

FINITE ELEMENT MODELLING AND ANALYSIS OF CONCRETE CONFINED BY STIRRUPS IN SQUARE RC COLUMNS

Xiang Zeng^{1,2}

¹ College of Civil Engineering and Architecture, Hainan University, No.58, Renmin Ave., Haikou 570228, China

² Institute of Development on International Tourist Destination, No.58, Renmin Ave., Haikou 570228, China
E-mail: zeng_t08@163.com

ABSTRACT

Concrete confined by stirrups with greater ductility than unconfined concrete has been used widely in reinforced concrete (RC) structures and its behavior is the classic topic. As the computer power is improving, an increasing number of modelling studies of the confined concrete using finite element (FE) methods have emerged in recent years. Aiming at developing a FE model to evaluate the behavior of concrete confined by stirrups in square RC columns, a new uniaxial compression stress-strain relation of concrete considering the confinement effect of stirrups was proposed. In the FE model, the behavior of confined concrete was described by combining the concrete damaged plasticity model with the proposed uniaxial compression stress-strain relation of confined concrete. Then, tested square RC columns confined by stirrups under axial load were simulated and the details of the FE model were described. Though the comparison between the predicted and measured curves of axial load N versus axial strain ε , the proposed uniaxial compressive model of confined concrete was verified. Finally, a parametric study of the effects of strength of stirrup and equal strength replacement of stirrups on the behavior of confined concrete was conducted.

KEYWORDS

Confined concrete; Confinement effect of stirrups; Uniaxial compression model; Finite element modelling; Parametric study

INTRODUCTION

Confined concrete with high volumetric stirrup ratio has been used widely in RC structures due to its higher strength and ductility caused by the confinement pressure of the stirrups. Many experiments on the behaviors of confined RC column under axial load have been carried out [1-4]. Based on the experimental studies, various empirical or semi-empirical formulas have been developed for describing the uniaxial compression stress-strain relation of stirrup-confined concrete by using the regression analysis or simplified theoretical studies [5-11]. As shown in Figure 1, those formulas describe the same characteristics of confined concrete, namely, higher peak stress and peak strain and much gentler descending branch of the stress versus strain relation than those of unconfined concrete. Most of the formulas are very practical.

With the advancement of the computer power, an increasing number of modeling studies of the confined concrete using FE methods have emerged in recent years [12-17]. Most of such

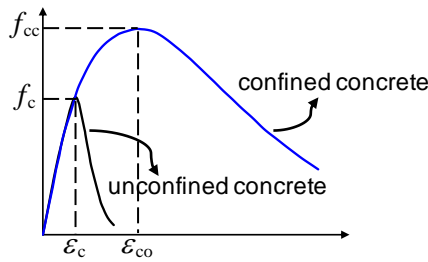


Fig. 1. - Uniaxial stress–strain curve of confined concrete based on experiments

studies have put the emphasis on the development of the constitutive model of confined concrete [12-15] and some of them investigated the effect of certain confinement arrangements on the behaviour of confined concrete [15-17]. As a matter of fact, high fidelity numerical simulation is a powerful means to investigate the mechanism of confining effects of stirrups and influential parameters and it is able to give more details about the mechanical responses of the confined concrete members under different loads. Now, FE modelling using the general-purpose simulation tools, which is much simpler and more accessible than using the complete program code compiled by the investigator self, has been popular in various research fields. So, developing the FE model of confined concrete based on the general-purpose simulation tools is very significant. However, few researchers have paid great attention to the topic [15-16].

Aiming at evaluating the behavior of confined concrete in square RC columns using a FE model based on the general-purpose simulation tool ABAQUS, a new uniaxial compression stress-strain relation of concrete with the confinement of stirrups was proposed in this study. Then, a FE model of RC column with consideration of the confinement effect of stirrups was developed. In the FE model, the behavior of confined concrete was described using the concrete damaged plasticity model into which the proposed uniaxial compression stress-strain relation of confined concrete was introduced. In order to verify the developed FE model, a comparison between the simulation and experimental results of the stirrup-confined RC columns under axial load was carried out. Finally, a parametric study of the effects of strength of stirrup and equal strength replacement of stirrups on the behavior of confined concrete was conducted.

BRIEF DESCRIPTION OF EXPERIMENT

In the paper, three RC columns with volumetric stirrup ratio ρ_v between 0.8% and 2.39% from the experiment conducted by Sheikh and Uzumeri [1] were simulated to verify the following FE model. The length of the test region of the columns, in which the stirrups were placed at specified spacing, was about 533 mm (Figure 2). The scheme of the stirrups in the test region and details of test specimens are presented in Figure 2. To ensure that the failure would occur in the test region, the tapered ends of the column were further confined with the help of welded boxes. In

Tab. 1. - Properties of tested columns

| Specimen | Cylinder strength of concrete f_c (Mpa) | Longitudinal steel | | | Stirrup | | | |
|----------|---|--------------------|---------------|-------------------------------|---------------|--------------|----------------------------------|---------------------------|
| | | Number and size | Diameter (mm) | Yielding strength f_y (Mpa) | Diameter (mm) | Spacing (mm) | Yielding strength f_{hy} (Mpa) | Volumetric ratio ρ_s |
| 2A1-1 | 37.5 | 8-No. 5 | 16 | 372 | 4.8 | 57 | 400 | 0.8% |
| 4A4-8 | 40.8 | 8-No. 7 | 22 | 385 | 4.8 | 29 | 400 | 1.6% |
| 2A5-14 | 31.5 | 8-No. 5 | 16 | 403 | 9.5 | 76 | 400 | 2.4% |

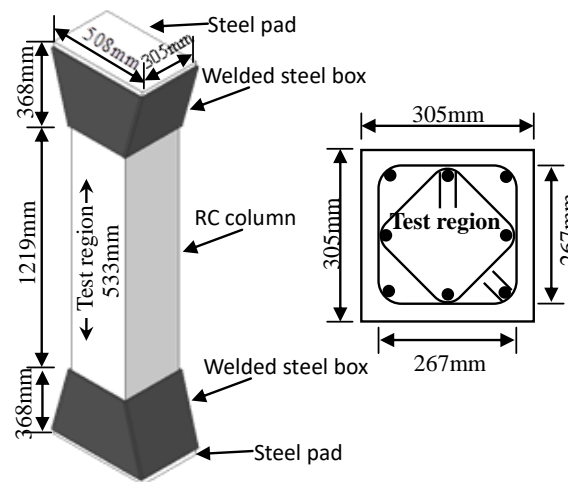


Fig. 2. - Details of test specimens

Table 1, the properties of tested columns were given in detail. All the specimens were applied on a concentric load. More details about the experiment could be seen in the paper by Sheikh and Uzumeri [1].

MATERIAL MODELLING OF CONCRETE

Concrete damaged plasticity model in ABAQUS was used to describe the behaviour of concrete [18]. The model is composed of plasticity model and linearly damaged model. The linearly damaged model can describe the stiffness degradation and stiffness recovery effects associated with stress reversals of concrete under cyclic loading. If no damage parameter is defined, the model is equal to a plasticity model. Under monotonic load, it is unimportant to use the damaged model. So, only the plasticity model is used under monotonic load in this study. The plasticity model is able to consider the strength improving at the state of triaxial loading by the definition of the yielding surface, and the description of the plastic behaviour is related to the equivalent stress-strain relationships of concrete, so taking the empirical or semi-empirical stress-strain relations of confined concrete based on experiments shown in Figure 1 as the equivalent uniaxial compression stress-strain relation in the concrete damaged plasticity model will predict the behaviour of confined concrete incorrectly. It seems that it is difficult to predict reasonably the post-peak behaviour of passively confined concrete in ABAQUS using the stress-strain relation of unconfined concrete [19]. By now, there is no proper uniaxial compression stress-strain relation for simulating the behaviour of stirrup-confined concrete in concrete damaged plasticity model in ABAQUS. Thus, a suitable equivalent uniaxial compressive stress-strain relation of stirrup-confined concrete is important. The author proposed a new equivalent uniaxial compressive stress-strain relation described in the following section. The basic innovation of the proposed compressive stress-strain relation is that it revised the peak strain and descending branch of the stress-strain relation of unconfined concrete by considering the confinement effect of stirrups and it is suitable for simulating the behaviour of stirrup-confined concrete in the concrete damaged plasticity model.

In the material model, the modulus of elasticity of concrete is assumed to be constant for an effective numerical implementation in ABAQUS and is equal to $4730f_c^{0.5}$ determined according to the building code compiled by ACI committee 318 [20], where f_c (N/mm^2) means the cylinder strength of concrete. The Poisson's ratio of concrete is deemed to be constant and is equal to 0.2 [21]. The plastic parameters in the material model for the unconfined and confined concrete including dilation angle, eccentricity, ratio of the biaxial compression strength to uniaxial compression strength of concrete, the ratio of the second stress invariant on the tensile meridian to

that on the compressive meridian and viscosity parameter are 30° , 0.1, 1.16, 0.667, 0.0001, respectively.

A new equivalent uniaxial compressive stress-strain relation of stirrup-confined concrete

A new equivalent uniaxial compressive stress-strain relation of confined concrete in prototype column (as shown in Figure 3) is proposed by the author as following:

$$y = \begin{cases} \alpha_a \cdot x + (3 - 2\alpha_a)x^2 + (\alpha_a - 2)x^3 & (x \leq 1) \\ \frac{x}{\alpha_d \cdot (x-1)^2 + x} & (x > 1) \end{cases} \quad (1)$$

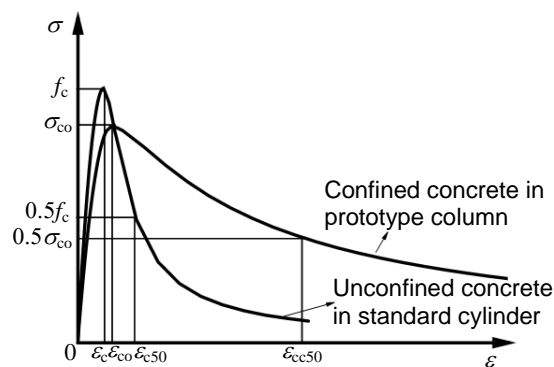


Fig. 3. - Stress-strain relation of confined concrete

In Eq. 1, $x = \varepsilon / \varepsilon_{c0}$ and $y = \sigma / \sigma_{c0}$, σ_{c0} is the peak strength of confined concrete and $\sigma_{c0} = 0.85f_c$ N/mm² by considering a strength-reduction factor related to the column shape, size and the difference between the strength of in situ concrete and the strength determined from standard cylinder tests [4, 9]. ε_{c0} is the peak strain of confined concrete, which is expressed as

$$\varepsilon_{c0} = \varepsilon_c + 800 \cdot I_e^{0.2} \cdot 10^{-6} \quad (2)$$

$$I_e = \rho_{se} f_h / \sigma_{c0} \quad (3)$$

where ε_c is the peak strain of unconfined concrete and its value is taken from *Fib Model Code for Concrete Structures 2010* [21]. I_e is the effective confinement index evaluated at ε_{c0} . f_h is the stress in confinement reinforcement at peak strength of confined concrete. f_h proposed by Le'geron and Paultre [9] is shown as follows:

$$f_h = \begin{cases} f_h = f_{hy} & \kappa \leq 10 \\ f_h = \min(f_{hy}, \frac{0.25\sigma_{c0}}{\rho_{se}(\kappa - 10)}) & \kappa > 10 \end{cases} \quad (4)$$

$$\rho_{se} = k_e A_{sh} / sc \quad (5)$$

$$\kappa = \sigma_{c0} / (\rho_{se} E_s \varepsilon_c) \quad (6)$$

where f_{hy} is the yield strength of stirrups. ρ_{se} is the effective sectional ratio of confinement reinforcement and κ is a parameter used to determine if yielding of transverse reinforcement occurs at peak strength of confined concrete. In Eq. 5, s and c are the spacing of stirrups and the diameter of the core measured centre-to-centre of hoops, respectively; A_{sh} is the total cross section

of stirrups in the one direction perpendicular to one side of the square column within spacing s . In Eq. 6, E_s is the modulus of elasticity of transverse reinforcement. k_e is the geometrical effectiveness coefficient of confinement, which represents the ratio of the smallest effectively confinement concrete area at midway between two layers of stirrups to the nominal concrete core area. k_e is proposed by Mander et al. [7]:

$$k_e = \frac{(1 - \frac{\sum w_i^2}{6c_x c_y})(1 - \frac{s'}{2c_x})(1 - \frac{s'}{2c_y})}{1 - \rho_c} \quad (7)$$

As shown in Figure 4, w_i in Eq. 7 is the i th clear distance between adjacent longitudinal bars. s' is the clear spacing of transverse reinforcement. ρ_c is the ratio of area of longitudinal steel to area of core section. c_x and c_y are the core dimensions to the centrelines of perimeter stirrups in two directions along the two sides of a RC column, respectively, and they are equal to c for square RC column.

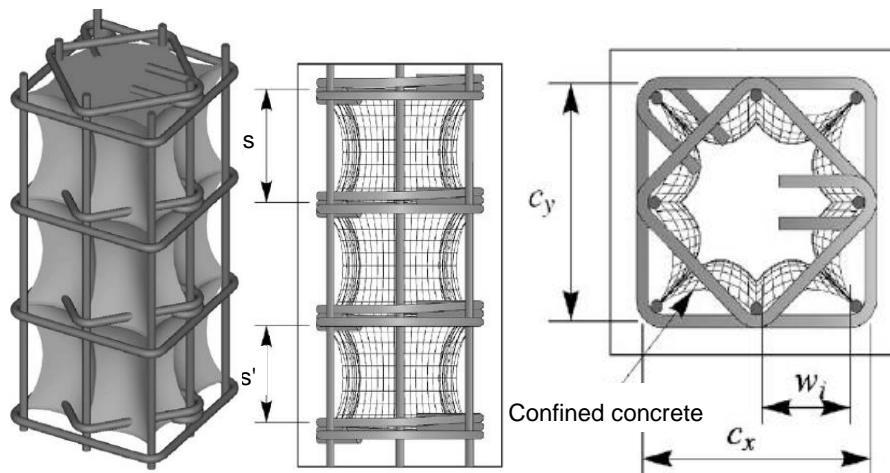


Fig. 4. - Diagram of partial parameters [22]

In Eq. 1, α_a and α_d control the slope of the ascending and descending branches of stress-strain curve. The expressions of α_a and α_d are shown as follows:

$$\alpha_a = 2.4 - 0.0125\sigma_{co} \quad (8)$$

$$\alpha_d = \frac{\varepsilon_{cc50} / \varepsilon_{co}}{(\varepsilon_{cc50} / \varepsilon_{co} - 1)^2} \quad (9)$$

where ε_{cc50} is the post-peak axial strain in confined concrete when capacity drops to 50% of confined strength. Based on the expression of ε_{cc50} proposed by Le'geron and Paultre [9], the modified expression of ε_{cc50} is suggested as:

$$\varepsilon_{cc50} = \varepsilon_{c50} (1 + 80I_{e50}) \quad (10)$$

$$I_{e50} = \rho_{se} f_{hy} / \sigma_{co} \quad (11)$$

in Eq. 10, ε_{c50} is post-peak axial strain of unconfined concrete when capacity drops to 50% of unconfined strength and $\varepsilon_{c50} = 0.004$ according to the proposal by Le'geron and Paultre [9]. I_{e50} is the effective confinement index evaluated at ε_{cc50} .

Uniaxial compression stress-strain relations of cover concrete and concrete confined by steel box

When the two effective confinement indexes I_e and I_{e50} are set to be zero, the Eq. 1 presents the uniaxial compressive stress-strain relation of unconfined concrete, which is used to simulate the behaviour of the cover concrete of RC column.

The uniaxial compression stress-strain relation of concrete confined by steel box presented by Han et al. [23], which considers the confinement effect of the steel tube on the plastic behaviour of concrete, is used to simulate the behaviour of confined concrete at the end of the specimens in Figure 1.

Uniaxial tensile behaviour

The tensile behaviour of concrete is assumed to be linear elastic until the tensile strength [18]. The post failure behaviour is specified by applying a fracture energy cracking criterion. The fracture energy is specified directly as a material property in the model and a linear loss of strength after cracking is assumed. The value of fracture energy G_F in N/m is determined by the expression proposed by the *Fib Model Code for Concrete Structures 2010* [21],

$$G_F = 73f_c^{0.18} \quad (12)$$

where f_c is the compressive strength in MPa.

MATERIAL MODELLING OF STEEL REBAR AND STEEL PAD

Isotropic elastic-plastic model was used to describe the behaviour of the rebar. The stress-strain relation for steel rebar consists of two linear stages (i.e. elastic and hardening) and the hardening modulus was $0.01E_s$, where E_s is the modulus of elasticity of steel rebar. The modulus of elasticity E_s is acquired from the material tests.

The steel pad is considered the elastic material with an elastic modulus of 2.06×10^5 MPa.

FE MODELLING OF CONFINED RC COLUMNS

The FE model was established based on the general-purpose FE software ABAQUS and the module of ABAQUS/Explicit is used to solve the static nonlinear problem.

The steel rebar is modelled using 2-node linear 3-D truss element (T3D2). Both the steel plate and the concrete were modelled as 8-node brick elements (C3D8R). The approximate global mesh size of 50 mm for the concrete body and the approximate global mesh size of 10 mm for the steel cage can provide precise simulation result. The FE model mesh is illustrated in Figure 5.

Embedded region constraint was employed in the FE model to embed the steel reinforcement cage (embedded elements) within the concrete block (host elements). That means the translational degrees of freedom of the embedded node are constrained to the interpolated values of the corresponding degrees of freedom of the host element, but these rotations are not constrained by the embedding [18].

As shown in Figure 5, one-half model with symmetry boundary on the X-Y plane was used to reduce the computation cost. The kinematic coupling constraint is used to constrain the motion of the end surface of the specimen to the motion of a reference point. The axial load was applied to the top reference point with translational degrees of freedom in the direction Y and Z and rotational degrees of freedom like spherical hinge. Pinned support boundary condition was set on the bottom reference point. The boundary conditions in FE model were chosen according to the real experimental boundary conditions.

General contact combining penalty friction formulation for the tangential behaviour and a contact pressure model in the normal direction was used to simulate the interaction between the

contact surfaces of steel boxes and the ends of concrete columns. A tie constrain was used to define the interaction between the steel pad at the end of the column and the corresponding end surface.

VERIFICATION

Figures 6 (a), (b) and (c) show the comparison of the axial load (N) versus the axial strain (ε) relations from the experiment and simulation. Here, the axial strain is an average value of the strain in the test region of specimens. It can be seen that the developed FE model is able to evaluate well the N - ε curves of RC columns with different volumetric ratio ρ_v varying in a range of 0.8%~2.39%.

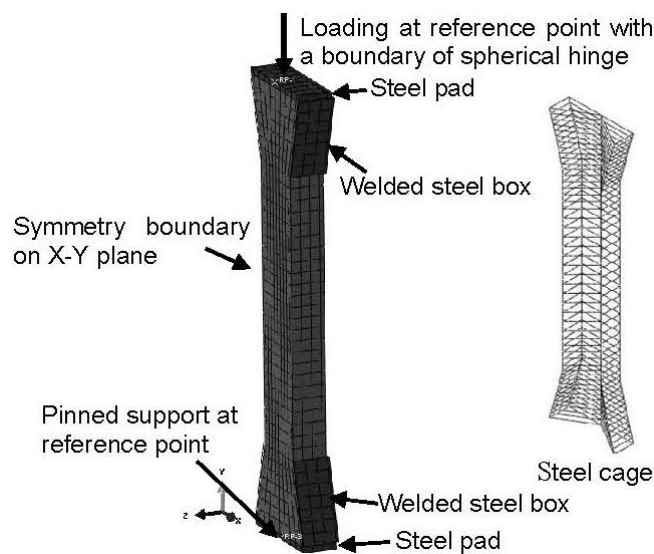


Fig. 5. - Boundary conditions and FEM meshes

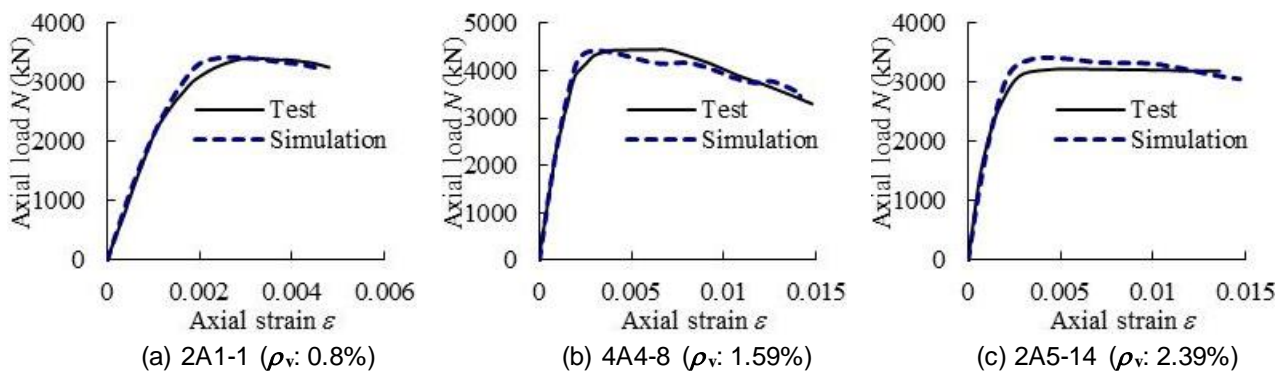


Fig. 6. - Comparison between experimental and predicted N - ε curves

EFFECTS OF STRENGTH AND EQUAL STRENGTH REPLACEMENT OF STIRRUPS

A parametric study was conducted based the verified FE model to investigate the effects of strength and equal strength replacement of stirrups. The details of all the specimens here are the same with the test specimens shown in Figure 2 except the investigated parameters illustrated in Table 2. As shown in Table 2, all specimens have the identical arrangement of longitudinal steel (8-No. 5) with the same yielding strength (400 MPa) and size (16 mm in diameter). The titles of the

specimens indicate the varied parameters. The alphanumeric characters in the titles of the specimens (e.g., C3S6-57R) have the following significance. The first letter C represents concrete and the number after the first letter indicates the cylinder strength of the concrete f_c . Number 3 and 6 represent 30 MPa and 60 MPa in cylinder strength of concrete, respectively. The second letter S refers to the stirrup and the following number represents the yielding strength of stirrup. Number 3 and 6 represent 300 MPa and 600 MPa in yielding strength of stirrup, respectively. The last two numbers are the spacing of stirrups. Numbers 57 and 29 mean 57 mm and 29 mm in spacing of stirrups, respectively. The last letter R shows the equal strength replacement of stirrups, which means that there is no change of spacing of stirrups and the product of the section area and yielding strength of steel bar retains the same when new steel rebar is instead of the old steel rebar. For example, specimen C3S6-57R represents the equal strength replacement of stirrups of specimen C3S6-57.

Tab. 2: Details of parameters

| Specimen | Longitudinal steel | | | Stirrup | | | | f_c (Mpa) |
|----------|--------------------|---------------|-------------------------------|---------------|--------------|----------------------------------|---------------------------|----------------|
| | Number and size | Diameter (mm) | Yielding strength f_y (Mpa) | Diameter (mm) | Spacing (mm) | Yielding strength f_{hy} (Mpa) | Volumetric ratio ρ_s | |
| C3S3-57 | 8-No. 5 | 16 | 400 | 4.8 | 57 | 300 | 0.8% | 30 |
| C3S6-57 | 8-No. 5 | 16 | 400 | 4.8 | 57 | 600 | 0.8% | 30 |
| C3S6-57R | 8-No. 5 | 16 | 400 | 6.8 | 57 | 300 | 1.6% | 30 |
| C6S3-57 | 8-No. 5 | 16 | 400 | 4.8 | 57 | 300 | 0.8% | 60 |
| C6S6-57 | 8-No. 5 | 16 | 400 | 4.8 | 57 | 600 | 0.8% | 60 |
| C6S6-57R | 8-No. 5 | 16 | 400 | 6.8 | 57 | 300 | 1.6% | 60 |
| C3S3-29 | 8-No. 5 | 16 | 400 | 4.8 | 29 | 300 | 1.6% | 30 |
| C3S6-29 | 8-No. 5 | 16 | 400 | 4.8 | 29 | 600 | 1.6% | 30 |
| C3S6-29R | 8-No. 5 | 16 | 400 | 6.8 | 29 | 300 | 3.2% | 30 |
| C6S3-29 | 8-No. 5 | 16 | 400 | 4.8 | 29 | 300 | 1.6% | 60 |
| C6S6-29 | 8-No. 5 | 16 | 400 | 4.8 | 29 | 600 | 1.6% | 60 |
| C6S6-29R | 8-No. 5 | 16 | 400 | 6.8 | 29 | 300 | 3.2% | 60 |

Figure 7 (a) and (b) show that the increase in strength of stirrups has little influence on the strength of core concrete, but it increases the ductility of the specimens indicated by the much gentler descending curves of $N-\varepsilon$. There exists a reasonable explanation for the phenomenon. The behaviour of confined concrete is related to the confinement stress from stirrups that is determined by the extent of lateral dilation of core concrete under axial load. It is found in the FE analysis that the stress of stirrups in specimens with varied strength of stirrups is close to each other at the peak axial loads, which leads to slight difference in strength of core concrete between the two contrasting specimens. That is because little lateral dilation of core concrete happens in the stage and limited stress of stirrups is produced leading to weakly exerting the high strength of stirrup. However, the extent of lateral dilation of core concrete increases with the axial deformation of column increasing in the descending stage of loading. Thus, higher stress in stirrups is produced for high strength stirrup, which causes higher confinement stress and the ductility of columns is improved. The high strength of stirrups is exerted considerably in this stage.

As shown in Table 2, the equal strength replacement of stirrup using the low strength steel in replacement of high strength steel (e.g., C3S6-57 and C3S6-57R) increases the volumetric ratio due to the size of the stirrups increasing. That improves the confinement effect of stirrups on core

concrete at peak axial load and gives rise to higher peak axial load, which can be observed in Figure 8. It appears that the smaller spacing of stirrups produces higher increase in peak axial load after the equal strength replacement of stirrup. Figure 8 (a) and (b) depict the reduction of ductility after the equal strength replacement of stirrup indicated by the steeper descending curves of $N-\varepsilon$.

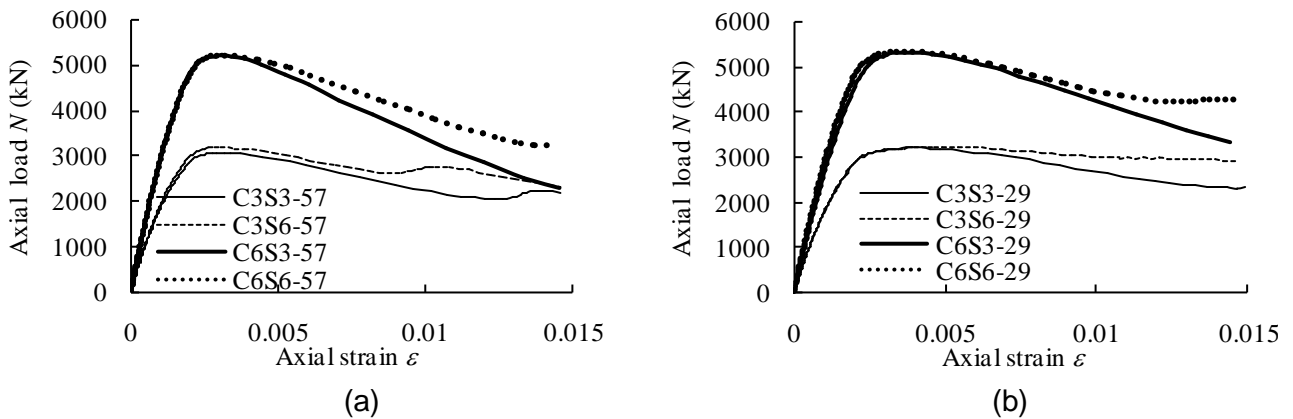


Fig. 7. - Effect of strength of stirrup

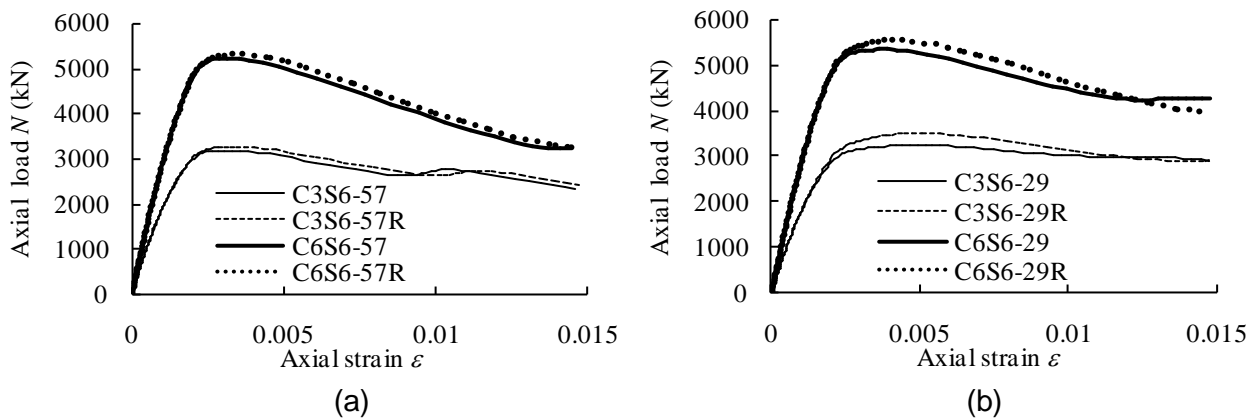


Fig. 8. - Effect of equal strength replacement of stirrup

CONCLUSION

In this paper, a new uniaxial compressive stress-strain relation of concrete confined by stirrups in square RC column was proposed to describe the behaviour of the confined concrete in three-dimension FE model by combining the concrete damaged plastic material model in ABAQUS. Based on the material model, a FE model for square confined RC columns under axial compression was developed. The FE model evaluates the $N-\varepsilon$ curves of tested confined RC columns well. Thus, the proposed uniaxial compressive stress-strain relation in conjunction with the concrete damaged plasticity model has the capability to evaluate the behaviour of concrete confined by stirrups and can be used to further investigate the behaviour of square confined RC members under different types of load.

A further parametric study shows that increasing the strength of stirrups has little effect on the strength of confined concrete, but it improves the ductility of the confined concrete. The equal strength replacement of stirrups using the low strength steel in replacement of high strength steel increases the strength of core concrete, but reduces the ductility of confined concrete.

ACKNOWLEDGEMENTS

The research reported in the paper is supported by the National Natural Science Foundation of China (No. 51608156), the Project of the Natural Science Foundation of Hainan Province (No. 20165208), the Scientific Research Starting Foundation of Hainan University (No. kyqd1534). All these sources of financial support are highly appreciated.

REFERENCES

- [1] Sheikh S.A., Uzumeri S.M., 1980. Strength and ductility of tied concrete columns. *Journal of the Structural Division*, 106(ST5): 1079-1102.
- [2] Scott B.D., Park R., Priestley M.J.N., 1982. Stress-strain behavior of concrete confined by overlapping hoops at low and high strain rates. *ACI Journal Proceedings*, 79(1): 13-27.
- [3] Moehle J.P., Cavanagh T., 1985. Confinement effectiveness of cross-ties in RC. *Journal of Structural Engineering*, 111(10): 2105-2120. [http://dx.doi.org/10.1061/\(ASCE\)0733-9445\(1985\)111:10\(2105\)](http://dx.doi.org/10.1061/(ASCE)0733-9445(1985)111:10(2105)).
- [4] Chung H.S., Yang K.H., Lee Y.H., Eun H.C., 2002. Strength and ductility of laterally confined concrete columns. *Canadian Journal of Civil Engineering*, 29(6): 820-830. <http://dx.doi.org/10.1139/I02-084>.
- [5] Park R., Priestley M.J.N., Gill W.D., 1982. Ductility of square confined concrete columns. *Journal of Structural Engineering*, 108(4): 929-950.
- [6] Sheikh S.A., Uzumeri S.M., 1982. Analytical model for concrete confinement in tied columns. *Journal of the Structural Division*, 108(12): 2703-2722.
- [7] Mander J.B., Priestley M.J.N., Park R., 1988. Theoretical stress-strain model for confined concrete. *Journal of Structural Engineering*, 114(8): 1804-1826. [http://dx.doi.org/10.1061/\(ASCE\)0733-9445\(1988\)114:8\(1804\)](http://dx.doi.org/10.1061/(ASCE)0733-9445(1988)114:8(1804)).
- [8] Saatcioglu M., Razvi S.R., 1992. Strength and ductility of confined concrete. *Journal of Structural Engineering*, 118(6): 1590-1607. [http://dx.doi.org/10.1061/\(ASCE\)0733-9445\(1992\)118:6\(1590\)](http://dx.doi.org/10.1061/(ASCE)0733-9445(1992)118:6(1590)).
- [9] Le'geron F., Paultre P., 2003. Uniaxial confinement model for normal- and high-strength concrete columns. *Journal of Structural Engineering*, 129(2): 241-252. [http://dx.doi.org/10.1061/\(ASCE\)0733-9445\(2003\)129:2\(241\)](http://dx.doi.org/10.1061/(ASCE)0733-9445(2003)129:2(241)).
- [10] Bousalem B., Chikh N., 2007. Development of a confined model for rectangular ordinary reinforced concrete columns. *Materials and Structures*, 40(6): 605-613. <http://dx.doi.org/10.1617/s11527-006-9172-2>.
- [11] Samani A.K., Attard M.M., 2012. A stress-strain model for uniaxial and confined concrete under compression. *Engineering Structures*, 41: 335-349. <http://dx.doi.org/10.1016/j.engstruct.2012.03.027>.
- [12] Liu J., Foster S.J., 2000. A three-dimensional finite element model for confined concrete structures. *Computers & Structures*, 77(5): 441-451. [http://dx.doi.org/10.1016/S0045-7949\(00\)00007-9](http://dx.doi.org/10.1016/S0045-7949(00)00007-9).
- [13] Kwon M., Spacone E., 2002. Three-dimensional finite element analyses of reinforced concrete

columns. *Computers & Structures*, 80(2): 199-212. [http://dx.doi.org/10.1016/S0045-7949\(01\)00155-9](http://dx.doi.org/10.1016/S0045-7949(01)00155-9).

[14] Yu T., Teng J.G., Wong Y.L., Dong S.L., 2010. Finite element modeling of confined concrete-I: drucker-prager type plasticity model. *Engineering Structures*, 32(3): 665-79. <http://dx.doi.org/10.1016/j.engstruct.2009.11.014>.

[15] Zeng X., Xu B., 2014. Numerical simulation on the dynamic behavior of short RC columns subjected to concentric rapid loading considering confinement effect of stirrups. *Engineering Mechanics*, 31, 190-197.

[16] Song Z.H., Lu Y., 2011. Numerical simulation of concrete confined by transverse reinforcement. *Computers and Concrete*, 8(1): 23-41. <http://dx.doi.org/10.12989/cac.2011.8.1.023>.

[17] Faria R., Pouca N.V., Delgado R., 2004. Simulation of the cyclic behaviour of R/C rectangular hollow section bridge piers via a detailed numerical model. *Journal of Earthquake Engineering*, 8(5): 725-748. <http://dx.doi.org/10.1080/13632460409350507>.

[18] Dassault Systemes Simulia Corp., 2014. Abaqus Version 6.14 Documentation- ABAQUS Theory Guide (Dassault Systemes Simulia Corporation).

[19] Tao Z., Wang Z.B., Yu Q., 2013. Finite element modelling of concrete-filled steel stub columns under axial compression. *Journal of Constructional Steel Research*, 89: 121-131. <http://dx.doi.org/10.1016/j.jcsr.2013.07.001>.

[20] ACI Committee 318, 2008. Building code requirements for structural concrete (ACI 318-08) and commentary (American Concrete Institute) 109 pp.

[21] Fib, 2013. Fib Model Code for Concrete Structures 2010 (Ernst & Sohn).

[22] Paultre P., Légeron F., 2008. Confinement reinforcement design for reinforced concrete columns. *Journal of Structural Engineering*, 134(5): 738-749. [http://dx.doi.org/10.1061/\(ASCE\)0733-9445\(2008\)134:5\(738\)](http://dx.doi.org/10.1061/(ASCE)0733-9445(2008)134:5(738)).

[23] Han L.H., Yao G.H., Tao Z., 2007. Performance of concrete-filled thin-walled steel tubes under pure torsion. *Thin Walled Structures*, 45(1): 24-36. <http://dx.doi.org/10.1016/j.tws.2007.01.008>.