STUDY ON THE SETTLEMENT OF TUNNEL BOTTOM AND PRESSURE OF ROCK MASS BASED ON CURVED BEAM ON ELASTIC FOUNDATION THEORY

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ABSTRACT

Tunnel invert is the weak section of curved beam on tunnel foundation, and it is easy to break down. Based on the curved beam theory on elastic foundation, the curved beam model of tunnel invert was established, the displacement equations of tunnel invert under external load were deduced, and the formulas of settlement of tunnel bottom and the pressure of rock mass were presented. By means of the calculating formulas, the distribution laws of settlement of tunnel bottom and pressure of rock mass were obtained when tunnel bottom was strengthened and not strengthened by high pressure jet grouting piles. The final formulas in the paper are precise to predict the settlement of tunnel bottom and pressure of rock mass, so they are of great value for tunnel design and construction.

KEYWORDS

Tunnel invert, Curved beam, Elastic foundation, Displacement equation, Pressure of rock mass

INTRODUCTION

There are many problems in the construction of curved beams, such as embankment culvert, underground pipeline and tunnel lining, and these problems can be used the curved beam model to solve [1-3]. At present, the researches of curved beams are mostly concentrated on the study of the basic theory [4-5]. Pagano et al. [6-7], Chien et al. [8], Shen et al. [9], and Tornabene et al. [10] used the linear elastic theory to study laminated composite plates under the condition of column bending and gave the exact solutions. Lekhnitskii et al. [11] gave the general solutions of anisotropic bending beam under moment load. This method was also adopted to calculate the interlaminar tensile stress for four point bending experiment of curved beam (ASTMD6415/D6415M-06a) [12]. Shenoi et al. [13], Arici et al. [14] established the model for bending behavior of elastically curved beam and obtained the elastic solutions of the anisotropic beam. Higher-order shear deformation theory can accurately calculate in-plane deformation and stress for shell structure with span to thickness ratio than four, but cannot calculate the interlaminar stresses for composite plates and shells [15, 16]. Aköz et al. [17] used the finite element method to analyze circular beam with variable cross section on elastic foundation under arbitrary load. Banan et al. [18] found the general finite element formula for spatial curved beams and arches on elastic foundation. Çalım et al. [19, 20] presented an effective method for analyzing the dynamic performance of straight beams and curved beams on elastic foundation. Wang et al. [21], Öz et al. [22], Chen et al. [23], Malekzadeh et al. [24], and Wei et al. [25] studied the vibration problem of curved beams on elastic foundation. Based on Euler beam theory, Zhong et al. [26], Adineh et al. [27], Cong et al. [28], Duc et al. [29, 30], and Dung et al. [31] studied the nonlinear transient thermal response for FGM beam with unstable heat conduction and revealed the mechanical characteristics of FGM beams on the basis of tension free. From the point of view, the academic researches on the curved beam on elastic foundation have made a wealth of

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research results, but there is little research in practical engineering applications.

In tunneling engineering, the tunnel lining is closely contacted with rock mass, the tunnel lining can be regarded as curved beam on elastic foundation. Dai et al. [32] put forward the calculation model of tunnel lining deformation and internal force on the basis of the Qingdao overlapping tunnel on the beam theory on elastic foundation. Based on principle of initial parameter method, Sun et al. [33] established the initial parameter matrix equations for solving the internal force and deformation of concrete lining on curved beam on elastic foundation. Through on-site monitoring of steel arch stress and pressure of rock mass, Wen et al. [34] deduced an analytical formula for internal force of the primary support of tunnel on curved beam on elastic foundation. Based on the beam model on double elastic foundation and influence of uneven settlement of soft soil, Li et al. [35] established a plane numerical model and analyzed the arrange of settlement joint and the longitudinal stress and deformation of the tunnel. In summary, more emphasis focuses on the analysis of the internal force and the uneven settlement in the longitudinal range of the superstructure of the tunnel, while the analysis of the tunnel invert is relatively rare.

In this paper, the curved beam model of the tunnel invert was established and the displacement equations of curved beam on elastic foundation under external load were deduced based on the differential equations of beam on elastic foundation. The displacement equations were used to calculate the settlement of tunnel bottom and the pressure of rock mass when the tunnel foundation of soft loess was strengthened and not strengthened by high pressure jet grouting piles.

DIFFERENTIAL EQUATIONS OF CURVED BEAM ON ELASTIC FOUNDATION

The Winkler Foundation assumes that the settlement at any point on the surface of the foundation is proportional to the pressure on the unit area of the point, this assumption is the foundation for a series of independent spring simulation on rigid base. When a point on the foundation surface is subjected to pressure p, because the springs are independent of each other, only the local settlement y is generated at this point, and no settlement occurs in other places. The foundation model is also called the local elastic foundation model. The formula is expressed in equation (1).

$$p=Ky \qquad\qquad (1)$$

Where the K is the coefficient of foundation reaction which represents the pressure intensity required to produce the unit deformation at a point on the foundation (kN/m^3); p is the pressure intensity at any point on the foundation (kPa) and y is the deformation of the foundation at the point of pressure (m).

The local elastic foundation model is still applicable in the derivation of curved beam on elastic foundation. There is a constant cross-section beam on the Winkler Foundation (Figure 1(a)). The inner and outer radius of curved beam is r_0 and r_1 respectively, and the thickness of curved beam is $h=r_1-r_0$, the sectional area is A, the sectional inertia moment is I, and the center radius is $ri=(r_1+r_0)/2$. Assuming that there is no friction effect, the resistance of the foundation is proportional to the radial displacement of the curved beam.
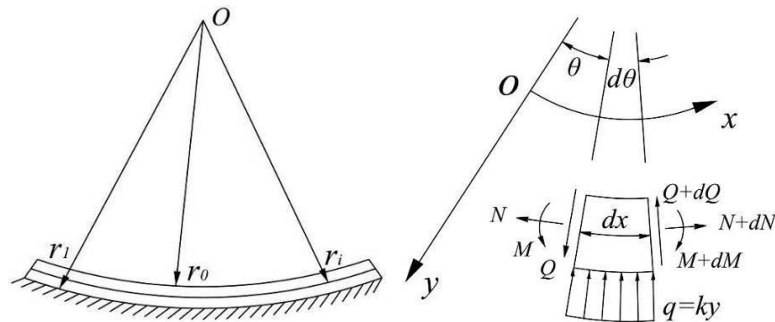
(a) Curved beam with constant cross section   (b) The force of element

Fig. 1 - Schematic diagram of curved beam on elastic foundation

We selected an arbitrary element $r_i d\theta$ from the beam as the object of analysis, as shown in Figure 1(b). The $y(\theta)$ is radial displacement; K is the coefficient of foundation reaction; the $M(\theta)$ is the bending moment of initial section; the $Q(\theta)$ is shear force and the $N(\theta)$ is axial force. All the forces are positive in the direction of the Figure 1.

According to the force state of the element, as shown in Figure 1(b), the radial, tangential and center moment balance equations of the infinitesimal element are established, as shown in equation (2) to Equation (4).

$$N(\theta)d\theta + Ky(\theta)r_1 d\theta + dQ(\theta) = 0 \tag{2}$$

$$Q(\theta)d\theta - dN(\theta) = 0 \tag{3}$$

$$dM(\theta) - Qr_i d\theta = 0 \tag{4}$$

The radial displacement $y(\theta)$ of curved beam has the following relationship with internal force [34]:

$$\frac{d^2 y(\theta)}{d\theta^2} + y(\theta) = \frac{M(\theta)r_i^2}{EI} + \frac{N(\theta)r_i}{EA} \tag{5}$$

Where E is the elastic modulus of curved beam.

To derive θ in the Equation (5) once and three times, we can get the following equations:

$$\frac{d^3 y(\theta)}{d\theta^3} + \frac{dy(\theta)}{d\theta} = \frac{r_i^2 dM(\theta)}{EI d\theta} + \frac{r_i dN(\theta)}{EA d\theta} \tag{6}$$

$$\frac{d^5 y(\theta)}{d\theta^5} + \frac{d^3 y(\theta)}{d\theta^3} = \frac{r_i^2 d^3 M(\theta)}{EI d\theta^3} + \frac{r_i d^3 N(\theta)}{EA d\theta^3} \tag{7}$$

Then we can obtain the Equation (8) by add Equation (6) and Equation (7):

$$\frac{d^5 y(\theta)}{d\theta^5} + 2\frac{d^3 y(\theta)}{d\theta^3} + \frac{dy(\theta)}{d\theta} = \frac{r_i^2 d^3 M(\theta)}{EI d\theta^3} + \frac{r_i d^3 N(\theta)}{EA d\theta^3} + \frac{r_i^2 dM(\theta)}{EI d\theta} + \frac{r_i dN(\theta)}{EA d\theta} \tag{8}$$

The Equation (9) can be obtained by simultaneous Equation (3), Equation (4) and Equation (6):

$$Q(\theta) = (\frac{r_i^3}{EI} + \frac{r_i}{EA})^{-1}[\frac{d^3 y(\theta)}{d\theta^3} + \frac{dy(\theta)}{d\theta}] \tag{9}$$

The Equation (10) can be got by simultaneous Equation (2) and Equation (9):

$$N(\theta) = -(\frac{r_i^3}{EI} + \frac{r_i}{EA})^{-1}[\frac{d^4 y(\theta)}{d\theta^4} + \frac{d^2 y(\theta)}{d\theta^2}] - Kr_1 y(\theta) \tag{10}$$

And the Equation (11) can be obtained by simultaneous Equation (5) and Equation (10):

$$M(\theta) = \frac{EI}{r_i^2}\left\{\frac{d^2y(\theta)}{d\theta^2} + y(\theta) + (\frac{r_i^2 A}{I}+1)^{-1}[\frac{d^4y(\theta)}{d\theta^4} + \frac{d^2y(\theta)}{d\theta^2}] + \frac{r_i}{EA}Kr_1 y(\theta)\right\}$$

(11)

We can get a five order radial displacement $y(\theta)$ differential equation with constant coefficients based on simultaneous Equation (2), Equation (4) and Equation (8):

$$\frac{d^5y(\theta)}{d\theta^5} + 2\frac{d^3y(\theta)}{d\theta^3} + n^2\frac{dy(\theta)}{d\theta} = 0$$

(12)

where $n^2 = 1 + Kr_1(\frac{r_i^3}{EI} + \frac{r_i}{EA})$

So the general solution of the Equation (12) is:

$$y(\theta) = C_0 + C_1 ch\alpha\theta cos\beta\theta + C_2 sh\alpha\theta cos\beta\theta + C_3 ch\alpha\theta sin\beta\theta + C_4 sh\alpha\theta sin\beta\theta$$

(13)

Finally, we can get analytical Equation of internal force of curved beam by simultaneous Equation (2) to Equation (6) and Equation (13):

$$Q(\theta) = T[C_1(A_1 sh\alpha\theta cos\beta\theta + A_2 ch\alpha\theta sin\beta\theta) + C_2(A_1 ch\alpha\theta cos\beta\theta + A_2 sh\alpha\theta sin\beta\theta) +$$
$$C_3(A_1 sh\alpha\theta sin\beta\theta - A_2 ch\alpha\theta cos\beta\theta) + C_4(A_1 ch\alpha\theta sin\beta\theta - A_2 sh\alpha\theta cos\beta\theta)]$$

(14)

$$N(\theta) = -Kr_1 C_0 - C_1(2\alpha\beta T sh\alpha\theta sin\beta\theta) - C_2(2\alpha\beta T ch\alpha\theta sin\beta\theta) +$$
$$C_3(2\alpha\beta T sh\alpha\theta cos\beta\theta) + C_4(2\alpha\beta T ch\alpha\theta cos\beta\theta)$$

(15)

$$M(\theta) = [(Kr_1 B_1 + 1)C_0 + C_1(2\alpha\beta)(B_1 T - 1)sh\alpha\theta sin\beta\theta + C_2(2\alpha\beta)(B_1 T - 1)ch\alpha\theta sin\beta\theta -$$
$$C_3(2\alpha\beta)(B_1 T - 1)sh\alpha\theta cos\beta\theta - C_4(2\alpha\beta)(B_1 T - 1)ch\alpha\theta cos\beta\theta] / B_2$$

(16)

Where $A_1 = -2\alpha\beta^2$; $A_2 = -2\alpha^2\beta$; $B_1 = r_i/(EA)$; $B_2 = r_i^2/(EI)$; $T = (\frac{r_i^3}{EI} + \frac{r_i}{EA})^{-1}$

THE DIFFERENTIAL EQUATIONS OF CURVED BEAM ON ELASTIC FOUNDATION UNDER SYMMETRIC LOAD

Section forms of the highway tunnel are generally symmetrical about the vertical axis, and no asymmetric loading tunnel, so it can be assumed that the load of tunnel rock mass is also symmetrical about vertical axis. The section where the tunnel center is located is taken as the initial section, the symmetry of structure and load can be used.

$$[\frac{dy(\theta)}{d(\theta)}]_{\theta=0} = 0$$

$$[Q(\theta)]_{\theta=0} = 0 \quad,$$

That is:

$$\alpha C_2 + \beta C_3 = 0$$

$$\beta C_2 - \alpha C_3 = 0$$

So we can get $C_2 = C_3 = 0$.

Then, Equation (13) to Equation (16) can be simplified as follows:

$$y(\theta) = C_0 + C_1 ch\alpha\theta cos\beta\theta + C_4 sh\alpha\theta sin\beta\theta$$

(17)

$$-Q_0 = T[C_1(A_1 sh\alpha\theta_0 cos\beta\theta_0 + A_2 ch\alpha\theta_0 sin\beta\theta_0) + C_4(A_1 ch\alpha\theta_0 sin\beta\theta_0 - A_2 sh\alpha\theta_0 cos\beta\theta_0)]$$

(18)

$$-Q_0 = T[C_1(A_1 sh\alpha\theta_0 cos\beta\theta_0 + A_2 ch\alpha\theta_0 sin\beta\theta_0) + C_4(A_1 ch\alpha\theta_0 sin\beta\theta_0 - A_2 sh\alpha\theta_0 cos\beta\theta_0)]$$

(19)

$$-Q_0 = T[C_1(A_1 sh\alpha\theta_0 cos\beta\theta_0 + A_2 ch\alpha\theta_0 sin\beta\theta_0) + C_4(A_1 ch\alpha\theta_0 sin\beta\theta_0 - A_2 sh\alpha\theta_0 cos\beta\theta_0)]$$

(20)

In Equation (17) ~ (20), the internal force analytical equation of curved beam on elastic foundation can be obtained after the integral constants C_0, C_1 and C_4 are calculated. And the integral constants C_0, C_1 and C_4 can be obtained by the boundary conditions at both ends of the curved beam.

ESTABLISHMENT OF CURVED BEAM MODEL ON ELASTIC FOUNDATION OF TUNNEL INVERT

When the tunnel invert was completed, it would be affected by the weight of the lining and the load of rock mass. The settlement of tunnel bottom would produce pressure on the foundation; similarly, the tunnel foundation would also produce the counterforce to the tunnel invert. Therefore, the tunnel invert could be regarded as a curved beam on the elastic foundation. The self-weight of tunnel lining and the pressure of rock mass could be equivalent to the load acting on both ends of tunnel invert, and the boundary conditions at the ends of the tunnel invert could be obtained by the equivalent load. Assuming that the tunnel invert was rigidly connected with the secondary lining of the tunnel, the equivalent load could be simplified as shear Q_0, axial force N_0 and bending moment M_0, as shown in Figure 2.
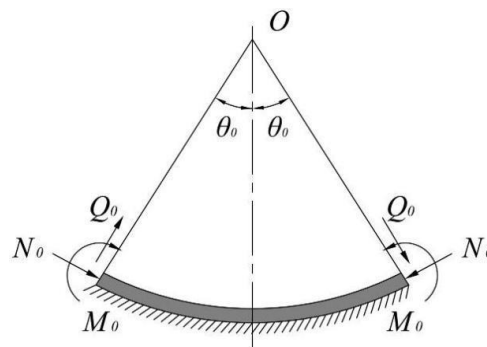

Fig. 2 - Simplified calculation model of tunnel invert

Then the boundary condition of the beam end was:

$$Q(\theta_0) = -Q_0$$
$$N(\theta_0) = -N_0$$
$$M(\theta_0) = -M_0 \tag{21}$$

So we could establish a three-variable linear equation by simultaneous Equation (18) ~ (20), (21), as shown in Equation (22) ~ (24):

$$-Q_0 = T[C_1(A_1 sh\alpha\theta_0 cos\beta\theta_0 + A_2 ch\alpha\theta_0 sin\beta\theta_0) + C_4(A_1 ch\alpha\theta_0 sin\beta\theta_0 - A_2 sh\alpha\theta_0 cos\beta\theta_0)] \tag{22}$$

$$-N_0 = -Kr_1 C_0 - C_1(2\alpha\beta T sh\alpha\theta_0 sin\beta\theta_0) + C_4(2\alpha\beta T ch\alpha\theta_0 cos\beta\theta_0) \tag{23}$$

$$-M_0 = [(Kr_1 B_1 + 1)C_0 + C_1(2\alpha\beta)(B_1 T - 1)sh\alpha\theta_0 sin\beta\theta_0 - C_4(2\alpha\beta)(B_1 T - 1)ch\alpha\theta_0 cos\beta\theta_0] / B_2 \tag{24}$$

The pressure of rock mass of tunnel invert could be calculated according to literature (JTGD70-2004) [36]. The internal force of the corner of tunnel invert could be obtained according to the elastic center method in Structural Mechanics. Then the shear force Q_0, axial force N_0 and bending moment M_0 acting on both ends of tunnel invert could be obtained.

CALCULATION EXAMPLES OF THE SETTLEMENT OF TUNNEL BOTTOM AND PRESSURE OF ROCK MASS IN THE DAYOUSHAN TUNNEL

The foundation reinforcement section of the Dayoushan tunnel was selected as the typical section, and the above theoretical formulas were tested by the engineering case. Firstly, the settlement of the tunnel bottom and pressure of rock mass were calculated when the foundation was not strengthened, and the law of the force and settlement of the tunnel bottom was explored. Then

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the settlement and pressure of rock mass of foundation reinforcement by high pressure jet grouting piles were calculated.

The composite lining was adopted in the Dayoushan tunnel, the excavation radius was 6.41m, and the excavation was constructed by bench cut method. The primary support of shallow section was sprayed with C25 concrete, and its thickness was 26cm; the secondary lining used C25 reinforced concrete, and its thickness was 50cm. The analysis of unit length of tunnel invert in shallow section was carried out. The geometric parameters and calculation parameters of tunnel invert were shown in Table 1 and Table 2 respectively.

Tab. 1 - Geometric parameters of tunnel invert

Thickness h (mm)	External radius r_1 (m)	Inner radius r_0 (m)	Central angle θ_0 (°)	Cross-sectional area A (m²)
50	12.561	12.061	26.9	5.87

Tab. 2 - Calculation parameters of tunnel invert

Parameters	Value	Material
Elastic modulus of concrete E_c (GPa)	29.5	C25 concrete
Elastic modulus of reinforcement E_s (GPa)	200	Φ8 double layer steel mesh
Sectional area of tunnel invert steel bar (m²)	5.03×10⁻⁵	Φ8 double layer steel mesh
Coefficient of foundation reaction K_1 (kN/m³)	8000	Soft foundation
Coefficient of foundation reaction K_2 (kN/m³)	60000	Foundation reinforcement by high pressure jet grouting pile

The composite elastic modulus of reinforced concrete could be calculated according to formula (25):

$$EA = E_c A_c + E_s A_s \tag{25}$$

Where E_c and E_s is the elastic modulus of concrete and steel bars of curved beam on foundation reinforcement, respectively; A_c and A_s is the sectional area of concrete and steel bars.

$$E = (E_c A_c + E_s A_s)/A = 29.54 \text{ (GPa)}$$

The section ZK3+160 of Dayoushan tunnel was selected for the study. The depth of this section was 30m, and the rock mass was 13m collapsible loess and 17m non collapsible loess. The lateral pressure coefficient was calculated according to the lateral pressure and vertical pressure obtained from the previous monitoring, and $\lambda=0.5$. The pressure of rock mass of shallow tunnel was calculated according to the literature (JTGD70-2004) [36], the vertical pressure $q=348kN/m$ and horizontal lateral

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pressure $e=174 \ kN/m$ were calculated, after the load was reduced by 60% (The pressure of rock mass acting on secondary lining accounted for about 60%), so $q_1=208kN/m$ and $e_1=104kN/m$. The internal force at the corner of tunnel invert could be obtained according to the elastic center method in Structural Mechanics. The shear force $Q_1=342kN$, the axial force $N_1=1233kN$ and the bending moment $M_0=591 \ kN\cdot m$ were obtained.

Considering the influence of the weight of secondary lining, the cross section area of secondary lining was $11.52m^2$, and the bulk density was $25kN/m^3$. So the weight of unit length of the secondary lining was $281.25kN$. The force produced by the weight of secondary lining acting on the tunnel invert could be calculated by the following formulas:

Shear force: $Q_2=(281.25/2)\times cos\theta_0=125$ (kN)
Axial force: $N_2=(281.25/2)\times sin\theta_0=63$ (kN)

Therefore, the total force acting on both ends of the tunnel invert were:

Shear force: $Q_0= Q_1+ Q_2=467$ (kN)
Axial force: $N_0= N_1+ N_2=1296$ (kN)
Bending moment: $M_0=591$ (kN·m)

Settlement of tunnel bottom and pressure of rock mass when tunnel foundation was not strengthened

When the tunnel foundation was not reinforced, the tunnel foundation was soft soil, and the foundation stiffness was small, and the coefficient of foundation reaction was selected according to the empirical value [1], $K_1=8000 \ kN/m^3$. The MATLAB program was compiled to solve the Equations (22) ~ (24), and the integral constant C_0, C_1 and C_4 in differential equations could be obtained.

$$C_0=12.39\times 10^{-3}, \quad C_1=4.16\times 10^{-3}, \quad C_4=7.95\times 10^{-3}$$

We could get the Equation (26) ~ (29) when we substitute C_0, C_1 and C_4 into equation (17) ~ (20):

$$y(\theta) = [12.39 + 4.16 ch\alpha\theta cos\beta\theta + 7.95 sh\alpha\theta sin\beta\theta]\times 10^{-3} \tag{26}$$

$$Q(\theta) = T[4.16(A_1 sh\alpha\theta cos\beta\theta + A_2 ch\alpha\theta sin\beta\theta) + 7.95(A_1 ch\alpha\theta sin\beta\theta - A_2 sh\alpha\theta cos\beta\theta)]\times 10^{-3} \tag{27}$$

$$N(\theta) = [-12.39 Kr_1 - 4.16(2\alpha\beta T sh\alpha\theta sin\beta\theta) + 7.95(2\alpha\beta T ch\alpha\theta cos\beta\theta)]\times 10^{-3} \tag{28}$$

$$M(\theta) = [12.39(Kr_1 B_1 +1) + 4.16(2\alpha\beta)(B_1 T -1) sh\alpha\theta sin\beta\theta - 7.95(2\alpha\beta)(B_1 T -1) ch\alpha\theta cos\beta\theta]\times 10^{-3} / B_2 \tag{29}$$

From Equations (26) to (29), the deflection, shear force, axial force and bending moment of any section of the tunnel invert can be obtained.

Internal force of tunnel invert

We calculated the internal force of any section of tunnel invert by Equation (27) ~ (29). Considering the symmetry of tunnel invert, half of the tunnel structure was calculated. The internal force diagrams were shown in Figure 3~ Figure 5.
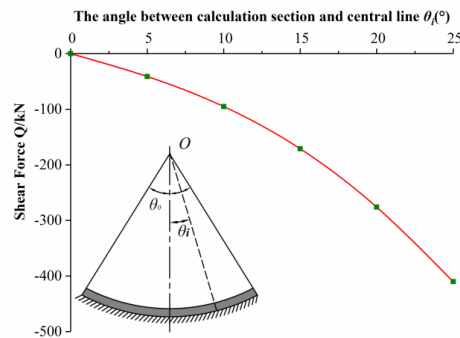

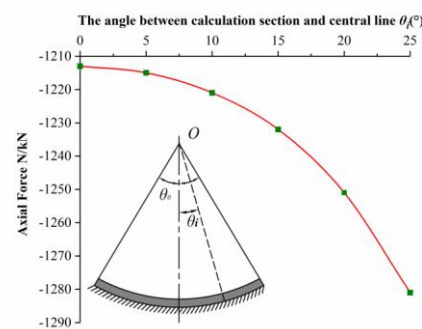*Fig.3 - The shear force of tunnel invert (kN)*

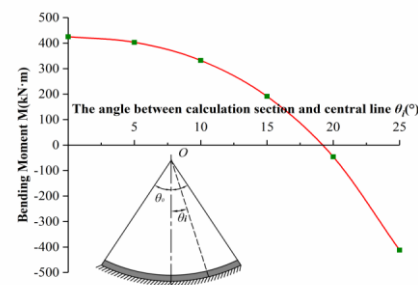*Fig.4 - The axial force of tunnel invert (kN)*

Fig.5 - The bending moment of tunnel invert (kN· m)

As shown in Figure 3 to 5, the center and the waist of tunnel invert bore the tension stress, and the lateral bore compressive stress. However, the outer sides bore tension stress and the inner sides bore compressive stress at the corner of tunnel invert. The internal force at the corner of tunnel invert was greater, and there was stress concentration phenomenon.

The axial force of the tunnel invert was relatively large, and the maximum axial force was located at the corner of tunnel invert, which was *-1282kN*, and the direction was pointing to the inside of tunnel. The shear force of the tunnel invert was relatively small, and the maximum shear force was located at the corner of tunnel invert, which was *-410kN*. The bending moment was relatively large, and the positive moment *425kN· m* was located at the center of the tunnel invert. The force form was not conducive to the stability of the tunnel invert, and the inside of the tunnel invert was easy to be pulled to crack.

Settlement of tunnel bottom

The radial displacement of any section of tunnel invert could be calculated by equation (26), the vertical displacement of different sections of tunnel invert could be obtained by formula $S_i = y_i \cos \theta_i$, the calculation results were shown in Table 3.

Tab.3 - Settlement of tunnel bottom (mm)

The angle between the calculation section and central line $\theta_i(°)$	Horizontal distance between calculation section and tunnel invert center L（m）	Radial displacement y（mm）	Settlement of tunnel bottom S（mm）
0	0	16.6	16.6
5	1.1	17.3	17.2
10	2.1	19.4	19.1
15	3.2	22.5	21.8
20	4.2	26.2	24.6
25	5.2	29.4	26.7

As shown in Table 3, the settlement of tunnel bottom was generally large when the tunnel foundation of soft loess was not reinforced. The settlement reached *26.7mm* at the corner of tunnel bottom, and the settlement at the center of the tunnel bottom reached *16.6mm*. The maximum differential settlement within the cross-section of tunnel bottom was as follows:

$$\Delta S_{\max} = 26.7\text{-}16.6 = 10.1 \text{(mm)}$$

The horizontal distance between the center and corner point of tunnel bottom was *5.2m*, therefore, the differential settlement of unit length was as follows:

$$\Delta S_{\max}{}' = \Delta S_{\max}/L = 1.94 \text{(mm/m)}$$

There is a big difference between the center and corner point of tunnel bottom, so the tunnel invert was easy to crack. According to the distribution of the bending moment of the tunnel invert, the risk of cracking at the center of tunnel invert was the biggest.

Pressure of rock mass of the tunnel bottom

After calculating the radial displacement *y* of the tunnel invert, the elastic resistance of the tunnel foundation to the tunnel invert could be obtained by the formula (1). The normal elastic resistance was approximately equal to the pressure of rock mass of the tunnel bottom, assuming that the frictional force was not taken into account. The calculation results of pressure of rock mass were shown in Figure 6.
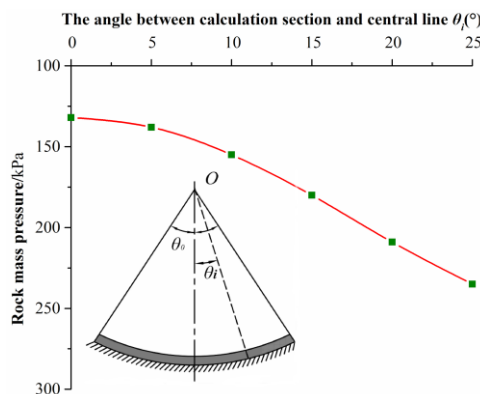

Fig.6 - pressure of rock mass of the tunnel bottom (kPa)

As shown in Figure 6, rock mass at the tunnel bottom was mainly subjected to compressive stress. The compressive stress at the center of tunnel invert center was the minimum, *132kPa*. The pressure of rock mass increased uniformly from the center to the corner of tunnel invert, and the maximum compressive stress was reached *235kPa* at the corner of tunnel invert. The stress concentration was easy to occur due to the structural reasons at the corner of tunnel invert. The weight of secondary lining and the pressure of rock mass acted at the corner of tunnel invert, and the corner of tunnel invert bore the large load. There should be smooth transition between side wall and the corner of tunnel invert, and the section size at the corner of tunnel invert could be increased appropriately to improve the force situation in the process of tunnel construction.

Settlement and pressure of rock mass of foundation reinforcement by high pressure jet grouting piles

When the tunnel foundation was reinforced by high pressure jet grouting piles, the bearing capacity and the stiffness of the tunnel foundation were greatly improved. The coefficient of foundation reaction was determined by field loading test, $K_2=60000\ kN/m^3$. The MATLAB program was compiled to solve the equations (22) ~ (24), and the integral constant C_0, C_1 and C_4 in differential equations could be obtained.

$$C_0=16.6\times10^{-4},\quad C_1= -1.4\times10^{-4},\quad C_4=6.8\times10^{-4}$$

We can get the equation (30) ~ (33) when substitute C_0, C_1 and C_4 into equation (17) ~ (20):

$$y(\theta) = [16.6 - 1.4ch\alpha\theta cos\beta\theta + 6.8sh\alpha\theta sin\beta\theta]\times10^{-4} \tag{30}$$

$$Q(\theta) = T[-1.4(A_1 sh\alpha\theta cos\beta\theta + A_2 ch\alpha\theta sin\beta\theta) + 6.8(A_1 ch\alpha\theta sin\beta\theta - A_2 sh\alpha\theta cos\beta\theta)]\times10^{-4} \tag{31}$$

$$N(\theta) = [-16.6Kr_1 + 1.4(2\alpha\beta Tsh\alpha\theta sin\beta\theta) + 6.8(2\alpha\beta Tch\alpha\theta cos\beta\theta)]\times10^{-4} \tag{32}$$

$$M(\theta) = [16.6(Kr_1B_1+1) - 1.4(2\alpha\beta)(B_1T-1)sh\alpha\theta sin\beta\theta - 6.8(2\alpha\beta)(B_1T-1)ch\alpha\theta cos\beta\theta]\times10^{-4} / B_2 \tag{33}$$

From equations (30) ~ (33), the deflection, shear force, axial force and bending moment of any section of the tunnel invert could be obtained.

Internal force of tunnel invert

We calculated the internal force of any section of tunnel invert by equation (31) ~ (33). Considering the symmetry of tunnel invert, half of the tunnel structure was calculated. The internal force diagram was shown in Figure 7~ Figure 9.
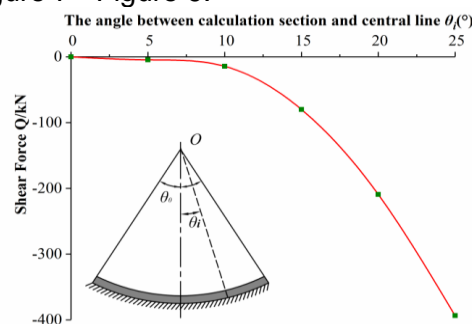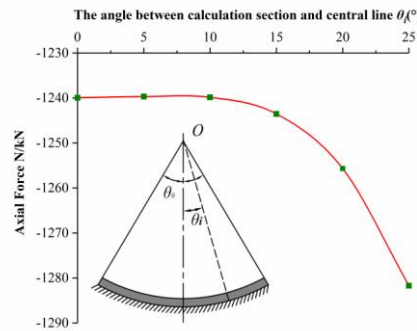

Fig.7 - The shear force of tunnel invert (kN)

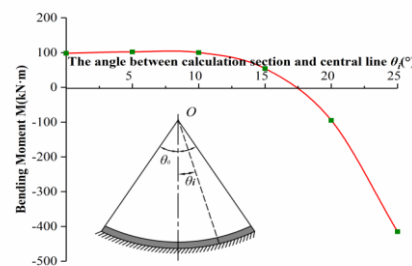*Fig.8 - The axial force of tunnel invert (kN)*

Fig. 9 - The bending moment of tunnel invert (kN.m)

As shown in Figure 7~ Figure 9, after the reinforcement of the tunnel foundation, the internal force of the tunnel invert was similar to that without reinforcement. The inside at the center and the waist of tunnel bottom bore the tension stress, and the lateral bore compressive stress. The outer side at the corner of tunnel invert bore tension stress, and the inner side bore compressive stress. However, the stress condition of tunnel invert had been improved. The axial force at the center the tunnel invert increased slightly and the shear force and bending moment decreased. The maximum positive bending moment at the center of the tunnel invert decreased to $98kN \cdot m$, which greatly improved the stress state of the tunnel invert.

Settlement of tunnel bottom

The radial displacement of any section of tunnel invert could be calculated by equation (30), the vertical displacement of different sections of tunnel invert could be obtained by formula $S_i = y_i \cos\theta_i$, the calculation results were shown in Table 4.

Tab. 4 - Settlement of tunnel bottom (mm)

The angle between the calculation section and central line θ_i(°)	Horizontal distance between calculation section and tunnel invert center L（m）	Radial displacement y（mm）	Settlement of tunnel invert S（mm）
0	0	1.5	1.5
5	1.1	1.7	1.7
10	2.1	2.2	2.2
15	3.2	3.1	3.0
20	4.2	4.1	3.9
25	5.2	4.7	4.3

As shown in Table 3 and Table 4, the settlement of the tunnel bottom was greatly reduced when the tunnel foundation was reinforced by jet grouting piles. The settlement at the corner of tunnel invert decreased to *4.3mm*, and the differential settlement of unit length was as follows:

$$\Delta S_{max}{}^{'} = \Delta S_{max} / L = 0.54 (mm/m)$$

The settlement of the tunnel invert was smaller and the overall settlement was more harmonious, which met the requirements of the standard. Thus, the bearing capacity and stiffness of tunnel foundation were greatly improved after the jet grouting piles was used to reinforce the tunnel foundation, and the reinforcement effect was good.

Pressure of rock mass of the tunnel bottom

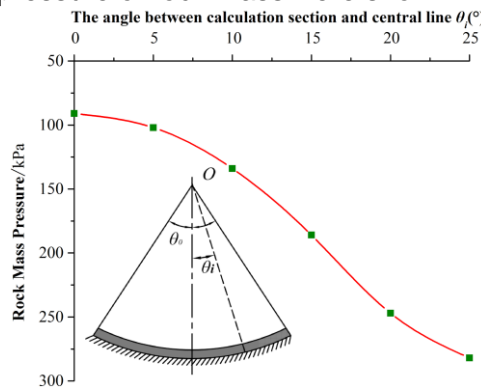The calculation results of pressure of rock mass were shown in Figure 10.

Fig.10 - Pressure of rock mass of the tunnel bottom (kPa)

As shown in Figure 10, the pressure of rock mass at tunnel bottom was similar to that without reinforcement, the compressive stress at the center of tunnel invert was the smallest, and the maximum compressive stress at the corner of tunnel invert, after foundation reinforcement, the compressive stress at the corner of tunnel invert increased more obviously, and increased from *235kPa* to *282kPa*. The stress concentration phenomenon at the corner of tunnel invert was more obvious, which was related to the substantial increase of the bearing capacity of the tunnel foundation after the foundation reinforcement.

CONCLUSION

(1)   When the tunnel foundation was not strengthened, the inside at the center and the waist of tunnel invert bore the tension stress, and the lateral bore compressive stress; the outside at the foot of tunnel invert bore tension stress and the inside bore compressive stress; the internal force at the foot of tunnel invert was greater, the axial force and bending moment of the tunnel invert were relatively large, and the greater positive moment was the inside at the center of the tunnel invert. The maximum positive bending moment at the center of the tunnel invert decreased greatly after the reinforcement of the tunnel foundation.

(2)   When the tunnel foundation was not strengthened, the settlement of tunnel bottom was generally large; the settlement at the corner of tunnel invert reached *26.7mm*, and the maximum differential settlement within the range of tunnel invert reached *10.1mm*; the risk of tension crack at the center of tunnel invert was larger. After tunnel foundation was strengthened, the settlement of tunnel bottom decreased greatly, and the overall settlement was more coordinated, and the reinforcement effect was good.

(3)   When the tunnel foundation was not strengthened, theoretical calculation results showed that the rock mass at tunnel bottom was mainly subjected to compressive stress; the compressive stress at the center of the tunnel invert was minimum, and it was large at the corner of tunnel invert. After the tunnel foundation was strengthened, the pressure of rock mass at tunnel bottom was similar to

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that without reinforcement; the compressive stress at the corner of tunnel invert increased obviously and the stress concentration phenomenon was more obvious.

OMPETING INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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