluation of the flow regime is to calculate the value of the Reynolds number and then compare the value to a critical one, which corresponds to point of transition between the regimes. In particular conditions of our experiments, equation (2) can be used:

$$Re_n = d_{por} \cdot u \cdot \frac{\rho}{\eta} \quad (2)$$

Where  $d_{por}$  is the effective pore diameter,  $u$  is the mean velocity of the flow in pore, which was calculated with the use of  $v_f$  and area of pore,  $\rho$  is the liquid density, and  $\eta$  its viscosity. [3]

The value of  $Re$  at the point of the transition is for the flow in straight tubes somewhere between 2000-4000, but in porous materials the value of  $Re$  should be smaller. [3,5]

Another equation, which can be used to calculate the value of Reynolds number, is Equation (3):

$$Re_f = \frac{1}{0.75 \cdot P + 0.23} \cdot \frac{v_f \cdot d_{ef}}{\nu} \quad (3)$$

Where  $P$  is the porosity,  $v_f$  is the mean velocity given by  $Q/A$ ,  $d_{ef}$  is the effective grain size (which, in case of materials of uniform grain size, can be substituted by medium size of the particles) and  $\nu$  is the kinematic viscosity ( $\nu = \eta/\rho$ ) (3). [4]

The value of  $Re$  at the point of transition valid for the flows of ground water is 7 – 10 according to the research by Pavlovsky [4,5].

In Fig. 9 hydraulic gradient  $i$  is plotted against the velocity of the flow through the sediment layer  $v_f$  for all of the particles used in our experiments. Continuous lines interlace the points for small values of velocity of the flow through the sediment layer, where linear relationship between  $v_f$  and  $i$  is applicable. All of the measured points where linear relationship is applicable are marked by crosses. The results show, that the rise of the velocity of the flow through the sediment layer is linear at first, but changes to non-linear with rising hydraulic gradient. It has been proven in the literature that the point at which the trend changes from linear to non-linear indicates the point of the transition between laminar and turbulent flow regimes. [3]



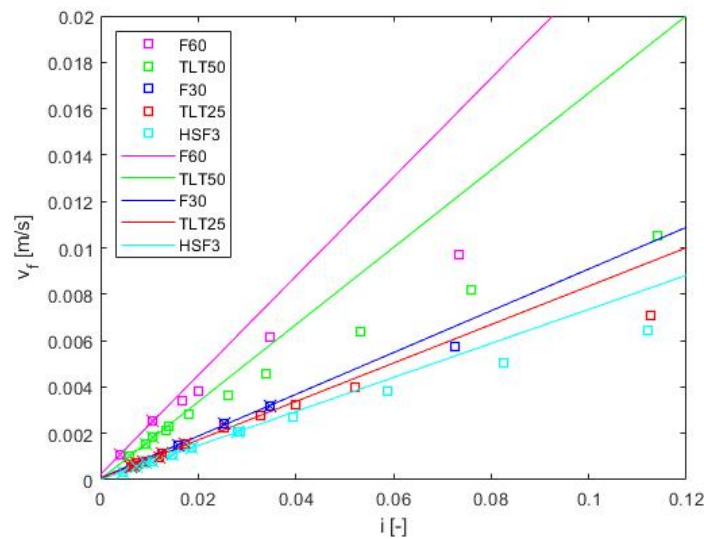


Fig. 9 – Relationship between hydraulic gradient and velocity of the flow through the sediment layer. Legend: continuous line – a linear relationship between  $v_f$  and  $i$  valid for laminar flow regime, square with a cross – points in laminar part, square – points in turbulent part

In order to determine the threshold between laminar and turbulent flow regimes, values of Reynolds number were calculated using the equations 2 and 3. The values of the velocity of the flow through the sediment layer used in the calculations were taken from Fig. 9 where the relationship of  $v_f - i$  changes from linear to non-linear. The point of transition was determined by linear interpolation of adjacent points, first, point 1, and 2, next 1, 2, 3 etc. When the line started changing the last point of penultimate step, was sign as the threshold. The results of the calculations were compared to the limits values reported in mentioned publication. [2, 3, 5]

In equation 2, the size of effective pore diameter is used. In our calculations, the parameter is defined by equation 6, which is derived from equation 4 and 5. Where  $V_{por}$  is volume of a pore,  $V_{par}$  is volume of particles,  $P$  is porosity,  $d_{por}$  is diameter of a pore and  $d_{par}$  is the diameter of the equivalent sphere particle.

$$P = \frac{V_{por}}{V_{por} + V_{par}} \quad (4)$$

$$\frac{V_{por}}{V_{par}} = \frac{d_{por}^3}{d_{par}^3} \quad (5)$$

$$d_{por} = d_{par} \cdot \left[ \frac{P}{1 - P} \right]^{1/3} \quad (6)$$

Tab. 2 – Calculated values of Reynolds number

Quantities	units	TLT25	TLT50	F30	F60	HSF3
$Re_f$	-	12.1	18.2	22.0	30.6	12.1
$Re_n$	-	14.0	21.1	25.4	35.3	14.0

Tab. 2 shows the values of Reynolds number ( $Re_n$ ) calculated from Equations 2 and 3. When the Equation 2 is used, resulting  $Re_n$  values for all particle fractions are between 14 – 36. These results correspond with the notice reported by Hillel. When the Equation 3 is used, resulting  $Re_n$  values for all particle fractions are between 12 – 31, which is very close to the values reported by Pavlovsky.[4,5] The value of  $Re$  at the point of transition for straight tube is 2000 – 4000, but in porous materials it should be significantly less. This is due to the fact that the pores are not connected to each other as straight tube, but they are randomly distributed what is given by their shapes, size etc. [3, 4], which can be seen in the Figure 10. [3,5]



Fig. 10 – Photo of F60

### Comparison of flow rates above and in the deposit layer

The ongoing research [1] to which is this experiment connected is focusing on investigation of internal structure of steady uniform, turbulent open-channel flows over an inclined granular bed in the upper plane regime with intense transport of sediment. The flow can be divided into three layers. Stationary bed of deposited particles at the bottom of the channel forms the first layer. A second layer above the stationary bed of particles, through which the particles are transported we call the transport layer. Third and last layer is that of clear water. It is necessary to know the flow rate of the mixture, which is equal to the total flow rate through the flume minus the flow rate of infiltrate water in order to evaluate the conducted experiments.

Mean velocity of the flow through the sediment can be determined for given hydraulic slope and material on the basis of relationship " $v_f - i$ ", which can be derived from Fig. 9, where velocity of the flow through the sediment layer is plotted against hydraulic gradient. Experiments in the tilting flume were performed under steady and uniform flow conditions (when constant height of the sediment bed was achieved across the whole length of the flume), therefore the hydraulic gradient is considered to be equal to the slope of the tilting flume. Ratio of velocity of the flow through the sediment layer  $v_f$  and velocity of the flow above the bed  $v_s$  against the hydraulic gradient can be seen in Figure 11. Velocity of the flow above the bed  $v_s$  is given by ration of flow rate and profile area above the bed. The flow rate above the bed is given by the difference between the total flow rate through flume and through the sediment, which was determined by using the procedures described in this paper. Based on the ratio of  $v_f/v_s$ , the ratio of heights of flow layers above the stationary bed and the thickness of the deposit layer, it is possible to determine the flow rate through sediment.

The experiments were conducted in rectangular tilting flume of width equal to 0.20 m. Resulting database includes various test runs that cover quite broad range of inclination angles, total flow rates and layer heights. To make some sense of investigated values, overview of the ranges can be seen below in Table 3. Further details on conducted experiments can be seen in authors previous work [1].

Tab. 3 – Measuring range ( $i$  – slope of tilting flume,  $Q_{flume}$  – value of whole flow in tilting flume,  $h_{deposit}$  – height of deposit layer,  $h_{transport}$  – height of transport layer)

Material	$i$ [-]		$Q_{flume}$ [l.s-1]		$h_{deposit}$ [mm]		$h_{transport}$ [mm]	
	min	max	min	max	min	max	min	max
F30	0.0268	0.0517	5.80	6.38	20	53	38	50
F60	0.0119	0.0794	8.24	10.26	19	64	53	77
HSF3	0.0050	0.0580	6.03	11.98	18	62	29	98
TLT25	0.0060	0.0671	6.16	8.09	18	55	44	100
TLT50	0.0080	0.0547	10.00	15.90	17	60	29	101

The Figure 11 shows that the ratio of velocities increases with increasing slope of water surface, which means, that with increasing slope the velocity of infiltrate water increases with higher rate, than the velocity in upper layers of the flow. This is caused by the increasing number of suspended particles in the flow above the deposit layer, which reduces the mean velocity of the transport layer as increasing concentration of particles provide more resistance to the flow.

Just to roughly quantify the  $v_f/v_s$  ratio, for higher slopes the values can reach up to a few percent.

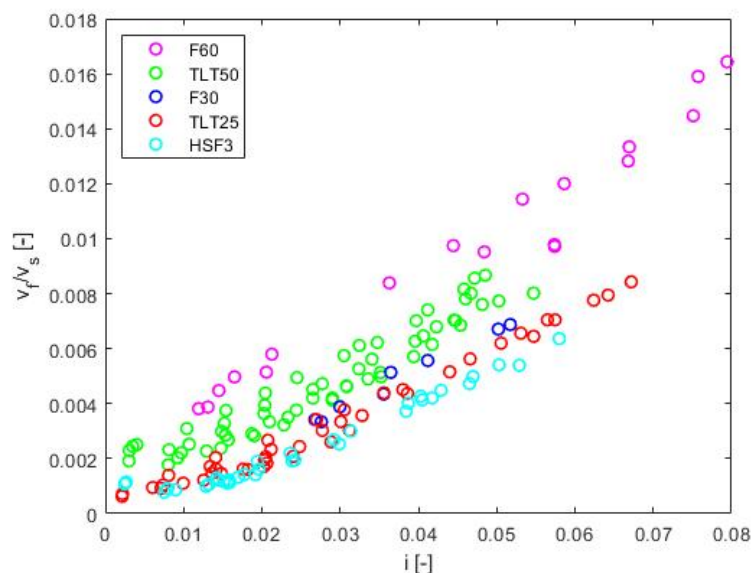


Fig. 11 - Ratio of  $v_f$  and  $v_s$  against the hydraulic gradient

## CONCLUSIONS

5 plastic particle fractions of different shapes and sizes were experimentally investigated in order to obtain their specific values of hydraulic conductivity. Analysis of the results showed, that



the values of hydraulic conductivity are by both, the size and the shape of the particles. While the size of the particle has a major effect on the value of the hydraulic conductivity, results also show that effect of the shape of the particles is not negligible.

Further analysis showed that the flow regime has also a strong effect on the values of hydraulic conductivity. While constant value of hydraulic conductivity can be applied under laminar conditions, in turbulent flow regime the value of hydraulic conductivity must be modified, as the corresponding value can be significantly lower, when the Darcy's law is used.

When experimentally identified values of hydraulic conductivity were used to calculate the amount of infiltrate water in a tilting flume for different inclination angles, results showed, that the ratio of the velocity in the sediment layer and the velocity above the stationary values increase with the increasing inclination of water surface.

### ACKNOWLEDGEMENTS

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