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DESIGN METHOD OF BENDING CAPACITY OF CONTINUOUS COMPOSITE SLAB

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ABSTRACT

This paper presents a calculation method for predicting the ultimate loading capacity of continuous composite slabs. Only the small scale slide block test was needed to determine few mechanical parameters, and less cost had to be paid, in comparison to the conventional m-k method. Various load conditions and parameters were considered. Comparisons between test results and predicted results have shown that the proposed method has enough precision. Furthermore, the simplified method was also proposed for practical design.

KEYWORDS

profield steel sheet; simplified design; bending capacity

1. INTRODUCTION

Composite slabs with profiled steel sheet have been widely used in practical structures since the 60's of the last century. In practice, compared with conventional reinforced concrete slabs, composite slabs have major advantages: no formworks and fewer scaffolds, higher loading capacity, lighter weight and faster construction speed. Furthermore, due to the composite action created by the bond and mechanical occlusion on the interface between the concrete and profiled steel sheet, the material strength of composite slabs can be fully utilized.

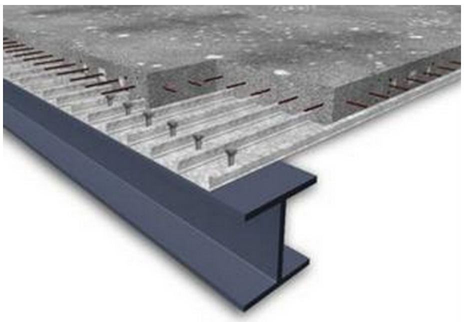

Fig. 1: Continuous composite floor with profiled steel sheet

Nowadays, there are many studies available about the simply supported composite slabs with single span [1 - 5], including experimental studies, theoretical models and design methods. However, as shown in Fig.1, continuous slabs often exist in practical structures, and some tests have been already carried out [6, 7], showing that the loading capacity of continuous composite slabs was much higher than that of simply supported slabs. Then the regression method, called m-k method, has been used for calculating the ultimate loading capacity of continuous composite

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slabs. In order to apply the m-k method shown in Fig.2, a large number of tests on full-scale slab specimens must be carried out by changing the geometrical dimensions and material strength levels and to determine the relation between the ultimate loading capacity versus various geometrical and material parameters of continuous composite slabs. Therefore, the high cost must be paid for obtaining the regression equation of composite slabs with various types of profiled steel sheet.

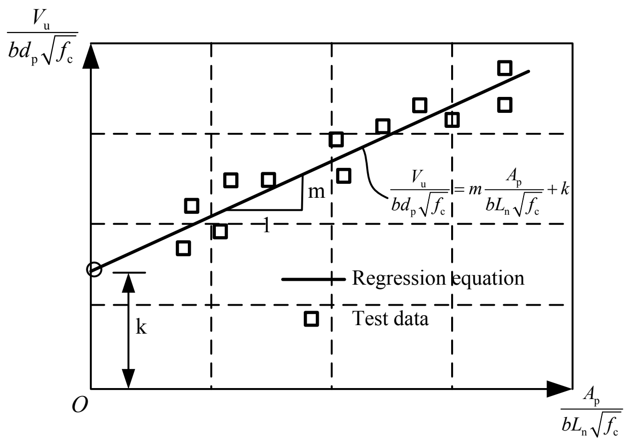

Fig. 2: m-k method for determining ultimate loading capacity of composite slabs

As shown in Fig.3, there are mainly four failure modes of composite slabs with profiled steel sheet [8]: (1) Flexure failure: at mid-span, the section of the profile steel sheet yield, and then the concrete crush, but the longitudinal interface between the steel sheet and concrete keeps good condition and little slip can be observed nearing the support. (2) Longitudinal shear failure: large slip exists on the interface, but the section at mid-span does not yield. (3) Vertical shear failure: when the ratio of the span to the height of the slab is relative small, the diagonal crack will appear near the support, similar to the shear failure mode of conventional reinforced concrete slabs. (4) Punching shear failure: when the thin slab bears heavy concentrated load, the cone shaped cracking occurs near the load point.

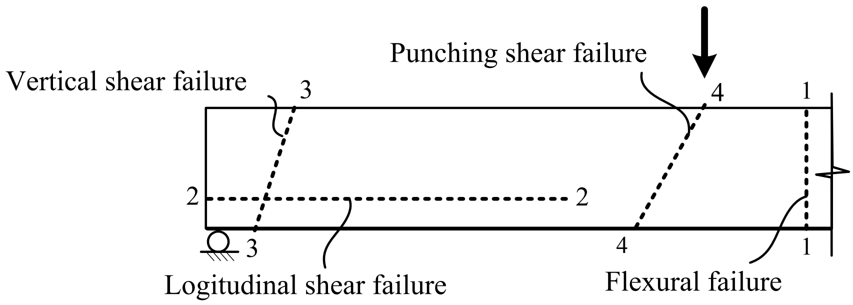

Fig. 3: Failure modes of composite slab

From the test results available now, it can be found that the flexure failure and longitudinal shear failure were the most common failure modes of composite slabs with steel sheet. Johnson has proposed simplified equations for calculating the ultimate capacity of composite slabs with vertical shear failure and punching shear failure modes [9], and the flexural capacity of the continuous composite slab can be directly obtained by the plastic equilibrium conditions on the section. Therefore, the method for calculating the longitudinal shear capacity of the continuous composite slab needed to be studied.

In this paper, a new theoretical model was developed for predicting the ultimate loading capacity of continuous composite slabs with profiled steel sheet, and the test validation was also

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made, in order to investigate the precision of the theoretical model. Finally, the simplified design methods were proposed for calculating the ultimate loading capacity of continuous composite slabs with profiled steel sheet in practice.

2 THEORETICAL MODEL

Firstly, in order to simplify the expression formulas in theoretical derivation, the section of the profiled steel sheet was equivalent to an I-shape section. As shown in Fig.4, the width and the thickness of the flange and the height of the web plate of the I-shape section were equal to the section of the profiled steel sheet, and the thickness of the web plate $t_w=2t/\cos\alpha$, where t was the thickness of the profiled steel sheet, and α was the inclining angle of the web of the profiled steel sheet.

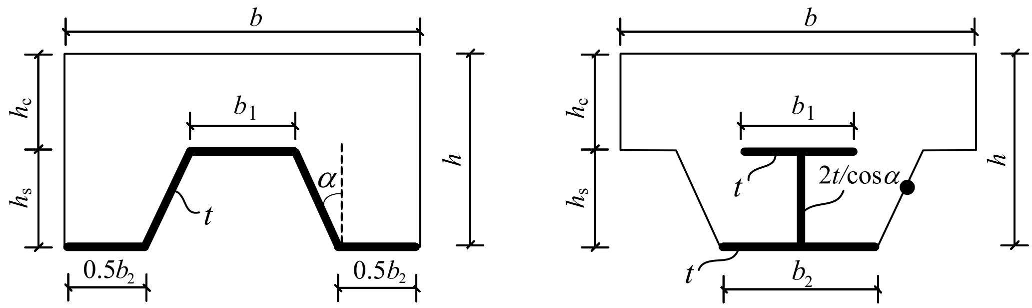
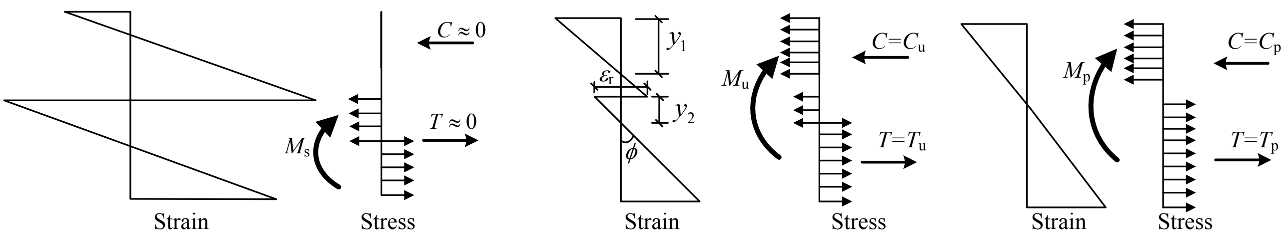


Fig. 4: Simplification for composite section

2.1 M_u-T_u relation

As illustrated in Fig.5, at ultimate state, the relation between the ultimate flexural capacity M_u of the section and the tensile force T_u of the profiled steel sheet could be determined by the slip strain ε_r on the interface. The M_u versus T_u relation had been assumed to be linear by Patrick [10]. However, the conclusions in the reference [3] indicated that the linear assumption of the M_u versus T_u relation would underestimate the longitudinal shear capacity of composite slabs with profiled steel sheet. Therefore, the M_u versus T_u relation should be discussed in detail.

Fig.5 showed three typical strain and stress distribution state of the composite section, corresponding to different shear connection degrees. When there was no composite action on the interface (Fig.5a), the concrete and steel sheet resisted the bending moment alone, and no axial force existed on the pure concrete section and the pure steel sheet section. If the concrete and steel sheet were completely composed (Fig.5c), there would be no slip on the interface, so the ultimate flexure loading capacity could be achieved. However, as shown in Fig.5b, in general, the slip often existed between the steel sheet and concrete, especially for slabs with longitudinal failure modes, so the theoretical model for calculating the ultimate loading capacity of continuous composite slabs should be established.



(a) no composite ($\varepsilon_r \to +\infty$) (b) general state (c) completely composite ($\varepsilon_r=0$)

Fig. 5: Strain and stress distribution on section

According to the axial force equilibrium condition on the section, the equation could be obtained as:

$$f_y t_w (h_s - 2t - y_2) + f_y b_2 t = f_c b y_1 + \int_0^{y_1} f_c b(y) \mathrm{d}y + f_y b_1 t + f_y t_w (y_2 - t) \tag{1}$$

where f_y was the yield strength of the steel and f_c was the compressive strength of the concrete. y_1 and y_2 were the height of the compressive zone of concrete and steel sheet respectively. $b(y)$ was the width of the concrete section below the top flange of the steel sheet.

Then the equations for calculating M_u and T_u could be expressed as:

$$\begin{cases} M_u = f_y b_2 t h + f_y t_w (h_s - y_2)(h_c + 0.5h_s + 0.5y_2) - 0.5 f_c b y_1^2 \\ \qquad - f_y b_1 t h_c - f_y t_w y_2 (h_c + 0.5y_2) - \int_0^{y_1} f_c b(y)(h_c + y) \mathrm{d}y \\ T_u = f_y t_w (h_s - 2y_2) + f_y (b_2 - b_1) t \end{cases} \tag{2}$$

Introducing equation (1) into equation (2), the parametric equation group about M_u and T_u with single parameter y_2 could be obtained. It could be observed that the tensile force T_u of the profiled steel sheet had a linear relation with y_2 and the ultimate flexural capacity M_u of the section had a parabolic relation with y_2, so there was parabolic relation between M_u and T_u, which was plotted in Fig.6.

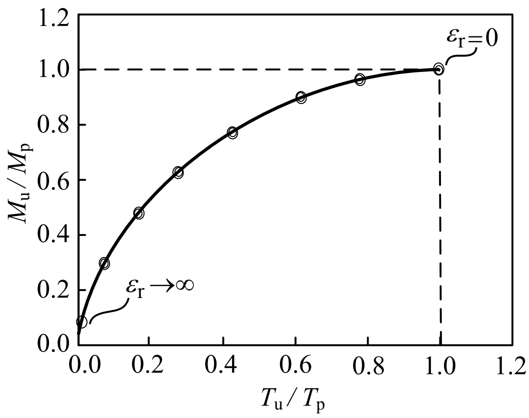


Fig. 6: Relation between M_u and T_u

As shown in Fig.5, there were totally two mechanical boundary conditions: (1) when the slip strain ε_r equalled to zero, the tensile force T_u of the profiled steel sheet was equivalent to its upper limit T_p, and the ultimate flexural capacity M_u of the section was equivalent to its upper limit M_p. (2) when there was no bond between the profiled steel sheet and concrete (ε_r approaching infinite), the profile steel sheet was at pure bending state, so the tensile force T_u of the profiled steel sheet was equivalent to zero, and $M_u = M_s$, where M_s was the flexural capacity of the pure section of the profiled steel sheet.

Based on a large number of numerical calculations, the M_u versus T_u relation could be obtained, as shown in Fig.6. Furthermore, the M_u versus T_u relation could be simply expressed as:

$$\frac{M_u}{M_p} = 1 - \left(1 - \frac{M_s}{M_p}\right)\left(\frac{T_u}{T_p} - 1\right)^2 \tag{3}$$

where the equations for calculating M_s and M_p could be expressed as:

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$$M_s = f_y W_p \tag{4}$$

$$M_p = f_y A_p (h - e_p - 0.5 f_y A_p / f_c b) \tag{5}$$

where W_p and A_p were the plastic moment and the section area of the profiled steel sheet, e_p was the distance from the bottom of the slab to the plastic neutral axis of the profiled steel sheet.

After the M_u versus T_u relation was obtained, the ultimate tensile force T_u of the profiled steel sheet section should be determined. In continuous slabs, as shown in Fig.7, the longitudinal shear failure surface might appear in two segments along the span: the segment A between the inflection point of the bending moment diagram and the load point, and the segment B between the side bearing and the load point. According to the force equilibrium condition, the ultimate tensile force T_u of the profiled steel sheet section was equivalent to the longitudinal shear force on the interface between the profiled steel sheet and concrete.

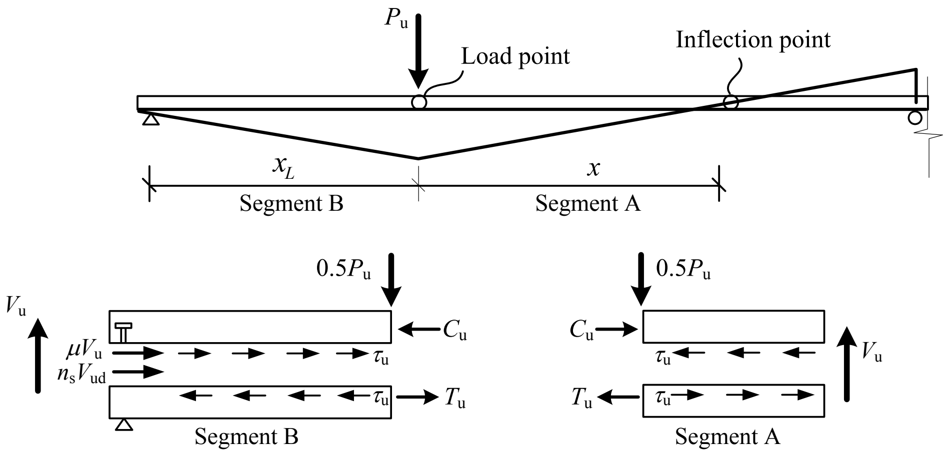


Fig. 7: Mechanical state of continuous composite slab at ultimate state

As shown in Fig.7, when the longitudinal interface of the segment A failed, the ultimate tensile force T_u of the section of the profiled steel sheet could be calculated as:

$$T_{u,A} = \tau_u b x \le T_p = f_y A_p \tag{6}$$

where τ_u was the shear strength of the interface between the profiled steel sheet and concrete, which could be calculated as [10]:

$$\tau_u = \nu \sqrt{f_c'} \tag{7}$$

where f_c' was the cylinder compressive strength of the concrete. ν was the bond coefficient of the interface determined by the types and constructions of the profiled steel sheet, which could be obtained by the small scale slide block tests with low cost and high precision (Fig.8).

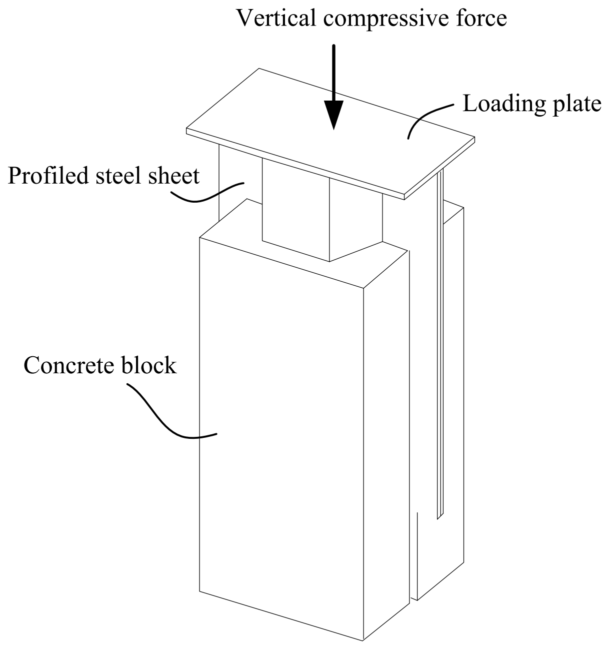


Fig. 8: Slide block test for determining bond coefficient between the steel sheet and concrete

As shown in Fig.7, when the longitudinal interface of the segment B failed, the ultimate tensile force T_u of the section of the profiled steel sheet could be regarded as the summation of three parts: (1) the shear stress on the interface, (2) the frictional force produced by the reaction force of the side bearing, (3) the shear capacity of studs at the edge. Therefore, the ultimate tensile force T_u of the section of the profiled steel sheet determined from the segment B could be obtained as:

$$T_{u,B} = \tau_u b x_L + \mu V_u + n_s V_{ud} \le f_y A_p \tag{8}$$

where μ was the frictional coefficient between the profiled steel sheet and concrete, which could be also determined by small scale slide block tests. V_u was the ultimate vertical shear force near the side bearing, n_s was the number of studs at the edge, and V_{ud} was the shear capacity of the stud, which could be calculated as [11]:

$$V_{ud} = 0.43 A_{st} \sqrt{E_c f_c} \le 0.7 f_u A_{st} \tag{9}$$

where A_{st} was the section area of the stud, f_u was the tensile strength of the stud, and E_c was the elastic modulus of the concrete.

Finally, the ultimate tensile force T_u of the profiled steel sheet section could be determined as:

$$T_u = \min\ (T_{u,A},\ T_{u,B}) \tag{10}$$

2.2 Ultimate loading capacity of continuous slab

Based on the M_u versus T_u relation of the composite section and the force equilibrium condition along the span, the ultimate loading capacity of the continuous composite slab could be calculated. Firstly, in order to obtain the area of the longitudinal shear failure surface, the position of the inflection point of the bending moment diagram should be determined, but it was related to the ultimate positive bending moment at the load point. Therefore, the iterative solution needed to be carried out. In the following parts, the methods for calculating the ultimate loading capacity of continuous composite slab under various load condition were discussed in detail.

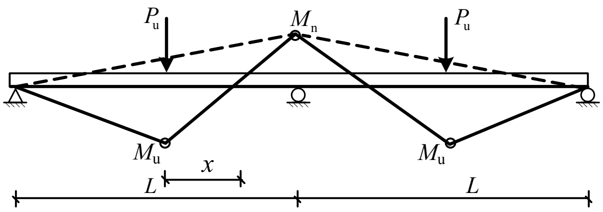
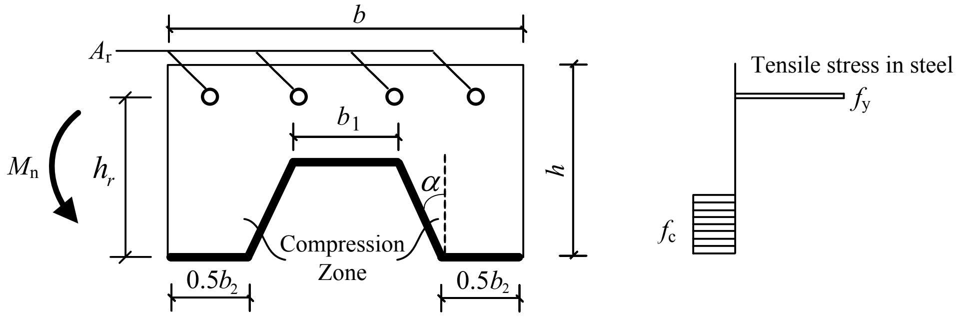


Fig. 9: Bending moment diagram of two-span continuous slab subjected to single point load

As shown in Fig.9, for the two-span continuous slab subjected to single concentrated load at mid-span of each span, according to the force equilibrium condition, the bending moment M_u at the load point could be calculated as:

$$M_u = 0.25 P_u L - 0.5 M_n \tag{11}$$

where M_n was the ultimate negative capacity of the composite section at the mid bearing.



(a) Geometrical relation (b) Stress diagram

Fig. 10: Calculation model for negative moment

The existing test results indicated that the local buckling of the profiled steel sheet always occurred when the section of the composite slab subjected to negative bending moment, so the contribution of the profiled steel sheet to the negative ultimate capacity M_n of the composite section must be neglected. As shown in Fig.10, the negative ultimate capacity M_n of the composite section could be calculated as:

$$M_n = f_{yr} A_r (h_r - 0.5 f_{yr} A_r / f_c b) \tag{12}$$

Where A_r and f_{yr} were the total area and yielding strength of the steel bars at the top of the section, h_r was the distance from the steel bars to the bottom of the composite section.

Based on the bending moment diagram shown in Fig.9, the position of the inflection point could be determined as:

$$x = 0.5L \frac{M_u}{M_u + M_n} \tag{13}$$

Taking equation (3), (6), (11), (12) and (13) as a non-linear equation group, the ultimate loading capacity of the continuous composite slab could be obtained by the iterative solution method, which was illustrated in Fig.11. Δ*M* was the incremental moment at each iterative step, which could be determined according to the solution precision, and *e* was the error limit for the iterative solution.

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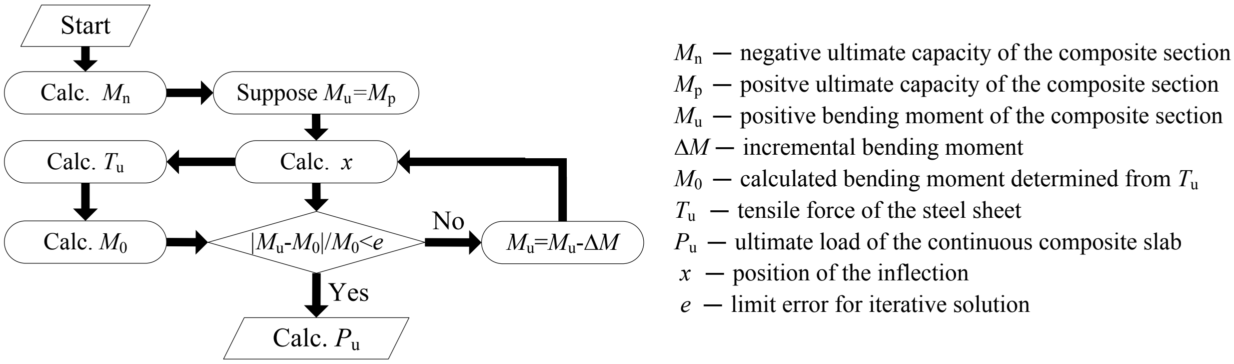

Fig. 11: Iterative flow for solving non-linear equation group

3 TEST VERIFICATION

In order to verify the proposed theoretical model, the comparative calculations were made based on tests in references [12] and [13]. The detailed results were illustrated in Table1. It could be observed that the proposed method has enough precision for predicting the ultimate loading capacity of continuous composite slab with steel sheet.

Tab. 1: Comparative results between the predicted results and the test results

Ref No.	Specimen No.	L/mm	h/mm	$P_{u,\ cal}$ / kN	$P_{u,\ test}$ / kN	$P_{u,\ cal}$/ $P_{u,\ test}$
Chen [12]	B-6	2600	165	138.33	146.80	0.94
	B-7a	2600	165	140.25	119.75	1.17
	B-7b	3200	165	106.58	119.75	0.89
Lee [13]	CSI-130-0.4(HS)	3000	130	91.57	96.00	0.95
	CSI-150-0.4(HS)	3000	150	110.81	110.00	1.00
	CSI-150-0.4(MS)	3000	150	111.42	128.00	0.87

4 CONCLUSION

Comparison with the test results showed that the theoretical method had enough precision for predicting the ultimate loading capacity of continuous composite slabs. Different from conventional m-k method with high cost, only the small scale slide block test with low cost had to be carried out to find the bond coefficient.

The simplified method could be used for continuous composite slabs subjected to the uniform distributed load in practical design, and the flexural failure mode and longitudinal shear failure mode were both considered.

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