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HYBRID CONTINUUM-DISCONTINUUM MODELLING OF ROCK FRACUTRE PROCESS IN BRAZILIAN TENSILE STRENGTH TEST

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ABSTRACT

A hybrid continuum-discontinuum method is introduced to model the rock failure process in Brazilian tensile strength (BTS) test. The key component of the hybrid continuum-discontinuum method, i.e. transition from continuum to discontinuum through fracture and fragmentation, is introduced in detail. A laboratory test is conducted first to capture the rock fracture pattern in the BTS test while the tensile strength is calculated according to the peak value of the loading forces. Then the proposed method is used to model the rock behaviour during BTS test. The stress propagation is modelled and compared with those modelled by finite element method in literatures. In addition, the crack initiation and propagation are captured and compared with the facture patter in laboratory test. Moreover, the force-loading displacement curve is obtained which represents a typical brittle material failure process. Furthermore, the stress distributions along the vertical direction are compared with the theoretical solution. It is concluded that the hybrid continuum-discontinuum method can model the stress propagation process and the entire rock failure process in BTS test. The proposed method is a valuable numerical tool for studying the rock behaviour involving the fracture and fragmentation processes.

KEYWORDS

Hybrid Continuum-Discontinuum, Brazilian Tensile Strength Test, Rock Failure Process, Crack Initiation and Propagation

INTRODUCTION

Understanding the rock failure behaviour can benefit many industrial operations, e.g. mining, tunnelling and demolition of urban structures. Nowadays, the rock behaviour is studied through laboratory test and numerical methods. With the development of the computer technologies, it is possible to carry out large-scale calculation in short time. Thus, numerical methods are widely employed to study the rock failure behaviours in various conditions [1-4].

Numerical methods can be classified according to the hypothesis that the rock is modelled as a continuous or discontinuous materials. Therefore, there are three kinds of numerical methods, i.e. continuum method, discontinuum method, and combined continuum-discontinuum method. As the rock can be considered as continuous material before fractures and cracks occur while it can be treated as discontinuous material after fracture and fragmentation, the combined continuum-discontinuum method might be a better choice to model the entire rock failure processes.

In the paper, a combined continuum-discontinuum method, i.e. hybrid finite-discrete element method is employed to model the rock failure process in Brazilian tensile strength (BTS) test. On the one hand, the modelling of the rock failure process in BTS test is used to calibrate the proposed method since the BTS test is widely studied and a laboratory test is conducted in this research. On the other hand, the modelled failure process of BTS test can be used to demonstrate the capabilities of the proposed method in capturing the rock behaviours under static loading.

Hybrid FEM/DEM Method

Since a hybrid finite-discrete element method merges finite element tools and techniques with discrete element algorithms [5], it is considered to combine the advantages of finite element method in modelling the continuum materials and the discrete element method in simulating the discontinuum materials. In the hybrid FEM/DEM, a typical numerical model is considered to consist of a single discrete body or a number of interactive discrete bodies within general shapes and sizes, each of which is represented by a single discrete element and the interaction of them is governed by contact law [6]. In addition to interacting with each other, each discrete element is discretized into finite elements to capture the transition from continuum to discontinuum i.e. deformability fracture and fragmentation [6]. As the fracture requires activation and growth of many flaws and cracks to complete the failure process, it is reasonable to describe the process in a continuous way considering the continuum and discontinuum of material and transition between them.

The key components of the hybrid FEM/DEM include contact detection and interaction among those individual discrete bodies, deformability and transition from continuum to discontinuum through fracture and fragmentation of individual discrete bodies, temporal integration scheme and computational fluid dynamics [6].

A hybrid FEM/DEM method consists of these essential components has been implemented by Liu et al. [6] on the basis of their previous enriched finite element codes RFPA-RT2D [7] and TunGeo3D [8], and the open-source combined finite-discrete element libraries Y2D and Y3D originally developed by Munjiza [5] and Xiang et al. [9], respectively, which is to be used in this study.

The modelling of rock fracture in Brazilian test involves the fracture and fragmentation of rocks. Correspondingly, the transition from continuum to discontinuum is the key components for the rock fracture modelling in Brazilian test. Therefore, it is introduced briefly here while the detailed introduction can be found in a recent paper published by the authors [10].

Transition from continuum to discontinuum

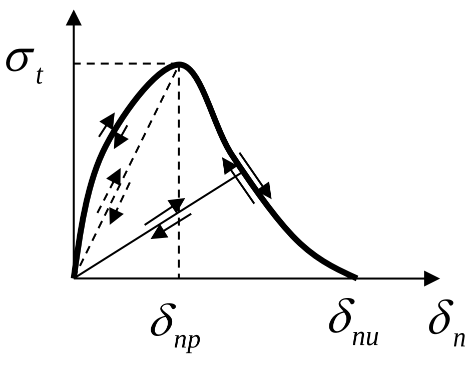
In the hybrid FEM/DEM, a discrete element is discretized into finite elements while those finite elements are bounded together through a four-node joint element. The transition from continuum to discontinuum, i.e. fracture and fragmentation of rock mass, is implemented through the separation of the joint elements while bonding stresses are involved in the separating processes of joint elements. In the hybrid FEM/DEM, a bonding stress is taken to be a function of the separation between the surfaces of joints elements and cracks are assumed to coincide at the surfaces of the joint elements during fracturing.

The separation δ at any point can be divided into two components in Equation 1

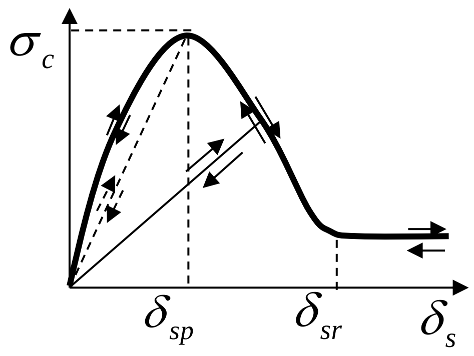
$$\delta = \delta_n n + \delta_s t \tag{1}$$

where n and t are the unit vectors in the normal and tangential directions, respectively, of the surface at such a point, δ_n and δ_s are the magnitudes of the components of δ on the normal and tangential directions, respectively.

Figure 1 illustrates the relationship between the bonding stress and opening/sliding displacement under tension and shear conditions. If the separations in the normal direction δ_n or in the tangential direction δ_s, reach critical displacements i.e. δ_{np} in normal direction or δ_{sp} in tangential direction, determined by the tensile strength σ_t or the shear strength σ_c of the element, damages are assumed to occur. During this period, the boding stresses are increasing. Then the separations of the joint element continue to increase, i.e. $\delta_n > \delta_{np}$ in normal direction or $\delta_s > \delta_{sp}$ in tangential direction, while the boding stresses gradually decrease. When the opening of the joint element in normal direction is beyond the ultimate opening displacement δ_{nu}, the joint element is broken and the tensile failure is assumed to occur. The opining of the joint element in tangential direction exceeds a residual opening displacement δ_{sr}, the shear failure is assumed to occur and the boding stress becomes a purely frictional resistance. Equation 2 illustrates the relationship between the bonding stress and the opening of the joint element in normal direction for tensile failure while the Equation 3 indicates the relationship between the bonding stress and the opening of joint element in tangential direction for shear failure.



a) Under tension conditions



b) Under shear conditions

Fig. 1 - Relationship between the bonding stress and opening/sliding displacement under tension and shear conditions

$$\sigma_n = \begin{cases} \left[2\dfrac{\delta_n}{\delta_{np}} - \left(\dfrac{\delta_n}{\delta_{np}} \right)^2 \right] \sigma_t, & if \quad 0 \le \delta_n \le \delta_{np} \\ f(D)\sigma_t, & if \quad 0 \le \delta \le \delta_{nu} \\ 0, & if \quad \delta \ge \delta_{nu} \end{cases} \tag{2}$$

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$$\tau = \begin{cases} \left[2\dfrac{\delta_s}{\delta_{sp}} - \left(\dfrac{\delta_s}{\delta_{sp}} \right)^2 \right] \sigma_c, & if \quad 0 \le \delta_s \le \delta_{sp} \\ g(D)\sigma_c, & if \quad \delta_{sp} \le \delta_s \le \delta_{sr} \\ \sigma_n \tan(\theta_f), & if \quad \delta_s \ge \delta_{sr} \end{cases} \tag{3}$$

where D is a damage variable between 0 and 1, $f(D)$ and $g(D)$ are damage functions described in the mechanical damage model [7], and θ_f is the joint residual friction angle.

Experimental study of the rock fracture in Brazilian dis test

In the last few decades, the BTS test has been developed mainly for determining the tensile strength of the rock and rock like materials [11, 12]. The BTS test is first proposed by AKazawa (1943) and Carneiro (1943) independently to calculate tensile strength of rock materials [11, 12]. Due to the relatively uniform distribution of the tensile stress along the loaded diameter [13] and its simplicity to prepare, this method is widely accepted as an indirect method to determine tensile strength of rocks.

The conventional equation for determining tensile strength using BTS test is given as follows [14, 15].

$$\sigma_t = \frac{2P}{\pi Dt} \tag{4}$$

where σ_t is the tensile, P is the load at failure, D is the diameter of the test specimen, t is the thickness of the test specimen measured at the centre.

In addition to the tensile strength, the stress distribution of BTS test is also widely studied. Hondros (1959) [16] gave a complete stress solution for Brazilian disc under diametrical compression valid for both plane stress and plan strain condition.

$$\sigma_{xx} = \frac{P}{\pi Rt\alpha} \left\{ \frac{\left[1-(\frac{r}{R})^2\right] Sin2\alpha}{1-2(\frac{r}{R})^2 Cos2\alpha + (\frac{r}{R})^4} - tan^{-1}\left[\frac{1+(\frac{r}{R})^2}{1-(\frac{r}{R})^2}\tan(\alpha)\right] \right\} \tag{5}$$

$$\sigma_{yy} = -\frac{P}{\pi Rt\alpha} \left\{ \frac{\left[1-(\frac{r}{R})^2\right] Sin2\alpha}{1-2(\frac{r}{R})^2 Cos2\alpha + (\frac{r}{R})^4} + tan^{-1}\left[\frac{1+(\frac{r}{R})^2}{1-(\frac{r}{R})^2} tan\alpha\right] \right\} \tag{6}$$

where P is the applied load, R is the disc radius, r is the distance from centre of the disc, t is the disc thickness, 2α is the angular distance of load arc, σ_{xx} and σ_{yy} are stresses along the horizontal and vertical directions respectively

The fracture initiation and propagation of Brazillian disc test were also investigated. For a valid Brazilain disc test for measuring the tensile strength, the failure is supposed to originate at the centre of the disc and the guarantee of crack initiation at the centre of the specimen is that the loading angle corresponding to the flat end width must be greater than a critical value. However, the fracture still may initiates away from the centre of the test disc [17] or occurs at the loading points [18].

In this section, the BTS test is conducted in the laboratory to capture the fracture pattern of Brazilian disc under diameter forces. Figure 1 shows the simple of granite rock. The diameter of the disc is 50mm while the thickness of the disc is 25mm.

Fig.1 - Rock simple for Brazilian test

Figure 2 illustrates hydraulic testing machine which is used to conduct the BTS test. The disc is placed between two curved loading jaws. The curved loading jaw moves at the speed of 0.02mm/min.

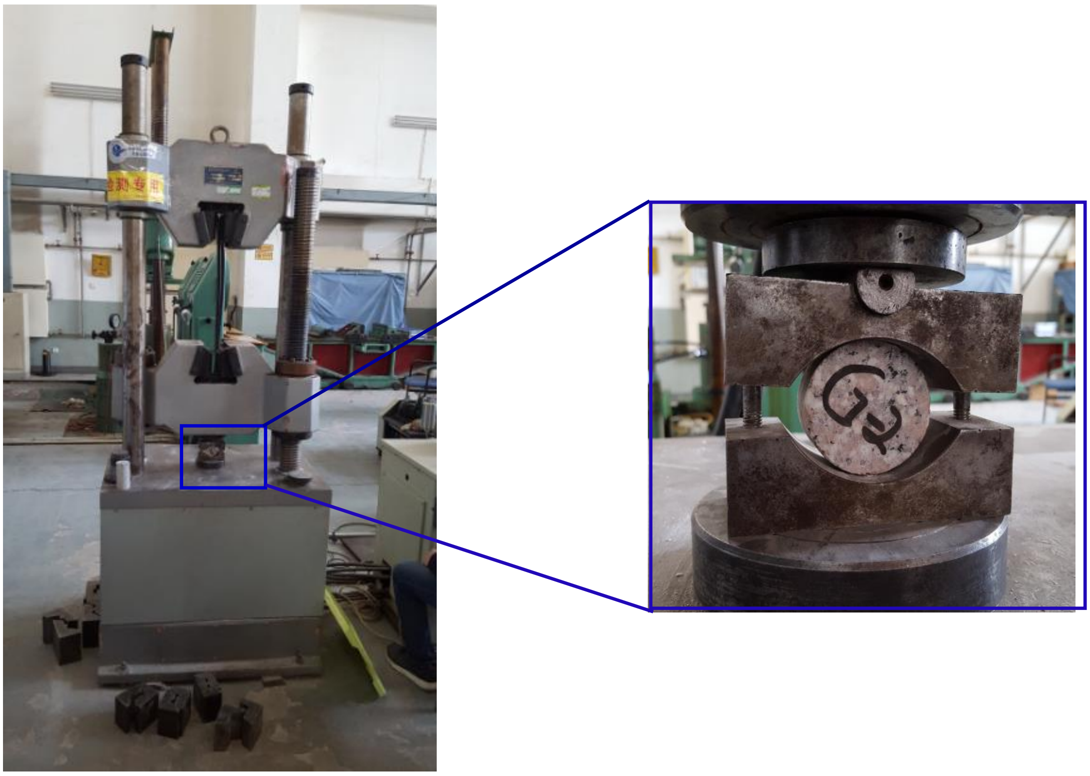


Fig.2 - Hydraulic testing machine for Brazilian test

Fig.3 - Fracture pattern of BTS test

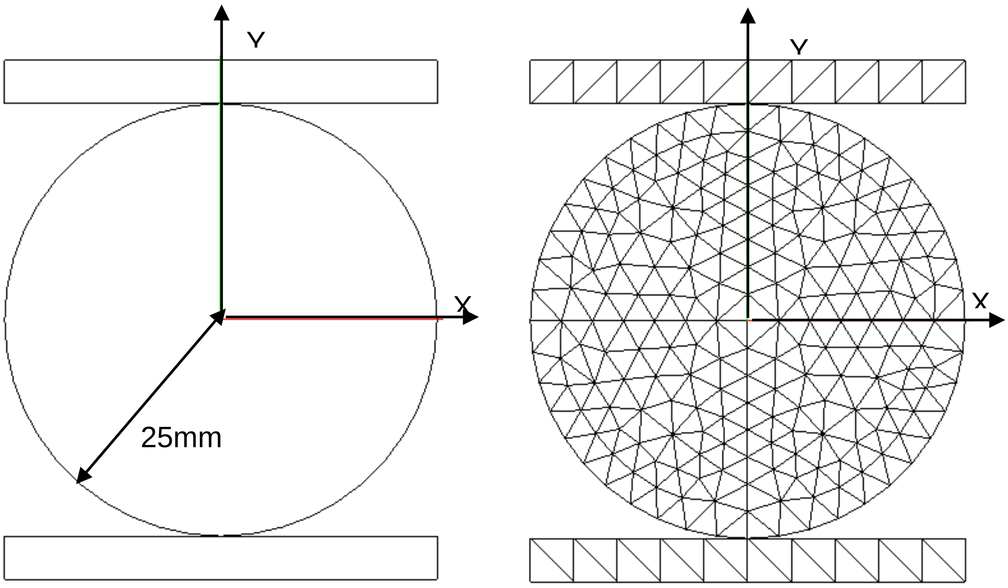
Figure 3 demonstrates rock failure pattern of the BTS test. It can be seen that the rock crack is the loading diameter. According to the fracture initiation and propagation of BTS test investigated by Colback (1966), the experimental result indicates that the experiment in this research is valid for calculating the tensile strength of the sample as the crack is along the loading diameter.

The recorded loading force is 24.79KN, which can be used to calculate the tensile strength of the sample. According to Equation 4, the tensile strength of the sample is calculated as follows.

$$\sigma_t = \frac{2P}{\pi Dt} = \frac{2\times 24.79KN}{3.14\times 0.05\times 0.025} = 12.63Mpa \tag{7}$$

Hybrid finite-discrete element modelling of the rock failure process in BTS test

In this section, a hybrid finite-discrete element method is employed to model the rock fracture initiation and propagation of the BTS test.



(a) Geometrical model *(b)Numerical model*

Fig.4 - Geometrical and Numerical model for BTS test

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The simulation of the BTS test is simplified as a plan strain problem and only the vertical direction is considered. Figure 4a indicates the geometrical model while Figure 4b illustrates the numerical model. It can be seen in Figure 4a that the diameter of the disc is 25mm and the sample is placed between two loading plates. The numerical model shown in Figure 4b is discretised into finite elements and the fracture and fragmentation modelling is implemented through separation of finite elements.

The material properties of the rock mass in the experimental test (Figure 1) are adopted for the numerical model and the detailed parameters are obtained through BTS test and uniaxial compression strength test. The materials properties of the rock specimen are Young's modulus $E = 60\,GPa$, Poisson's ratio $v = 0.26$, density $\rho = 2600\,Kg\,m^{-3}$, tensile strength $\sigma_t = 12\,MPa$, compressive strength $\sigma_c = 120\,MPa$, internal friction angle $\Phi = 30^{\circ}$, surface friction coefficient $u = 0.1$. The material properties of the loading plates follow those of standard steels. During the modelling, two palates move towards each other at 0.1m/s. It should be noted that loading rate is bigger than that in laboratory test. However, the applied loading rate will not significantly influenced the rock properties according to the dynamic rock fracture test by Zhang (2001) [19].

Numerical modelling of the stress wave propagation process

The modelled temporal and spatial distribution of the stress wave under constant displacement increment loading of 0.1m/s are depicted in Figure 5. The stress distributions in the Figure 5 correspond to the minor principal stress while the size can be refereed to colour legend shown at the top of the Figure 5.

As the loading plates contact the sample, the stress waves are induced immediately at the two contact areas as shown in Figure 5 at 0.5μs. Then the stresses at the two loading ends propagate toward each other at meet at 32.5μs. As can be seen that the stress is mainly distributed along the loading line and the stress becomes more and more intensive as shown in Figure 5 at 220.5μs. When the stress along the loading is strong enough, i.e. the tensile stress along the horizontal direction meets the tensile strength of the sample, tensile failure occurs as shown in Figure 5 at 260μs. After that, the stress still is mainly distributed along the loading line while it is scattered in the disc after the failure completes (Figure 5 from320~450μs).

Numerical modelling of the crack initiation and propagation

Figure 6 demonstrates the crack initiation and propagation process in the BTS test. In Figure 6, the red colour represents the tensile failure while the blue colour represents the shear cracks and the boundary of the sample.

As indicated in Figure 5 that the tensile stresses exceed the tensile strength of the material at 261.5μs, the tensile failure occurs at almost the centre of the disc (Figure 6 at 261.5μs).Then the cracks propagate along the loading diameter toward to the loading plates. At 290 μs, the cracks initiated from the centre of the disc arrived at the loading areas. As the stresses continue to accumulate at the loading areas in the top and bottom of the disc, the compressive stress exceeds the compressive strength of the sample. Thus, shear failure occurs at the loading vicinity, i.e. the contact areas of the loading palates with the disc, as shown in Figure 6 at 305μs. As the loading plates continue to move toward each other, the tensile cracks along the loading diameter expand due to the tensile stresses along the horizontal direction induced by compression of the disc on the vertical direction. In addition, the shear cracks at the loading vicinities propagate approximately parallel to the loading diameter (Figure 6 from 320 to 450μs).

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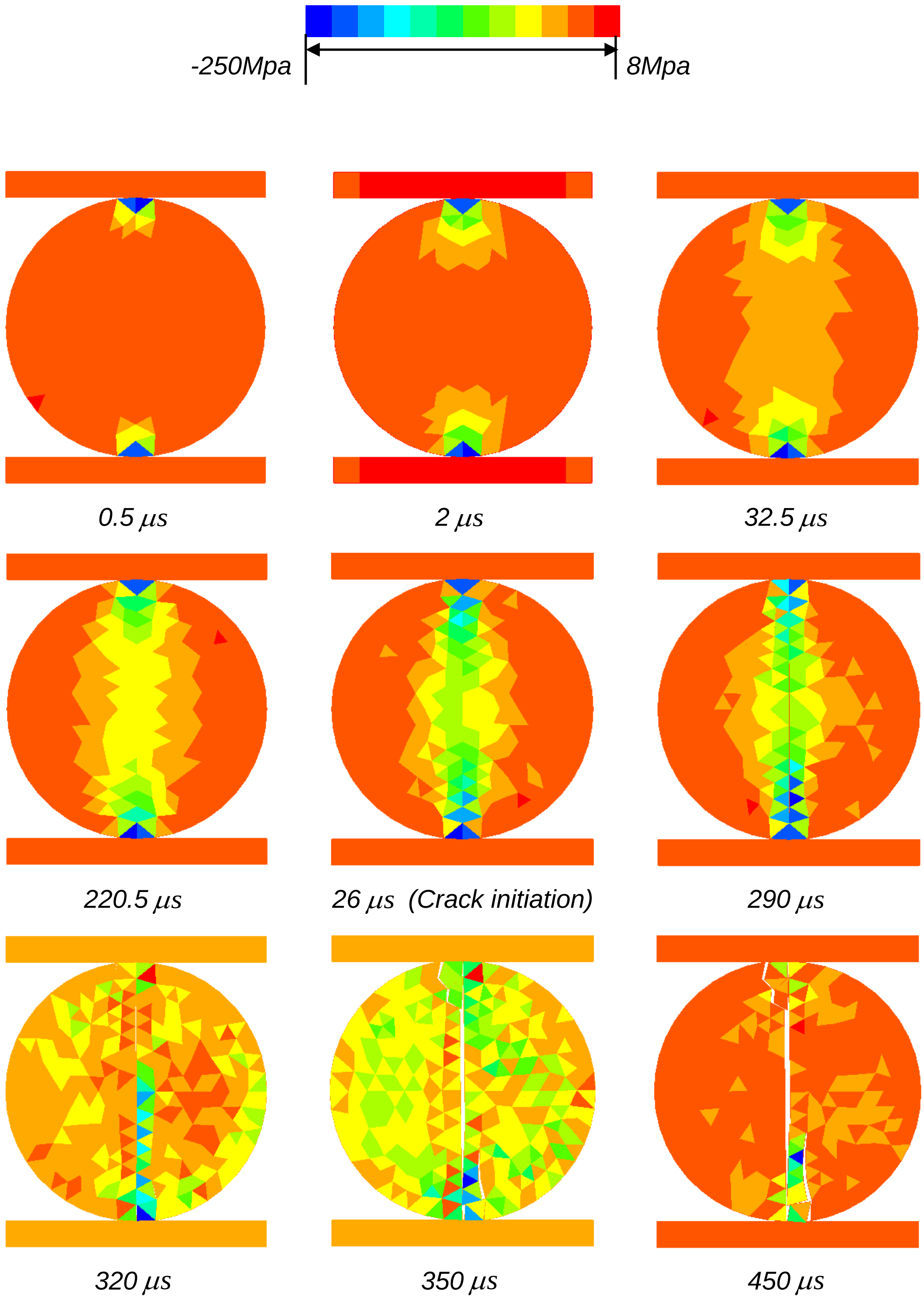


Fig.5 - Stress propgation process of BTS test

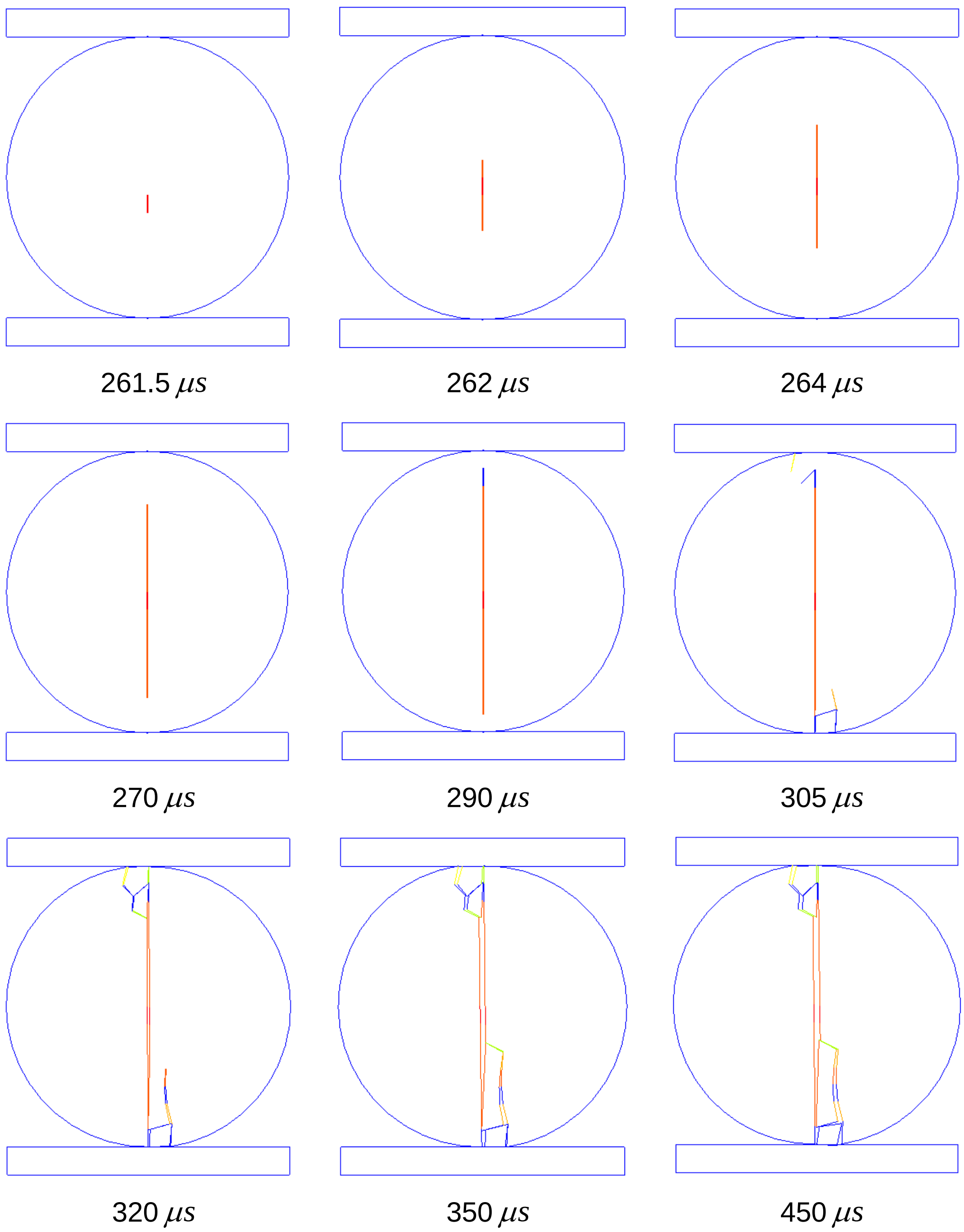
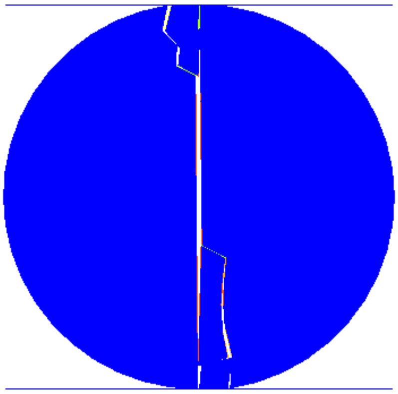


Fig.6 - Crack initiation and propagation of BTS test



(a) Numerical result

(b) Experimental result

Fig.7 - Comparison of the numerical and experiential results

According to experiments conducted by Malan et al. (1994) [20], a typical fracture propagation in BTS test includes primary tensile cracks along the loading diameter and secondary shear cracks from the loading sides parallel to the primary cracks. Thus hybrid finite-discrete element method obtains a valid failure pattern in BTS test. Figure 7 compares the numerical result with the experimental result. In Figure 7a, the blue colour represents rock mass while the white colour represents cracks. As shown in Figure 7, the modelled result agrees well with the experiential result as follows:

- Both the main cracks are along the loading diameters.
- Both the secondary cracks occur at the loading vicinities
- Both the discs are separated into two halves along the vertical diameters.

Force-loading and Stress along the loading diameter analysis

In this section, the relationship between the force-loading from the top plates and the corresponding displacement is analysed while the stresses distribution along the loading line are compared with the theoretical solutions.

Force-loading displacement analysis

Figure 8 shows the relationship between displacements and loading from the top plate. As shown in the Figure 8, the force increases rapidly as the loading plate moves. Initially, the force-loading displacement is almost linearly (Figure 8 A-B-C). Then the force reaches its peak (Figure 8 D). After that the force decreases gradually for a while (Figure 8 D-E) and decreases sharply to the bottom (Figure 8 E-F). Finally, the force drops to approximately zero as the sample completely loses its bearing capability (Figure 8 G).

The force-loading displacement curve of the BTS test represents a typical behaviour of brittle rock under compression: a compressive deformation region (AB), a linear-elastic deformation region

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(BC), a non-linear deformation region (CD), a strain-softening deformation region (DEF) and a residual deformation region (FG).

According to the recorded peak load, i.e. 1.01MN, the tensile strength can be obtained on the basis of Equation 4.

$$\sigma_t = \frac{2P}{\pi Dt} = \frac{2\times1.01\times10^6}{3.14\times0.050\times1} = 12.87\text{Mpa} \qquad (8)$$

The modelled tensile strength, i.e. 12.87Mpa is bigger than the input parameter (12Mpa), which is caused by a relatively bigger loading rate (0.1m/s) adopted.

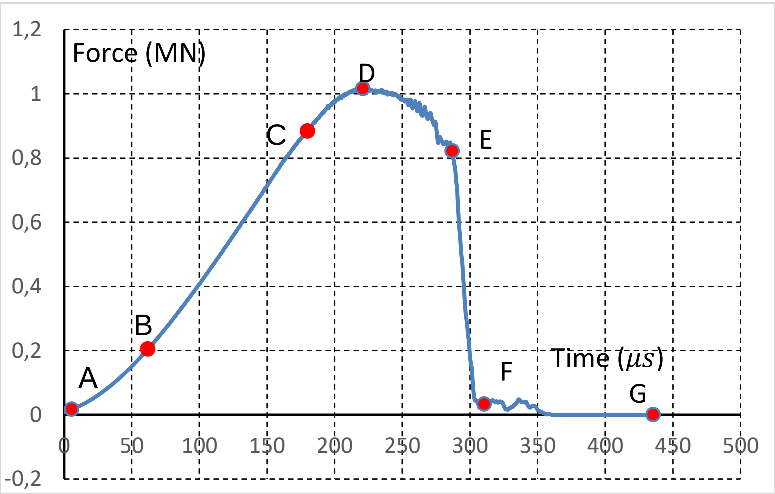

Fig.8 - Force-loading displacement curve

Comparison of the numerical and analytical stresses distribution along the loading diameter

Since the complete stress distribution of BTS test along the loading diameter are given by Hondros (1959), the numerical solution along the loading line is compared with the analytical solution to calibrate the hybrid FEM/DEM method.

The Equation 5 and Equation 6 are used to generate the stress distributions along the loading diameter. In the two equations, P is the peak load, i.e. 1.01MN while 2α is 0.1047, i.e. 6^0 . Figure 9 illustrates the comparison of Hondros` solution according to Equation 5 and Equation 6 and the numerical solution. In terms of stress in the horizontal direction (σ_{xx}), the analytical or theoretical stress along the loading diameter keeps constant and that is the reason BTS test is used to obtain tensile strength indirectly. The numerical stress in the horizontal direction, i.e. the tensile stress, agrees with the analytical solution as the numerical tensile stress keeps almost constant although there is a slightly fluctuation. Additionally, the horizontal stresses for both numerical and analytical solutions become compressive stresses instead of tensile stresses at the loading points probably due the stress concertation. For the stresses in the horizontal direction (σ_{yy}), the numerical solution also shows a good agreement with the analytical or theoretical solution as they both compressive stress and gradually become strong from the centre of the disc to the loading points.

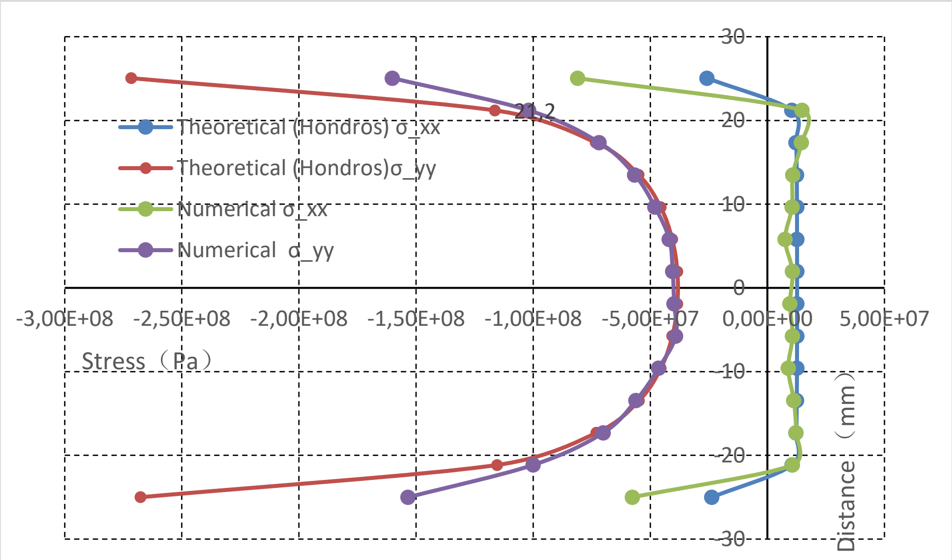

Fig.9 - Theoretical and numerical results of stresses distribution across loaded diameter for dis specimen

CONCLUSION

A combined continuum-discontinuum method, i.e. hybrid finite-discrete element method, is proposed to model the crack initiation and propagation of rock in BTS test. Since the key component, i.e. transition from continuum to discontinuum from fracture and fragmentation, makes the hybrid method superior to the continuum based method, e.g. finite element method, and the discontinuum based method, e.g. discrete element method, the transition from continuum to discontinuum is introduced in detail. Then the proposed method is employed to model the rock failure process in BTS test. In order to calibrate the proposed method, an experimental study is done first to capture the rock failure pattern and to calculate the tensile strength of the rock in BTS test.

The stress propagation is modelled and compared with those well documented in literatures. The crack initiation and prorogation are simulated and the modelled result, i.e. rock failure pattern, shows a good agreement with the experimental study. Additionally, the force-loading displacement curve for the simulation is drawn and it represents a typical brittle material failure process. Furthermore, theoretical and numerical solutions for the stresses on the loading line are compared. The numerical solution agrees well with the theoretical solution although there is a slight fluctuation for the tensile stress along the loading line. It is concluded that

- The hybrid method is able to model stress propagation process in BTS test. The stress mainly concentrates on the loading line and the tensile strength is almost constant although the rock is not an elastic material.
- The hybrid method can capture the entire rock failure process in BTS test and the modelled results show good agreements with the experimental result. Additionally, the crack initially generates at the centre of the loading line and propagates toward the two loading points.
- The hybrid method is a valuable numerical tool in modelling the rock initiation and propagation process.

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