LAMINAR SETTLING OF GLASS BEADS IN VISCO-PLASTIC LIQUIDS

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ABSTRACT

The paper deals with a determination of the terminal settling velocity of coarse particles in quiescent visco-plastic liquids of Herschel-Bulkley type. Experiments on laminar settling of glass beads of different sizes were conducted in transparent Carbopol solutions of various rheological properties in a sedimentation column. The terminal settling velocity of a solitude bead was determined together with the rheological parameters of the Carbopol liquid. An evaluation of the experimental results confirms the existence of the laminar regime for all tests and compares the measured velocities with predictions by Wilson et al. method. Furthermore, an alternative method is proposed for a prediction of the terminal settling velocity in the laminar regime which uses a particle-based determination of the strain rate in the expression for the equivalent viscosity. A comparison with our experimental results shows that the predictions using the proposed method agree well with the experiments and the proposed method is in the laminar settling regime more accurate than the Wilson et al. method.

KEYWORDS

Non-Newtonian liquid, rheological properties, terminal settling velocity, equivalent viscosity

INTRODUCTION

Many slurries of industrial interest are homogeneous mixtures of fine solid particles and water (or other liquid). If concentration of fines is high, then the mixtures exhibit a non-Newtonian behaviour. If coarse particles are present in the mixture as well (as for instance in mixtures of tailings and process water in mining operations), it is important to know whether the coarse particles are suspended in the non-Newtonian carrier, or they settle down. In slurry practice, a majority of non-Newtonian carriers are visco-plastic and hence exhibit a yield stress. If carrier liquid of this sort is subjected to acting of the shear stress, no strain rate is produced until the applied shear stress exceeds the yield stress. In case that a discrete particle is placed in a quiescent visco-plastic medium, the particle will not settle unless it is heavy enough to produce the shear stress sufficient to trigger particle settling. The fall of spherical particles in visco-plastic fluids was subject of numerous experimental studies over the last 50 years. Pioneer works built on methods for predicting the terminal settling velocity of spherical particles in Newtonian medium and modified Reynolds number or drag coefficient in order to use the methods for non-Newtonian fluids may be found, [1-2]. At the same time, an alternative approach was suggested to use graphs relating the drag coefficient with relevant dimensionless number based on rheological properties of the fluid, [3-4]. Later, detailed numerical simulations of the flow field surrounding a falling particle were published, [5-7]. Wilson et al.[8] proposed a direct (explicit) method for a prediction of the terminal settling velocity of spheres in both Newtonian and non-Newtonian liquids. Recently, Arabi and

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Sanders [9] reported on their measurements of terminal settling velocities of metal spheres in clay-water suspensions with yield stress.

The aim of this paper is to collect own experimental results of terminal settling velocities of spherical glass beads in visco-plastic liquids in the laminar regime and to use them first to evaluate a predicting ability of the Wilson et al. [8] method and second to test an alternative method based on a different formulation of the equivalent viscosity required for settling velocity predictions.

METHODS FOR PREDICTION OF TERMINAL SETTLING VELOCITY

General equations

The terminal settling velocity of a spherical particle in quiescent fluid, V_{ts}, is expressed by using the following general equation, based on the balance of forces acting the settling particle,

$$V_{ts} = \sqrt{\frac{4\left(\rho_s - \rho_f\right) g d}{3 \rho_f C_D}} \tag{1}$$

where ρ_s is the density of particle, ρ_f is the density of the fluid, and d is the diameter of a spherical particle. The equation (1) is valid for both Newtonian and non-Newtonian liquids. For the drag coefficient C_D, an analytical formula covering settling in a laminar regime is general too and it reads,

$$C_D = \frac{24}{\mathrm{Re}_p} \tag{2}$$

In the drag-formula, C_D is related to the particle Reynolds number,

$$\mathrm{Re}_p = \frac{\rho_f V_{ts} d}{\mu_{eq}} \tag{3}$$

which contains the equivalent (apparent, secant) viscosity, μ_{eq}, representing rheology of the fluid. It is defined as

$$\mu_{eq} = \frac{\tau}{\left(du/dy\right)} \tag{4}$$

where τ is the shear stress and du/dy is the strain rate. If the fluid is Newtonian, the equivalent viscosity is constant equal to the dynamic viscosity. If the fluid exhibits non-Newtonian behaviour, however, μ_{eq} varies with the local shear rate associated with the particle settling. Moreover, μ_{eq} needs to be related to other rheological parameters characteristic for particular non-Newtonian fluid. A solution of Equations 1-3 is iterative, which makes calculations of V_{ts} user-unfriendly.

In literature, efforts have been made to produce direct methods for the terminal settling velocity related to the Reynolds number.

Wilson et al. method

The method by Wilson et al.[8] gives an explicit relationship between two dimensionless numbers to predict the terminal settling velocity of a spherical particle for all three regimes of settling (laminar, transitional, turbulent).

Based on an analogy of settling in Newtonian and non-Newtonian fluids, the method relates the dimensionless settling velocity V_{ts}/V^* to the sphere shear Reynolds number Re^* defined as

$$\mathrm{Re}^* = \frac{\rho_f V^* d}{\mu_{eq}} \tag{5}$$

where V^* is the mean shear velocity, which is defined using the mean surficial shear stress $\bar{\tau}$ of a settling particle. This stress is expressed as the submerged weight divided by the surface area of the particle, which for a sphere is:

$$\bar{\tau} = \frac{\left(\rho_s - \rho_f\right) g d}{6} \tag{6}$$

As a result, the mean shear velocity is:

$$V^* = \sqrt{\frac{\bar{\tau}}{\rho_f}} = \sqrt{\frac{\left(\rho_s - \rho_f\right) g d}{6 \rho_f}} \tag{7}$$

A relationship between V_{ts}/V^* and Re^* is calibrated so that it gives the same results as the indirect method by Turton and Levenspiel [10] for settling in Newtonian fluids. For the laminar regime ($Re^* \leq 10$), the relationship is

$$\frac{V_{ts}}{V^*} = \frac{\mathrm{Re}^*}{\left[3\left(1 + 0.08\,\mathrm{Re}^{*^{1.2}}\right)\right]} + \frac{280}{\left[1 + 3 \cdot 10^4\,\mathrm{Re}^{*^{-3.2}}\right]} \tag{8}$$

Wilson et al.[8] suggested to relate shear stress to the mean surficial shear stress at some reference level and to use a rheological model as an intermediary for a specification of a corresponding strain rate. If non-Newtonian fluid fits the Herschel-Bulkley rheological model, the equivalent viscosity is

$$\mu_{eq} = \frac{\tau_{ref}}{\left[\left(\tau_{ref} - \tau_y\right) / K\right]^{1/n}} \tag{9}$$

where τ_y, K, n are rheological parameters of the Herschel-Bulkley model (τ_y is the yield stress, K is the coefficient of consistency, n is the flow index). The reference shear stress, $\tau_{ref} = \zeta \cdot \bar{\tau}$, where the parameter $\zeta = 0.3$ was based on authors' calibration using a set of 189 experimental data points. This solution means that the use of the method is limited to $\tau_y < 0.3 \cdot \bar{\tau}$ as the value of equivalent viscosity would be less than 0 otherwise [9].

Particle strain-rate based method

In the framework of the AMIRA P1087 project, V. Matoušek tested an alternative method for a determination of the equivalent viscosity at the shearing condition caused by particle settling. The method is based on a simple assumption that the strain rate for the rheological model of the fluid sheared by a settling particle is expressed as V_{ts}/d. Hence, for visco-plastic fluid which obeys the Herschel-Bulkley rheological model, the shear stress relevant to a spherical particle of the diameter d settling with the velocity V_{ts} is

$$\tau = \tau_y + K\left(\frac{du}{dy}\right)^n = \tau_y + K\left(\frac{V_{ts}}{d}\right)^n \tag{10}$$

The equivalent viscosity is according to Equation 4

$$\mu_{eq} = \frac{\tau_y + K\left(V_{ts}/d\right)^n}{V_{ts}/d} \tag{11}$$

This equivalent viscosity is used in Equations 1-3 to predict V_{ts}. This method is implicit.

EXPERIMENTS

Experiments were carried out in the Water Engineering Laboratory of the Czech Technical University in Prague. Details of the experiments are given in the first-author's MSc thesis [11].

Materials

As a non-Newtonian medium Carbopol (Ultrez 10) solutions were used. Carbopol is an acidic powder of particle size from 2 to 7 microns which, after dispersion in water and neutralization process, forms a non-Newtonian solution of Herschel-Bulkley type (rheology typical for thickened tailings). Values of the rheological parameters (τ_y, K, n) depend on a concentration of the powder in the solution. An advantage of Carbopol is its transparency and a quite simple preparation of solutions of various concentrations.

Coarse particles from four different fractions of narrow-graded glass beads were tested. The fraction TK2.0 with median size d_{50} = 2.01 mm (particle sizes from 1.86 to 2.12 millimetres) and density ρ_s = 2426 kg/m^3. Slightly coarser fractions TK2.9 with particle sizes from 2.73 to 3.11 mm (d_{50} = 2.98 mm, ρ_s = 2496 kg/m^3) and TK3.0 from 2.90 to 3.13 mm with d_{50} = 3.02 mm and ρ_s = 2497 kg/m^3. The last fraction TK7.0 was narrow graded with all particle sizes finer than 7.65 mm and all particles coarser than 6.37 mm, d_{50} = 7.06 and ρ_s = 2481 kg/m^3. All beads were considered spherical.

Collected data set

In total, 33 experimental data points based on 124 measurements of V_{ts} in 10 Carbopol solutions of various rheological properties were collected (see Table 1).

Tab. 1: Experimental results

Test no.	**V_{ts} [cm/s]**				**Rheological parameters**		
	TK2.0	**TK2.9**	**TK3.0**	**TK7.0**	**τ_y [Pa]**	**K [Pa.s^n]**	**n**
1	0	0	0	2.40	2.27	1.43	0.53
2	0	0	0.02	2.42	2.25	1.34	0.54
3	0	0.02	0.07	4.31	2.07	1.07	0.55
4	0	0.13	0.09	6.25	1.64	0.87	0.57
5	0.01	0.18	0.23	8.93	1.29	1.04	0.54
6	0.02	0.25	0.30	10.65	1.26	0.94	0.55
7	0.04	0.28	0.33	9.74	1.39	1.19	0.53
8	0.11	0.51	0.60	12.50	0.77	0.84	0.55
9	0.15	0.77	0.83	14.35	0.63	0.71	0.57
10	0.51	2.12	2.08	199.01	0.33	0.55	0.58

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Test procedure

Terminal settling velocity was calculated by dividing the fixed vertical distance (333 mm) between two monitored planes in the sedimentation column with the measured time a particle needed to travel the distance between the two planes. At the top of the column the particle was carefully placed and released into the fluid paying attention not to induce any rotation or add initial force during the release. Each solid particle travelled a distance of 67 mm before reaching the first monitored plane. For every test series, at least 3 identical particles from each fraction were released into the sedimentation column and the average of measured velocities was reported as a single data point. Fluid samples were collected and rheological parameters measured in the rotational viscometer HAAKE VT 550 before and after the test series to ensure that the rheological properties of the fluid remained constant during the series.

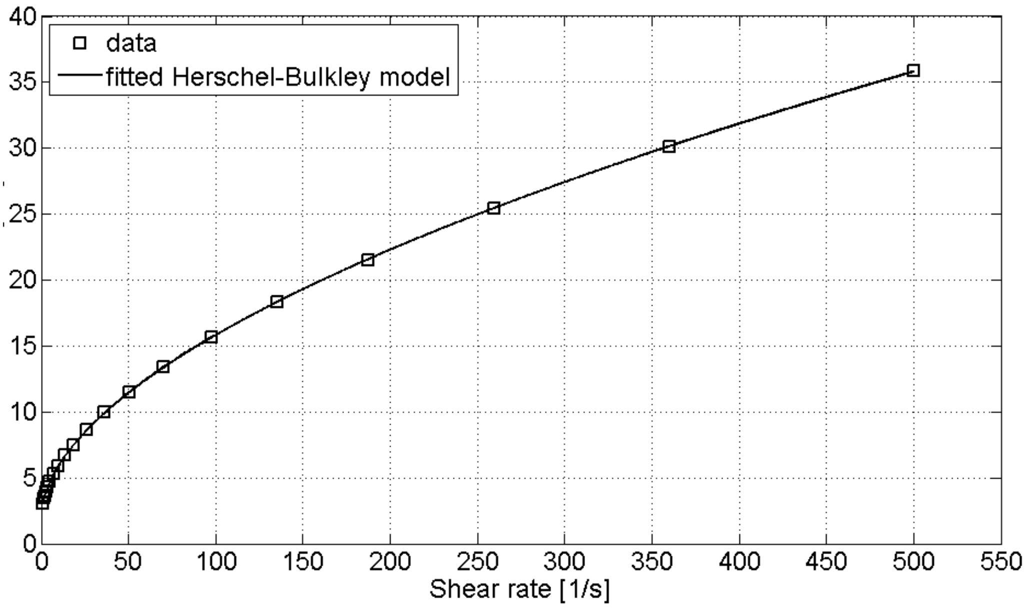

Fig. 1: Rheogram for Test series No. 3

DISCUSSION OF RESULTS

The measured parameters (τ_y, K, n, d, ρ_s, V_{ts}) are employed to evaluate an accuracy of the above described predictive methods for V_{ts}. In Figures 2 and 3, the results are presented in plots using the dimensionless groups proposed by Wilson et al.[8]. In Figure 2, the experimental results are compared to the predictions by the original Wilson et al.[8] method (Equations 5-9). Just 31 out of total 33 experimental data points could be used for the comparison due to the limitation of the method (it does not work at $\tau_y \geq 0.3 \cdot \bar{\tau}$). Although some agreement is observed, the scatter is rather big. In Figure 3, the experimental results are compared to the predictions using the Wilson et al.[8] method (Equations 5-8) in which the particle strain-rate based method is used to calculate μ_{eq} (Equation 11). This modified approach considerably improves the accuracy of the predictions (reduces the scatter). For $Re^* < 1$, the agreement between the measurements and predictions is excellent. At higher $Re^*(1 < Re^* < 10)$, the modified method tends to slightly underestimate values of V_{ts}.

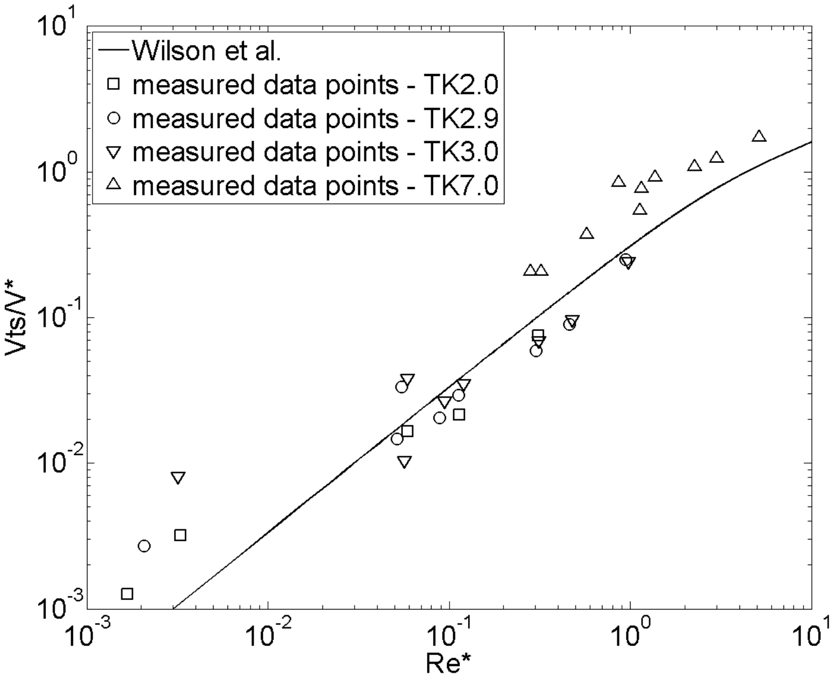

Fig. 2: Comparison of experimental data with Wilson et al.[8] technique

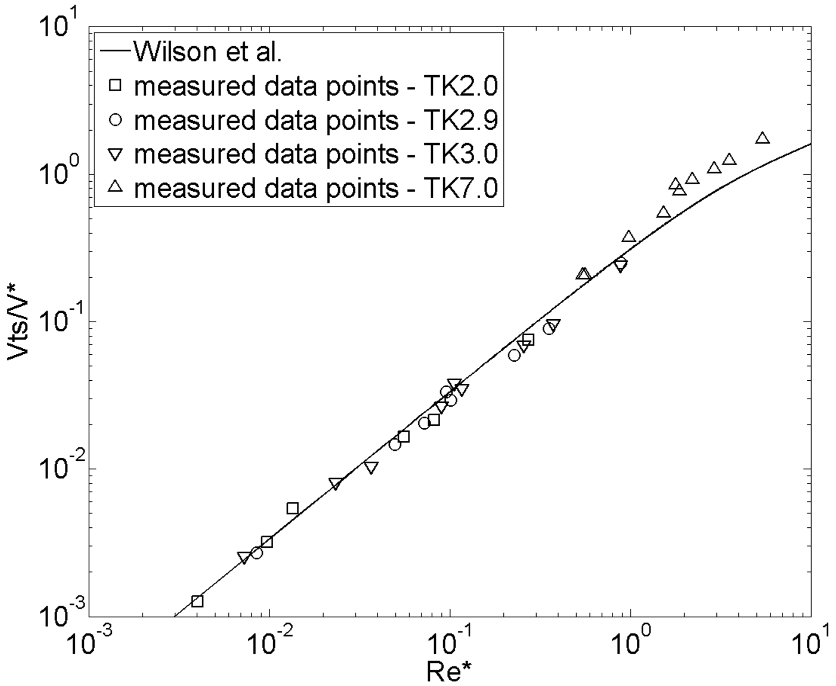

Fig. 3: Comparison of experimental data with particle strain-rate based method

Figure 4 compares experiments to predictions in the plot using the general equations (Equations 2-3). The plot confirms a validity of the Stokes law and hence the presence of the laminar regime in the entire range of measured settling conditions. This is consistent with our visual observations which did not recognize any wakes developed behind the settling particles.

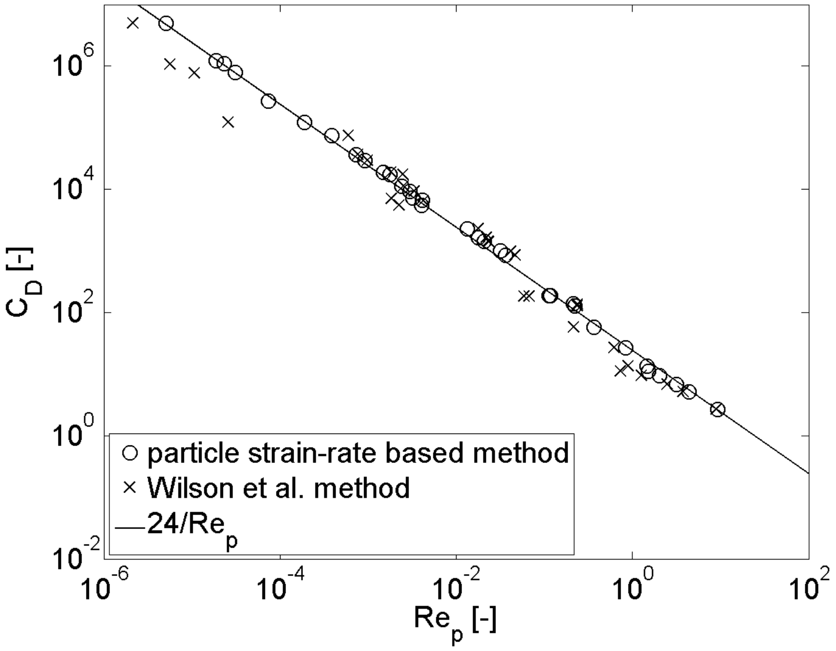

Fig. 4: Relation between drag coefficient C_D and particle Reynolds number Re_p

The parity plot in Figure 5 shows that the deviation of the predicted V_{ts} from the measured V_{ts} is confined to ±30 per cent if the particle strain-rate based method is used for the prediction. The deviation tends to be considerably bigger with the original Wilson et al.[8] technique used as a predictive method. The plot confirms that the proposed alternative method produces generally a better match.

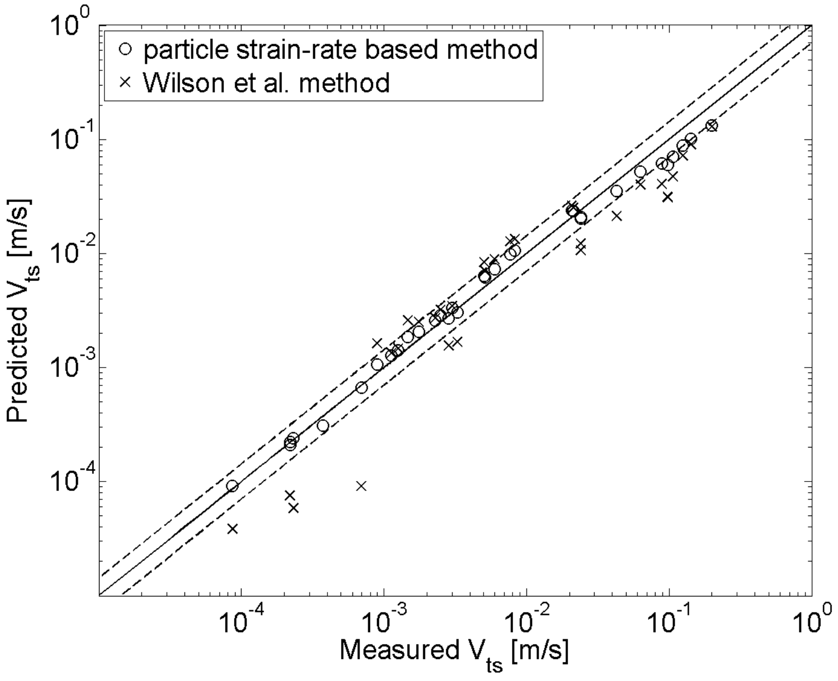

Fig. 5: Parity plot of measured and predicted terminal settling velocity. Legend: solid line = perfect match, dotted lines = +/- 30% deviation

CONCLUSIONS

Our experimental investigation on laminar settling of glass beads in visco-plastic liquids reveals that the experimentally obtained terminal settling velocities can deviate from those predicted by the Wilson et al. [8] method with more than 30 per cent.

Overall accuracy of a prediction significantly improves in case that an alternative particle strain-rate based method is used for determining of the equivalent viscosity associated with shearing due to particle settling. On the other hand, the use of the particle strain-rate method makes the prediction of the terminal settling velocity an implicit procedure while the Wilson et al.[8] method is direct.

In the future work, the promising predictive ability of the particle-strain-rate-based method should be confirmed by comparing with a larger number of experimental data in the laminar regime. The method should be tested also in the transitional regime associated with higher values of particle Reynolds number.

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REFERENCES

[1] Valentik L., Whitmore R. L., 1965. Terminal Velocity of Spheres in Bingham Plastics. British Journal of Applied Physics, vol. 16: 1197-1203

[2] Ansley R.W., Smith T.N., 1967. Motion of Spherical Particles in a Bingham Plastic. American Institute of Chemical Engineers, vol. 13: 1193-1196

[3] Anders U.Ts., 1961. Equilibrium and Motion of Spheres in a Viscoplastic Liquid. Soviet Physics-Doklady, vol. 5: 723

[4] du Plessis M.P., Ansley R.W., 1967. Settling Parameters In Solid Pipelining. Journal of the Pipeline Division (American Society of Civil Engineers), vol. 93: 1

[5] Beris A.N., Tsamopoulos J., Armstrong, R.C., Brown, R.A., 1985. Creeping Motion of a Sphere Through a Bingham Plastic. Journal of Fluid Mechanics, vol. 158: 44-219

[6] Blackery J., Mitsoulis, E., 1997. Creeping Motion of a Sphere in Tubes Filled with a Bingham Plastic Material, Journal of Non-Newtonian Fluid Mechanics, vol. 70: 59-77

[7] Beaulne M., Mitsoulis E., 1997. Creeping Motion of a Sphere in Tubes Filled with Herschel-Bulkley Fluids. Journal of Non-Newtonian Fluid Mechanics, vol. 72: 55

[8] Wilson K.C., Horsley R.R., Kealy T., Reizes J.A., Horsley M., 2003. Direct Prediction of Fall Velocities in Non-Newtonian Materials. International Journal of Mineral Processing, vol. 71: 17-30

[9] Arabi A.S., Sanders R.S., 2014. Particle terminal settling velocities in non-Newtonian, viscoplastic fluids. In: Hydrotranport 19, edited by VirtualPiE Ltd t/aBHR Group, 449-458

[10] Turton R., Levenspiel O., 1986. A Short Note on Drag Correlation for Spheres. Powder Technology, vol. 47: 83

[11] Kesely M., 2016. Evaluation of Settling Velocity of Coarse Particles in Visco-plastic Fluid and Frictional Loss in Complex Slurry Flow (MSc thesis at Czech Technical University in Prague) 51 pp.