

INFLUENCE OF CYCLIC LOADING ON THE DEFLECTION DEVELOPMENT OF CONCRETE SPECIMENS

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ABSTRACT

Durability of the structures is one of the most discussed issues of last decades. It is one of the most easily measured properties for analysis during the structural lifetime. Concrete deflections increase over time due to rheological effects (creep and shrinkage) in addition cyclic creep can be observed on the cyclically loaded structures. The deflection increase due to the cyclic creep is not properly quantified. The fatigue damage function presented in this paper provides an analytical solution for the deflection development due to cyclic loading. The evaluation of the deflection is based on the reduction of the initial modulus of elasticity.

Main principles of the function are discussed and compared with the standardized approaches for the fatigue assessment. Experimental verification of the fatigue damage function was carried out on reinforced concrete specimens and on prestressed concrete slab. To improve the standardized approaches, the real stress distribution was considered with the use of newly-developed method of partial integration over the height of the specimen compressive zone.

The deflection increase due to cyclic loading was measured regularly with inductive displacement transducer. Comparison of the measured values and the values calculated using the presented function shows good agreement. The fatigue damage function can be used easily in "in-hand" calculations, or can be inserted into FEM-based software and used in practical applications for assessing the increase in the deformations of concrete structural elements caused by cyclic loading.

KEYWORDS

Concrete, fatigue, cyclic loading, strain, deflection, experimental methods

INTRODUCTION

Fatigue is commonly defined as a process of permanent progressive changes in the structure of a material exposed to cyclic loading. Concrete is nowadays one of the most widely-used building materials for various kinds of structures. Along with the improvements in its properties, the use of high strength concrete has resulted in the design of slenderer structures. These structures are subjected to a higher live load proportion of the total load, and greater

vulnerability to fatigue failure can therefore be assumed. Many common structures, bridges and crane tracks are exposed to cyclic loading, which can result in accelerated crack propagation, greater deflections, reduction in structural stiffness and consequently fatigue failure.

Fatigue of concrete and concrete structures was first described at the beginning of the 20th century, and became a significant topic in the 1920s with the development of highways, concrete railway bridges and airport pavements.

History of fatigue of concrete

The first publications dealing with fatigue of concrete came from German authors [1, 2]. These publications were focused on concrete bridges and airport pavements, structures which had to resist 10^5 - 10^7 load cycles during their lifetime. According to present terminology, this phenomenon is called high-cycle fatigue. In the late 1950s, research was focused mainly on fatigue caused by seismic loading (10^0 - 10^3 loading cycles). This phenomenon is called low-cycle fatigue. With the rapid growth of mass transport structures in the 1970s, the number of cycles that structures have to resist increased to about 10 to 50 million load cycles.

Fatigue testing of concrete was mainly developed in the 1960s, e.g. [3], with the goal of providing a proper description of the stress-strain curve of concrete exposed to cyclic loading. In the late 1970s, Holmen [4] focused his research on strain development under cyclic loading with different amplitudes.

The developments in concrete fatigue testing show similarities with the developments in static testing of concrete. These studies were performed as compressive tests. They were followed by testing the fatigue of concrete in tension [5], fatigue under varying loads [6] and frequencies [7], and fatigue in flexure [8].

Nowadays, testing in flexure is the most popular type of test, together with biaxial or triaxial compression [9]; the fracture mechanics approach is now widely used in fatigue of concrete.

During the 20th century, many different approaches for fatigue analysis of concrete structural elements were proposed. Hsu [10] extended the S-N curve to the S-N-T-R system, which takes into account the time of loading and the ratio of the minimum and maximum stress applied to the strength of the concrete. Petryna [11] developed a method for assessing the reliability of reinforced concrete structures subjected to fatigue loading. Material models of concrete subjected to cyclic loading are often based on fatigue damage accumulation and micro-crack propagation. Horii [12] pointed out that the fatigue crack initiation mechanism and crack growth are different for static and fatigue loading. This section should describe in detail the study material, procedures and methods used.

The presented paper is focused on the deflection increase due to compressive fatigue of concrete. Influence of the tensile fatigue was neglected due to crack development in reinforced concrete specimens. For the deflection calculation of prestressed specimen the tensile fatigue was simplified as stated in the text.

STRAIN DEVELOPMENT UNDER CYCLIC LOADING

The concrete failure mechanism and strain development under cyclic loading

Concrete is a heterogeneous three-phase material. It is full of flaws and initial stress concentrations, and the fatigue process in this kind of material is much more complex than in homogeneous ferrous materials. The development of the secant modulus of elasticity in concrete subjected to cyclic loading, the cyclic creep curve, consists of three phases as can be seen in Figure 1. Phase 1 - the initiation phase; microcracks develop in the weaker parts of the cement paste and the strain increases rapidly (5-10% of the limit number of applied cycles N). Phase 2 - the cracks propagate in a stable manner, and the strain increases approximately linearly with the number of

applied load cycles (about 80% of N). Phase 3 - represents unstable crack growth, which leads to fatigue failure of the specimen (remaining 10-15% of N).

Development of the secant modulus of concrete under cyclic loading

The development of the secant modulus of elasticity under cyclic loading reflects the development of strain or deformation, and vice-versa. The development of the secant modulus of elasticity under cyclic loading was described by Holmen in 1979 [4], see Fig. 1.

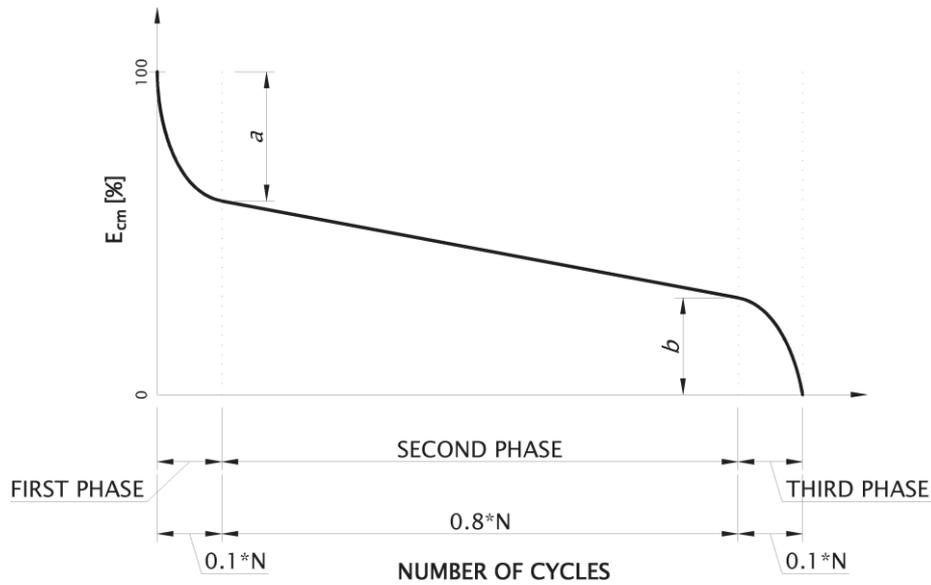


Fig. 1: Development of the secant modulus of elasticity of concrete under cyclic loading [4]

In his research, Holmen used a loading frequency of 5Hz and minimum stress equal to $0.05f_{cm}$. The maximum stress varied from $0.675f_{cm}$ to $0.95f_{cm}$. The first phase of development of the secant modulus of elasticity finished at 75-95% of E_{cm} , and the second phase finished at 68-75% of E_{cm} , depending on the maximum stress applied, see Fig. 2.

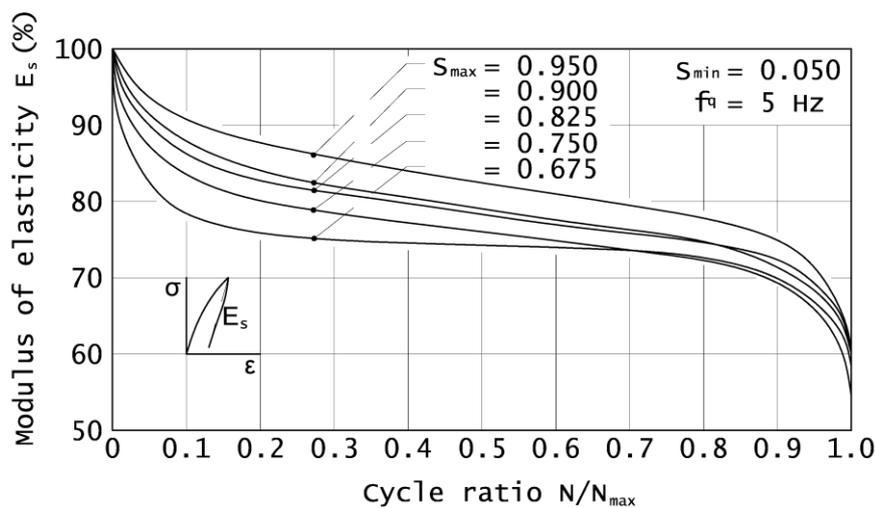


Fig. 2: Percentage reduction of the secant modulus of elasticity with the cycle ratio. Mean curves for different stress levels (from [4])

Parametric description of the secant modulus development under cyclic loading

A parametric description of the development of the secant modulus of elasticity of concrete under cyclic loading is proposed on the basis of experiments carried out by Holmen [4]. These experiments were performed on concrete probe cylinders with dimensions of 100x250 mm (diameter x height) subjected to cyclic compressive loading.

For the purposes of parameterization, the duration of the first phase and also the third phase of the strain development is assumed to be 10% of the total number of load cycles that the structural element is able to resist. Simultaneously, according to the Holmen experiment [4] the contribution of the first and the third phase of the development was substituted by constants for the given compressive stress level S_{max} . S_{max} is the maximum stress level defined as the ratio between maximum compressive stress and design fatigue endurance $f_{cd,fat}$, as in eq. (1). Coefficient η_c is the averaging factor of concrete stresses in the compression zone considering the stress gradient and it applies only in case of Model Codes [13, 14].

$$S_{max} = \eta_c |\sigma_{c,max}| / f_{cd,fat} \quad (1)$$

$$f_{cd,fat} = \beta_{cc,sus}(t, t_0) \beta_{cc}(t_0) f_{cd} \left(1 - \frac{f_{ck}}{250} \right) \quad (2)$$

Constants a and b are introduced, a for the decrease in the secant modulus of elasticity in the first phase of its development under cyclic loading, b for the remaining proportion of the original secant modulus of elasticity at the beginning of the third phase of its development. The graphical meaning of the constants is explained in Figure 1. The phenomenon of the deflection increase can be as well assumed for structural elements which are exposed to cyclic bending, as described further in this text. The formulas for constants a and b were obtained by linear regression:

$$a = 0.47 - 0.4S_{max} \quad (3)$$

$$b = 0.57 + 0.17S_{max} \quad (4)$$

Due to the method used for assessing the formulas, and input data based on higher stress levels, the formulas for constants a and b are valid only for stress levels $S_{max} > 0.174$. For stress levels $S_{max} < 0.174$ the increase in deflections due to fatigue loading can be neglected, or data from the research carried out by Holmen [4] can be extrapolated. Holmen dynamically tested 462 specimens, thus the dependence of the constants only on stress level S_{max} based on his research can be considered as correct.

Fatigue damage function

Formulation of the problem, motivation

A mathematical function for describing the strain development in a concrete specimen under cyclic loading is sought for. This function should be able to give a decreasing multiplier of the initial modulus of elasticity at each particular moment of the cyclic loading (after n_i loading cycles), thus respecting the three phases in strain development under cyclic loading. The reduced value of the initial modulus of elasticity can then be used for calculating deflections increased by damage accumulation caused by cyclic loading.

Boundary conditions, simplifications, our approach

The fatigue damage function uses the parametric description of the development of the secant modulus of elasticity of concrete under cyclic loading, as proposed above.

Some further assumptions are added and listed:

- The function is set up (on the x-axis) to the ratio of the number of load cycles that the structural element has already resisted (n) to the total number of load cycles that the structural element is able to resist at the particular load level (this value can be calculated by procedures given e.g. in Eurocode 2 [15] or in the Model Codes [13, 14]).
- The value of the fatigue damage function after the end of the first phase of strain development is equal to $(1-a)$.
- The value of the fatigue damage function at the start of the third phase of strain development is equal to b .

According to the chosen form of the fatigue damage function, the function is the sum of a power function and an exponential function.

Power part of the fatigue damage function

The power part represents the rapid decrease in the modulus of elasticity at the start of cyclic loading, i.e. the first phase of cyclic loading, and the stable progressive decrease in the modulus of elasticity during the majority of the service life of a structural element subjected to cyclic loading, i.e. the second phase of cyclic loading.

The result of the power part has to fulfil the following criteria:

- Its value at $n/N = 0.1$ has to be equal to a .
- Its value at $n/N = 0.9$ has to be equal to $(1-b)$.
- Due to the variation in differences between $(1-b)$ and a for various load levels, which determines the increase of the function between $n/N = 0.1$ and $n/N = 0.9$, the power has to be a function of S_{max} . The power part of the fatigue damage function can be followed in Fig. 3.

Exponential part of the fatigue damage function

The exponential part is independent of S_{max} . It represents the rapid decrease in the modulus of elasticity at the end of the service life of a structural element (third phase).

The result of the exponential part of the function has to fulfil the following criteria:

- Its value for $n/N = 0.0$ to 0.9 has to be insignificant.
- Its value for $n/N = 0.9$ to 1 has to be dominant.
- Its value at $n/N = 1$ has to be equal to b , so that the sum of the power and exponential part is equal to 1. The exponential part has behaviour as in Fig. 3.

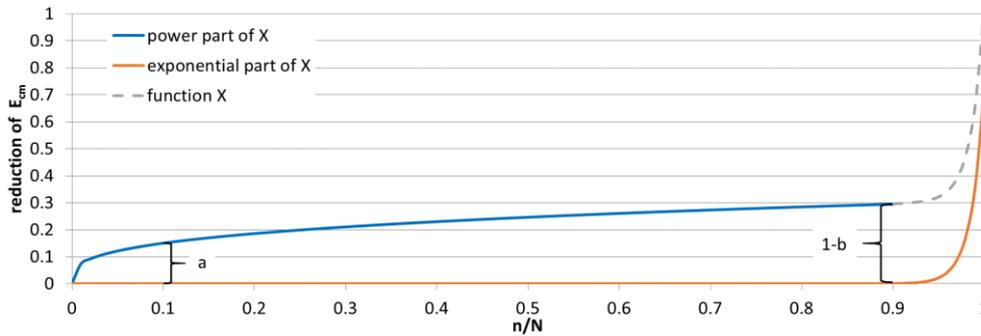


Fig. 3: Example of the power and exponential part of function X

Limit number of applied cycles N – usage and discussion of standards

EN 1992-1 [15] states that the fatigue should be evaluated in the most stressed fibres. Model Code 2010 [13] partially reflects the stress distribution in compression zone with η_c coefficient. Experimental results show that the calculated design values of the limit number of applied cycles N according to these standards differs significantly (eq. 5 for Model Code 2010 and eq. 6 for EN 1992-2).

$$\log N_1 = \frac{8}{Y-1} (S_{c,max} - 1)$$

$$\log N_2 = 8 + \frac{8 \cdot \ln(10)}{Y-1} (Y - S_{c,min}) \cdot \log \left(\frac{S_{c,max} - S_{c,min}}{Y - S_{c,min}} \right) \tag{5}$$

with

$$Y = \frac{0.45 + 1.8 \cdot S_{c,min}}{1 + 1.8 \cdot S_{c,min} - 0.3 \cdot S_{c,min}^2}$$

(a) if $\log N_1 \leq 8$, then $\log N = \log N_1$

(b) if $\log N_1 > 8$, then $\log N = \log N_2$

$$\log N = \left(14 \frac{1 - S_{c,max}}{\sqrt{1 - \sigma_{c,min} / \sigma_{c,max}}} \right) \tag{6}$$

Experimental testing of reinforced concrete specimens (described further) did not show any visible damage after 400 thousand cycles, this may correspond with the value of N calculated according to Model Codes – $N = \sim 1.3 \cdot 10^7$, whereas for the calculation according the Eurocode the limit number of applied cycles $N = 1900$ (both values were calculated using material safety factor $\gamma_M = 1.0$). Based on these findings the Model Code is used in the proposed model.

For the evaluation of the prestressed specimen in the experiment the limit number of applied cycles for concrete in tension or in compression-tension were used as in Model Code 2010 [13].

$$\begin{aligned} \log N &= 9(1 - S_{c,\max}) && \text{for } \sigma_{ct,\max} \leq 0.026 |S_{c,\max}| \\ \log N &= 12(1 - S_{c,\max}) && \text{for } \sigma_{ct,\max} > 0.026 |S_{c,\max}| \end{aligned} \quad (7)$$

The fatigue damage function

The development described in previous sections leads to the following formulation of the fatigue damage function (eq. 8). Fig. 4 gives an example of the fatigue damage function for various load levels.

$$\omega_{F_i} = 1 - \left\{ a \cdot \left(\frac{n_i}{c_1 N} \right)^{\frac{S_{\max}^{c_3}}{c_4}} + b \cdot \exp \left[\left(\frac{n_i}{N} - 1 \right) \cdot c_2 \right] \right\} \quad (8)$$

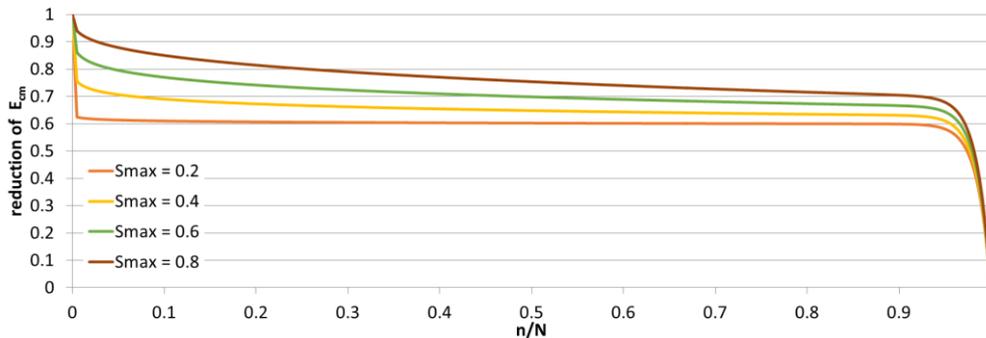


Fig. 4: Example of the fatigue damage function for various load levels

As stated above, the increase in the deflections caused by cyclic loading can be as much as 40% of the initial static deflections. This phenomenon should be taken into consideration especially in evaluating the deflections of existing structures by means of a repeated load test when assessing their remaining useful lifetime (for example bridges with the loading test at the start of the operation, common practice in some countries e.g. the Czech Republic).

With regard to the number of samples tested by Holmen and limitations of described approach only for macro-scale fatigue assessment other factors influencing modulus of elasticity (material texture, porosity etc.) are neglected.

EXPERIMENTAL VERIFICATION OF THE FATIGUE DAMAGE FUNCTION ON REINFORCED AND PRESTRESSED CONCRETE ELEMENTS

General approach to the experimental program with respect to its applicability to real structures

Two types of concrete specimens were prepared for an experimental verification of the fatigue damage function: reinforced concrete specimens and a pre-stressed concrete specimen. The two types are described in the following sections. The preliminary results were presented in [16] and [17]. The results presented here have been widened and evaluated using a new alternative approach for fatigue assessment.

Test arrangement specifications

The four-point bending tests were chosen for their advantage of zone subjected to pure bending (decomposed to pure compression/tension) without the influence of shear. This material point zone, i.e. the crack localization zone corresponds to the behaviour of common structures. The principle described above and the stress distribution on an ideal cross-section is illustrated in Fig. 5.

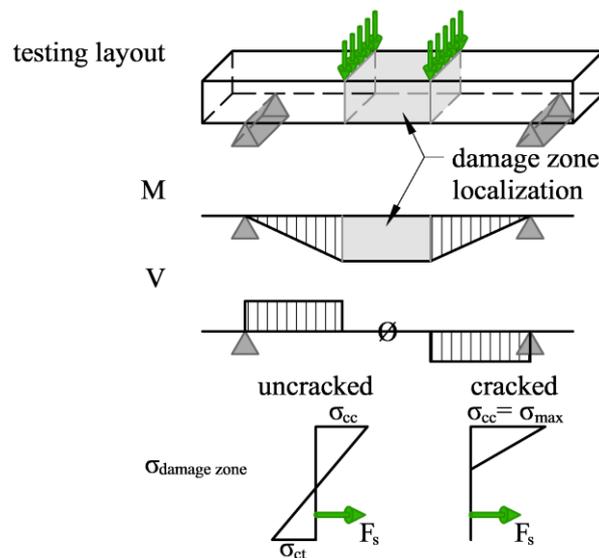


Fig. 5: Four-point bending arrangement and localization zone principle with the stress distribution in the damage zone

Evaluation of the variously stressed fibres of the cross-section

The stress distribution in the calibration experiments [4] was constant, so that each part of the cylinder was exposed to the same conditions. This assumption is not valid for specimens subjected to bending, where each part of the cross-section is exposed to a different compressive or tensile stress. It can be assumed that most of the structures subjected to cyclic loading are exposed to bending.

Dividing the compressive zone over the height of the cross-section was therefore chosen to include a different behaviour of variously stressed fibres, thus presenting a new and less conservative approach to fatigue assessment of a structural element subjected to bending. The analysed cross-section decomposes into an ideal cross-section of layers with a different modulus of elasticity depending on the stresses that it is subjected to. This approach should reflect the differences between the fatigue behaviour of the compressive cyclic loading of the cylinders, which the fatigue damage function is based on, and specimens subjected to cyclic bending.

Calculation of the deflections

For an evaluation of the deflections, partial integration over the compressive zone height was used to reflect the real stress distribution. The method used here is based on the average stress in each layer, and not on the maximal stress in the top fibres, as e.g. in Eurocode 2 [15]. The compressive zone of the cracked concrete specimens was divided into 20 layers (in the case

of 300x150x1300 specimens, each layer has about 3 mm in thickness). This division was based on a sensitivity analysis.

Using coarse division, up to 5 layers, the small average stresses in the wide integration layers lead to results on the unsafe side (in contrast with the standardized approaches). The calculated deflections did not show good agreement with the measured values, and were distinctly lower than the measured values.

With a finer division, up to 15 layers, the average stresses in the layers are higher (and correspond more to the real stresses). Thus some of the layers may exceed the limit number of applied cycles, which leads to a significant reduction in the modulus of elasticity (see below). The division is still coarse, and the height of the layer represents a considerable part of the cross-section. This significantly influences the ideal moment of inertia of the cross section, which may lead to the very conservative results.

Dividing the compressed zone of the ideal cross-section into 20 layers proved to be the optimal solution for these experimental settings. Finer division into more than 20 layers is not necessary, as the difference in the results is negligible in comparison with the possible increase in computational time requirements.

In the comparison of the calculated and measured data (Figure 9 – Figure 12), which will be discussed later, instant increases in the calculated deflections can be observed. This phenomenon is caused by the incorporation of the differences in fatigue behaviour between probe cylinders and beams in bending into the evaluation. From the definition of the fatigue damage function, the modulus of elasticity decreases to zero at the limit number of applied cycles N (this corresponds to the behaviour of the probe cylinders). In the case of cylinders, the specimen fails as a whole element after N cycles; but in the case of specimens subjected to bending, the bond between differently stressed neighbouring parts will influence the behaviour of the specimen. When one layer reaches the limit number of load cycles corresponding to the applied S_{max} , there is still a bond with the neighbouring part. When one of the layers deteriorates, the other layers take over its role. Due to this stress redistribution, some residual values of the modulus of elasticity of the extremely damaged part can be assumed. The position of the neutral axis changes with the deterioration of the layers, and thus changes the maximum stress applied to the layer.

To include the facts mentioned above, the constant multiplier of the modulus of elasticity is used when the fatigue damage function reaches the third phase according to Figure 1. This constant multiplier was taken as an average value of the fatigue damage function at the start of the third phase of the strain development, where the function takes values between approximately 0.65 and 0. Thus the modulus of elasticity value is 0.33 ($E_{residual} = 0.33 \cdot E_{cm}$) for $S_{max} \geq 0.2$. For low maximum stresses, it can be assumed that the influence of fatigue is lower. For that reason, the upper limit value of the fatigue damage function ($\omega = 0.65$) was chosen for $S_{max} < 0.2$ ($E_{residual} = 0.65 \cdot E_{cm}$). These instant changes in modulus of elasticity lead to instant increases in the calculated deflections, as was mentioned above (Fig. 9 – Fig. 12).

The principle of partial integration described above, and the use of different modulus of elasticity for calculating the ideal moment of inertia, is illustrated in Fig. 6.

In order to calculate the deflections, the ideal moment of inertia is needed. An evaluation of this cross-sectional characteristic was carried out in 4 steps:

- 1) From the fatigue analysis, the maximum stress levels S_{max} were obtained for each partial height (layer) within each specimen.
- 2) The fatigue damage function was evaluated depending on the maximum stress levels S_{max} .
- 3) The decreased modulus of elasticity was calculated with the fatigue damage function for each layer.

- 4) The ideal moment of inertia was calculated on the basis of the decreased modulus of elasticity of each layer.

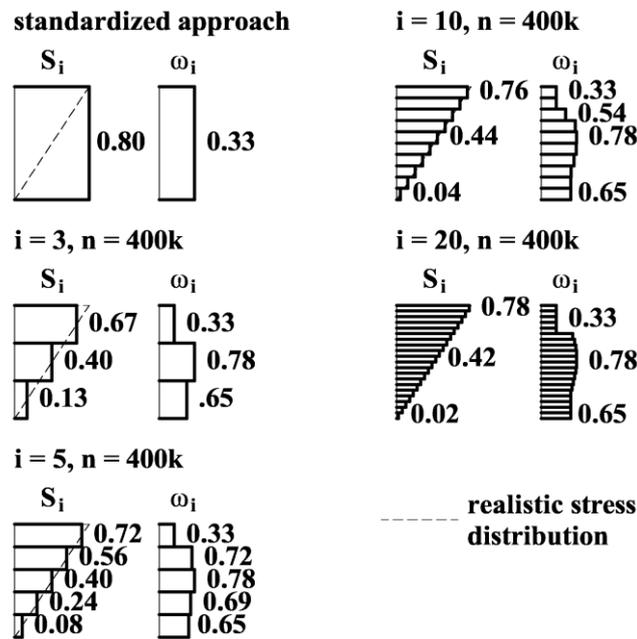


Fig. 6: Principle of partial integration of the compressive zone with the results of sensitive analysis for $n = 400,000$ cycles

Reinforced concrete specimens

Several sets of specimens were prepared for an experimental verification of the fatigue damage function on reinforced concrete. Each set contains a specimen for cyclic loading (300x150x1300 mm), specimens for evaluating the modulus of elasticity of concrete, tensile strength in bending and compressive strength (100x100x400 mm), and additional 150 mm probe cubes for evaluating the compressive strength.

All specimens were made with concrete strength class C25/30 (for the mix proportions, see Table 1 **Chyba! Nenalezen zdroj odkazů.**). The specimens proposed for cyclic loading were designed as over-reinforced (6Ø16 grade B500 reinforcing steel). Thus failure by compressive-zone crushing should occur and fatigue failure of the concrete can be assumed. The scheme of the dimensions and the reinforcement of the specimen is shown in Fig. 7.

Tab. 1: Mix proportions of the concrete mixture

Constituent	kg/m3
CEM II 32.5	320
Sand	836
Fine aggregate	495
Coarse aggregate	443
Water	185

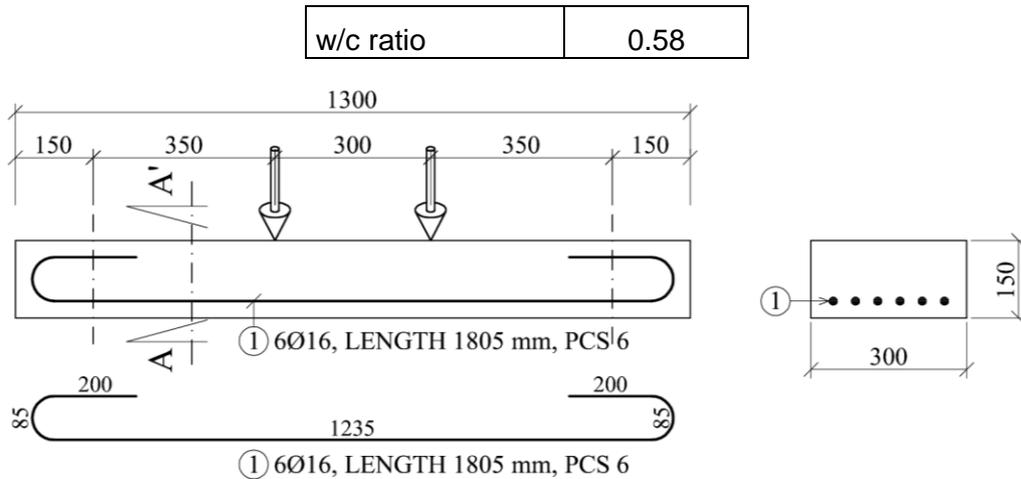


Fig. 7: Scheme of the fatigue testing specimen and its reinforcement

Testing layout

The arrangement of the cyclic loading is four-point bending with a span length of 1000 mm and overhangs 150 mm in length. This testing layout was chosen because it offers several advantages, as discussed in the previous sections. The experiments were conducted in the Experimental Center of the Faculty of Civil Engineering, Czech Technical University in Prague. The parameters of the fatigue testing are presented in Table 2. The arrangement is shown in Fig. 8.

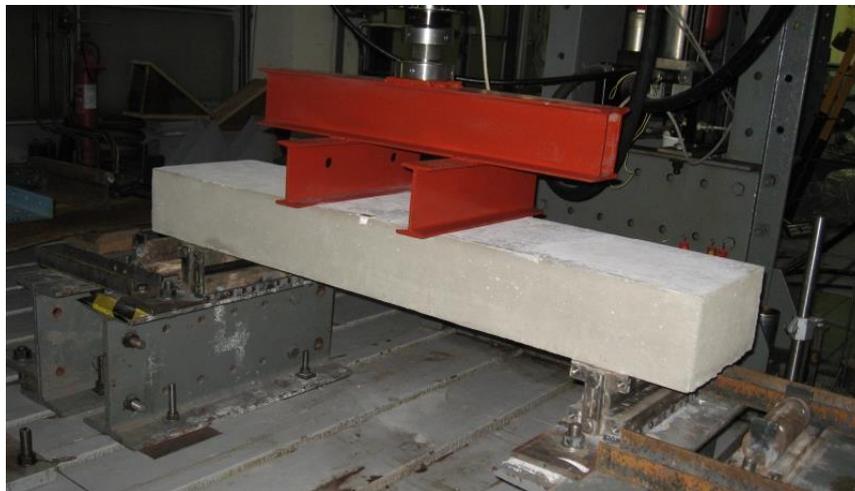


Fig. 8: Arrangement of the fatigue testing

Tab. 2: Specification of the fatigue testing settings

Loading frequency	Cyclic force		Eccentricity
	min	max	
Hz	kN		m
5	5	100	0.3

Deflection measurements and loading history

Two types of deflection measurements were performed during the fatigue testing. The first type - static deflection measurements - takes place after every hour of cyclic loading (circa 18000 load cycles). The second type - dynamic deflection measurement - was carried out during the fatigue testing immediately after the static deflection measurements. In order to obtain the exact deflection in the middle of the span, the settlement of the supports was also measured.

Each specimen was tested for one week, which corresponds to about 350-450 thousand cycles.

The deflections of the reinforced concrete specimens were measured by the inductive displacement transducer. Table 3 presents the measured deflections and the number of load cycles which they were measured at.

Tab. 3: Deflection measurements on reinforced concrete specimens

Specimen #							
1		2		3		4	
Age	32 days	Age	61 days	Age	29 days	Age	64 days
Test. date	21.9.2009	Test. date	20.10.2009	Test. date	24.9.2013	Test. date	29.10.2013
n_i	δ_i [mm]						
0	1.094	0	1.177	0	2.015	0	1.343
28080	1.334	7760	1.125	78772	1.730	18250	1.216
40440	1.406	14960	1.202	78772	1.380	40398	1.287
60930	1.654	21820	1.228	138225	1.404	54442	1.348
90450	1.514	40040	1.275	197512	1.393	68644	1.371
90450	1.511	60120	1.297	197512	1.400	83847	1.380
110560	1.576	60120	1.314	249298	1.406	92780	1.383
110560	1.562	78330	1.306	306619	1.407	92780	1.389
130970	1.552	97920	1.371	306619	1.416	112695	1.448
151200	1.578	117200	1.401	367726	1.380	129417	1.554
173990	1.592	134850	1.433	407785	1.430	146949	1.603
191900	1.506	134850	1.429			162622	1.624
211640	1.639	155700	1.409			180676	1.667
211640	1.544	174770	1.453			195123	1.680
231690	1.700	194160	1.497			205526	1.714
252500	1.578	209080	1.485			205526	1.738
271720	1.674					229617	1.761
292120	1.479					248947	1.747
312170	1.515					268999	1.776

312170	1.518			288429	1.793
332970	1.600			304231	1.825
360530	1.676			304231	1.850
392380	1.538			322571	1.845
				350076	1.828
				375004	1.841
				392653	1.857

Comparison of the measured deflection values of the reinforced concrete specimens and the values calculated using the fatigue damage function

Four specimens have already been tested. Specimen No. 1 in September 2009, No. 2 in November 2009, No. 3 in September 2013 and No. 4 in November 2013. Properties of the specimens are listed in Table 4. A comparison between the calculated and measured values is shown in Fig. 9 – Fig. 12.

The displacement was not measured continuously to prevent the fatigue damage of the inductive displacement transducer, thus each measurement had to be reinstrumented. This may result in the measurement errors as can be seen in Fig. 9. Still the trend is clearly observable.

Tab. 4: Properties of the tested specimens

Specimen no.	Age	f_{ck}	E_{init}
	[days]	[MPa]	[GPa]
#1	32	33.1	30.0
#2	61	34.2	34.3
#3	28	29.1	29.7
#4	60	34.0	30.0

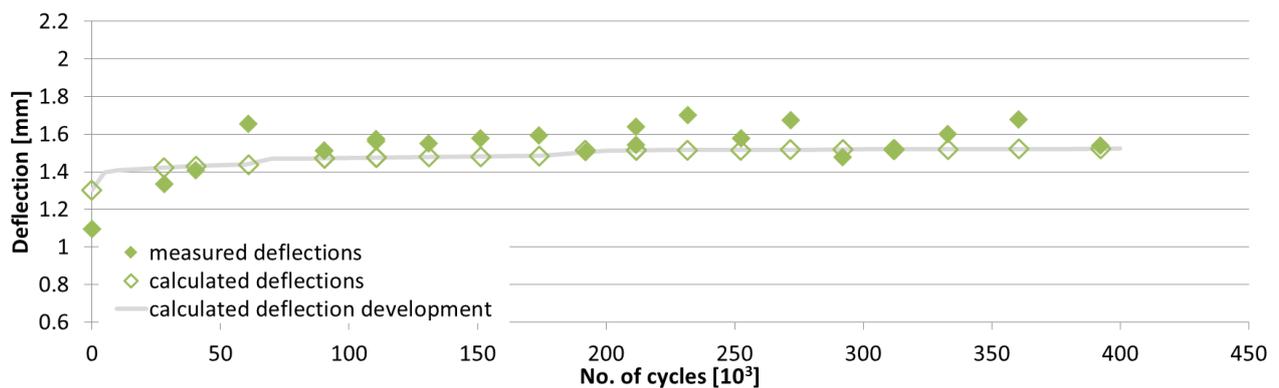


Fig. 9: Comparison of measured and calculated deflections for set #1

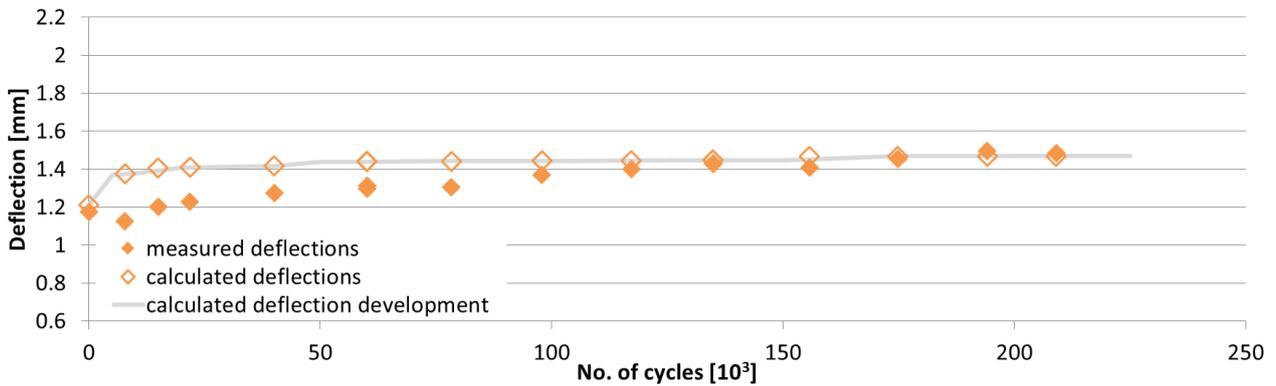


Fig. 10: Comparison of measured and calculated deflections for set #2

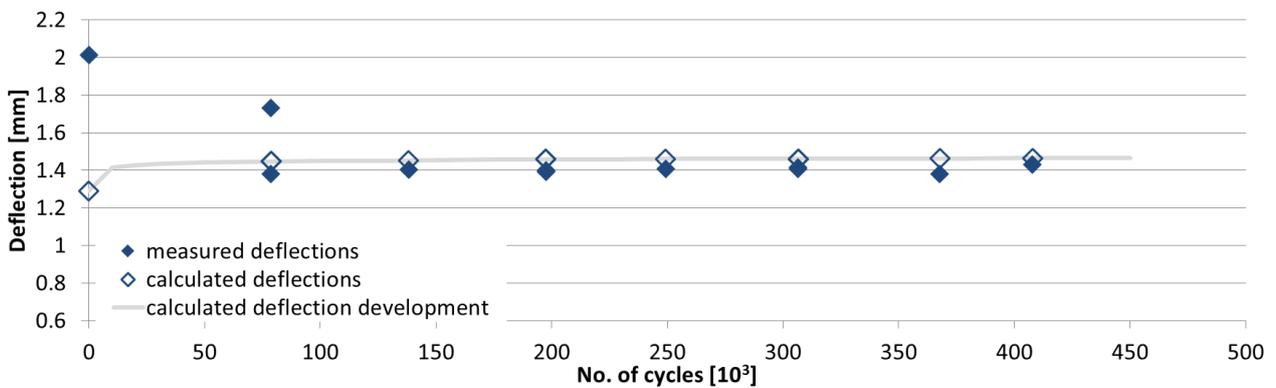


Fig. 11: Comparison of measured and calculated deflections for set #3

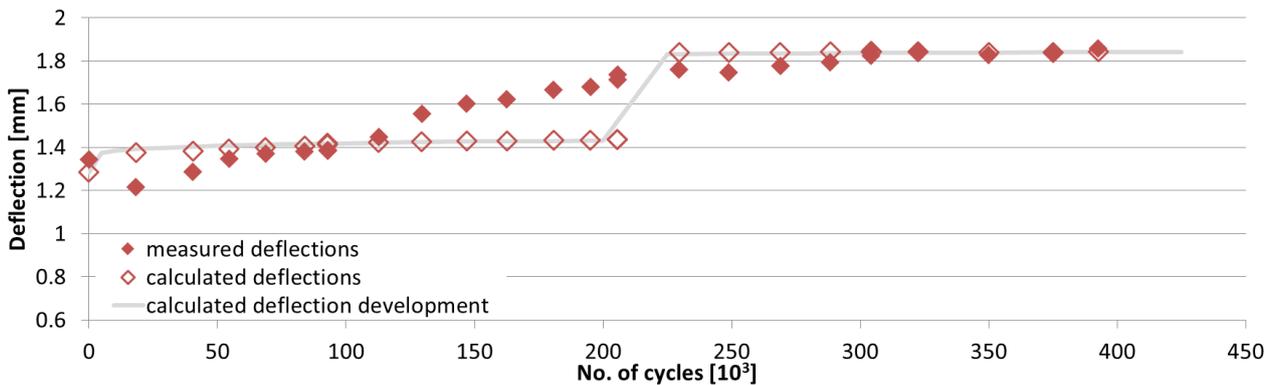


Fig. 12: Comparison of measured and calculated deflections for set #4

In Fig. 11 the decreasing slope at the beginning of cyclic loading and the difference in the second data point is probably a result of error in the first two measurements.

During the fatigue testing of the fourth specimen, a crack appeared in the direction of the principal tensile stress close to the support. This crack influenced the deflection measurements, as can be seen in Fig. 12. The crack opening developed between 110 thousand cycles and 230 thousand cycles. The increase in the deflection is about 0.4 mm. This increase was included in the comparison with the fatigue damage function.

Conclusions from the experimental verification of the fatigue damage function on the reinforced concrete specimens

The behaviour of the specimens corresponds with the trend calculated by the fatigue damage function, especially after first 100 thousand cycles. Within the first 100 thousand cycles, the values calculated with the fatigue damage function are higher than the measured values. It can be assumed, that this phenomenon is caused by the method of deflection calculation which uses moment of inertia of fully cracked cross section, when the cracks in specimens are not fully opened yet as described in [18] and Model Code 2010 [13].

The motivation for the approach presented here is to reflect the realistic distribution of the stress using the partial integration over the height. With this method, dividing the compressive zone into 20 layers provided optimal settings for the fatigue testing arrangement, and there was good agreement with the measured data. This approach should remove the conservativeness of the standardized model of fatigue assessment based on the analysis of the most stressed fibres.

Prestressed concrete specimen

The authors were allowed to incorporate their measurement system into the setup for the experiments described in [19].

The specimen was prestressed from one side by eleven 15.7mm prestressing tendons. The additional reinforcement was grade B500A. The strength class of the concrete of the prestressed slabs was prescribed as C 45/55. For the exact dimensions of the prestressed slab, see Fig. 13.

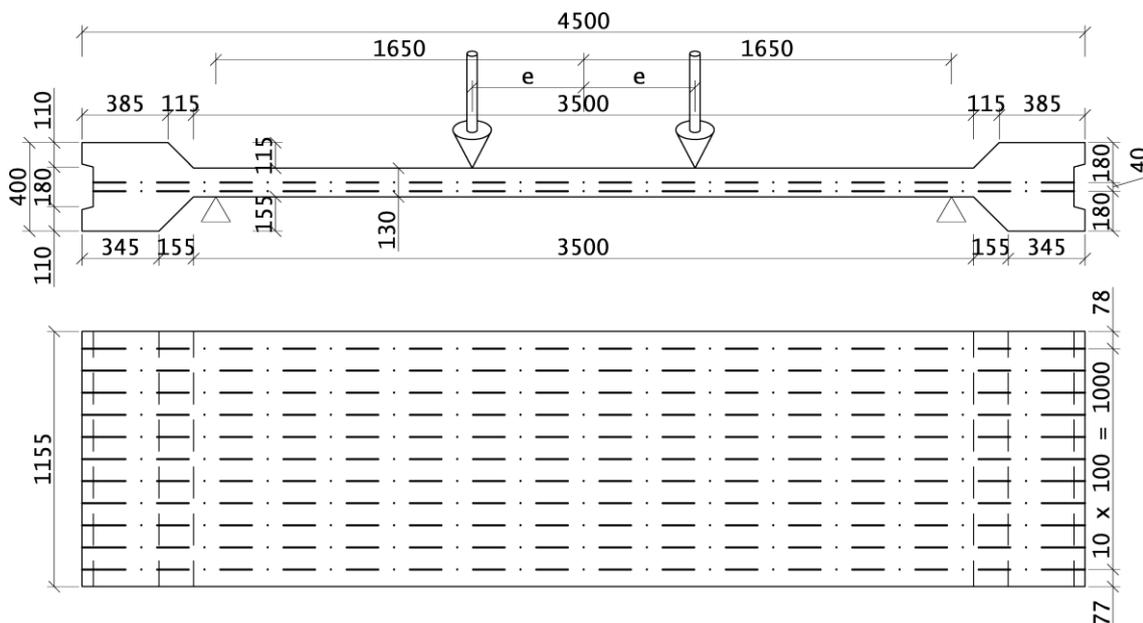


Fig. 13: Static scheme with the dimensions and the prestressing scheme in the slabs

For the evaluation of the deflections using the fatigue damage function, partial integration over the height of the specimen was used, as described above. However, the part of the cross-section subjected to tension was included. Values of the maximum of applied cycles N for the part in tension were evaluated according to Model Code 2010 [13] (eq. (7)), and are listed in Table 5 (positive values for compressive stress).

Tab. 5: Calculated values of the maximum applied cycles in tension according to Model Code 2010

Distance from the bottom fibres [m]	σ_{DL} [MPa]	σ_{DL+LL} [MPa]	N_k [-]
0.021	16.966	-0.331	1321789
0.018	17.441	-1.138	6.80E+09
0.015	17.915	-1.945	1.98E+08
0.011	18.390	-2.751	5744556
0.008	18.864	-3.558	167009
0.005	19.339	-4.365	4855
0.002	19.814	-5.171	141

The height of the prestressed specimen was divided into 40 layers to obtain similar precision as for the reinforced concrete specimens (each layer is ~3 mm in thickness). This division was based on a sensitivity analysis of the calculated deflection. When using coarse division, 5-10 layers, the difference between the calculated deflections was up to 10%. With a higher division, 20-25 layers, the difference was up to 4%. When using smooth division, 95-100 layers, the calculated difference was smaller than 1%. With the selected precision of 40 layers, the difference of the calculated deflections was up to 2%.

The influence of the cracks on the development of the increase in deflections due to cyclic loading needs to be discussed. The prestressed specimen passes through three stress distribution stages during cyclic loading. In the first stage, all fibres of the cross-section are subjected to compressive stresses; in the second stage, there is decompression in the bottom fibres; and, finally, in the third stage, the cross-section is divided into two zones - concrete subjected to compression, and concrete subjected to tension. The time sequence of these stages based on the characteristics of cyclic loading (sinusoidal loading) is illustrated in Fig. 14.

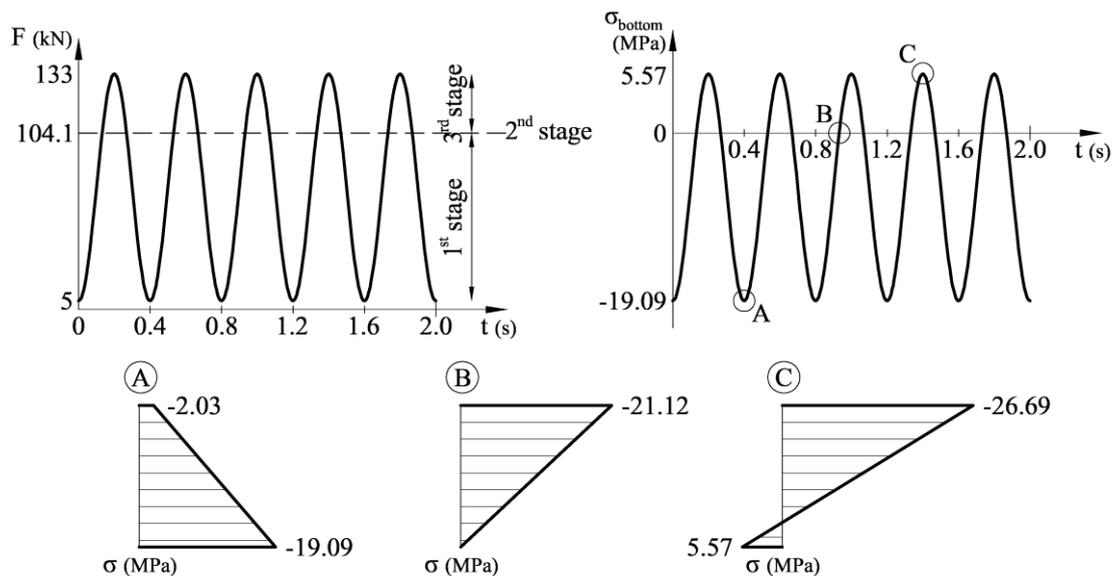


Fig. 14: Loading and stress development with emphasized stages

The major part of the stress distribution during cyclic loading corresponds to the first stage. In the third stage, the cracks open and the compressive zone gets smaller with the increase in the applied force. This means that, in the third stage, a stress reversal appears in the bottom fibres. In the approach presented here, the influence of the stress-reversal is taken into consideration.

In the approach followed by Eurocode 2 [15] (eq. (9)), the tensile stress is neglected and the minimum compressive stress is assumed to be 0 (for tensile stresses in the checked fibres). However, when assessing the fatigue endurance of a concrete specimen, the authors assumed that the difference in the stress distribution due to the opening of cracks has a low influence on the development of the fatigue damage function, or on the increase in deflections due to fatigue.

$$\frac{\sigma_{c,\max}}{f_{cd,\text{fat}}} \leq 0.5 + 0.45 \frac{\sigma_{c,\min}}{f_{cd,\text{fat}}} \quad (9)$$

The evaluation of the decreased modulus of elasticity for each layer was based on the stress distribution for an uncracked cross-section with stress ranges corresponding to all three stages. Thus the bottom fibres are exposed to stress reversal.

The influence of the cracked cross-section on deflections was included by increase in the calculated values. The increment was calculated as the difference between the calculated deflections of the uncracked specimen and the cracked specimen at the time of cracking ($n = 1\,350\,000$ cycles). For both calculated deflections the deteriorative effect of the 1 350 000 applied cycles was taken into account.

To verify these assumptions, two approaches were compared. In the first approach, an evaluation was made of the fatigue damage function for the cracked cross-section (with the influence of a crack opening, thus without the stress-reversal in the bottom fibres after $n = 1\,350\,000$ cycles); in the second approach, an evaluation was made of the fatigue damage function for the uncracked cross-section (without the influence of a crack opening, thus with stress-reversal in the bottom fibres for all evaluated cycles). The results obtained with the first approach were extremely conservative (when compared with the measured data). The results obtained with the second approach showed good agreement with the measured data. The assumptions were verified for the testing arrangement of the experiment (described below).

Test layout

The specimen was subjected to four-point bending with a span length of 3300 mm. The arrangement is shown in Fig. 15.

Deflection measurements and loading history

During the experimental program, a total of five deflection measurements were made by an inductive track recorder. The measurements were conducted as dynamic deflection measurements, i.e. during the fatigue testing. The loading history of the test specimens was not continuous. This was due to the limitations of the laboratories.

Static analysis of prestressed slab specimens

An independent static analysis of the specimen was made in order to evaluate the maximum stresses S_{\max} from the dead load and from cyclic loading.

The specimens were designed from concrete strength class C45/55, but the mean measured compressive strength value was 80 MPa. The modulus of elasticity was corresponding to Eurocode 2 for C80/95 ($E_{cm} = 42$ GPa). The measured static deflection was +6 mm.

The detailed FEM time-dependent analysis showed that the deformation of the specimen should be +5.8 mm in the middle of the span when loaded only by permanent loads, which corresponded to the deflections mentioned above.

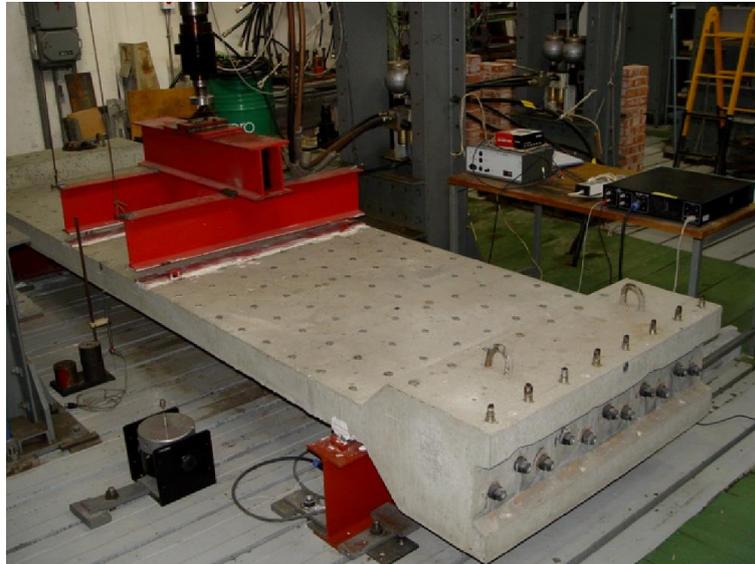


Fig. 15: Schematic layout of the experiments

Fatigue analysis of the prestressed specimens

Detailed fatigue analysis of the prestressed specimen was performed to predict the fatigue behaviour of the slab.

Considerations for the prestressed specimen were the same as in the case of the reinforced concrete specimens except that the fatigue endurance was calculated for the entire height of the specimen.

Table 6 shows the stress range for the load case applied to the prestressed specimen, together with the fatigue endurance in both characteristic and design values.

Tab. 6: Stress range due to dead load, dead load + cyclic load and fatigue endurance in design values with $\gamma_M = 1.5$ and $\gamma_M = 1.0$ according to Model Code 2010

Stress	Dead load	DL+LL	N $\gamma_M = 1.5$	N $\gamma_M = 1.0$
$\sigma_{top, fibers}$	-1.07	-26.69	7	4.49E+06
$\sigma_{bottom, fibers}$	-20.05	5.57		

Similarly as for the reinforced concrete specimens, the characteristic material properties values are used for verifying the fatigue endurance.

The verification of the fatigue endurance of the prestressed specimen according to Eurocode 2 [15] and Model Code 2010 [13] shows that the slab could not have experienced a compressive fatigue failure in the top fibres during the 2 268 570 load cases that it resisted.

Comparison of measured and calculated deflection values

A total of five deflection measurements were made. The time of the measurements, the measured deflections and the number of load cycles that were measured are shown in Table 7.

Tab. 7: Deflection measurements on Slab No. 2

Measurement No.	Time [days]	Days between measurements [days]	No. of load cycles at measurement [-]	Measured deflections [mm]
1	89	-	1 280 040	14.437
2	104	15	1 504 000	17.937
3	160	56	1 740 360	18.303
4	187	27	2 019 370	17.688
5	217	30	2 268 570	17.991

Fig. 16 shows the loading history of a prestressed specimen with the locations of the measurements marked.

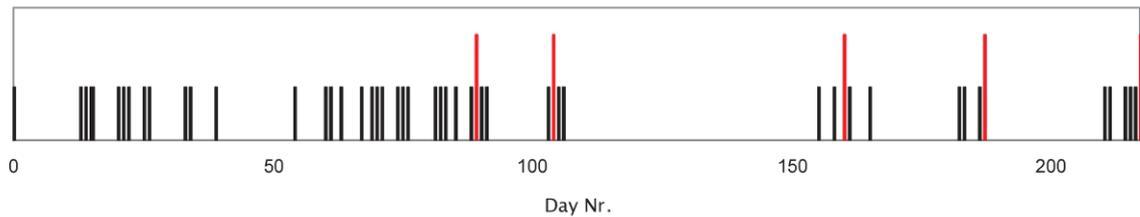


Fig. 16: Loading history of the prestressed slab with time of the deflection measurements marked

As was mentioned above, cracks propagated on the soffit of the slab after approximately 1 350 000 load cycles. These cracks resulted in an irreversible increase in the deformations between measurements 1 and 2 (see **Chyba! Nenalezen zdroj odkazů.**able 7).

Measured deflection values and values calculated using the fatigue damage function are summarized in Fig. 17.

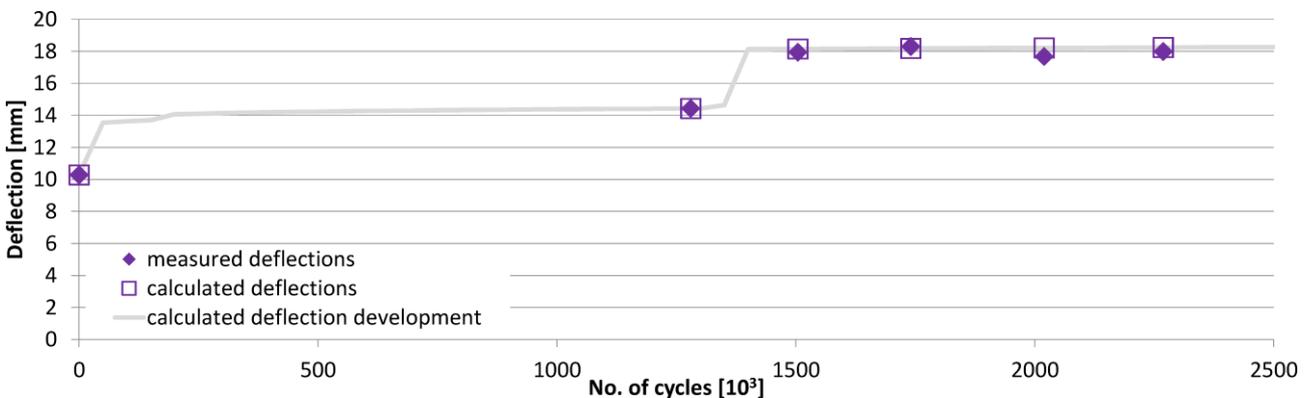


Fig. 17: Comparison of measured and calculated deflections of a prestressed specimen

With precision of 40 layers, the difference between the measured and calculated deflections is 5%. As was stated in section 3.3, the choice of this number of layers means that the difference in the calculated values is around 2%. The maximal potential error between the calculated and measured values is therefore assumed to be up to 7%.

The crack development in the bottom fibers of the prestressed specimen increased the uncertainty of the calculation. Within the rest periods, which were quite long (see Fig. 16), self-healing processes take place in the concrete:

- the stress concentrations on the tips of the cracks decrease due to relaxation of stresses,
- according to stress distribution within the prestressed specimen, the cracks close and can be healed by pore water reacting with unbonded cement.

A positive effect of rest periods on fatigue performance was observed in many cases, see e.g. [20].

The difference between static deflections and deflections caused by cyclic loading is emphasized in Table 8. There is a significant increase of 40% for the uncracked cross section, and 78% after crack development.

Tab. 8: Difference between static deflections and deflections within cyclic loading

Measurement No.	Static deflection [mm]	Measured deflections [mm]	Calculated deflections [mm]	Measured / calculated deflections [mm]	Measured / static deflections [mm]
1	10.286	14.437	14.430	1.001	140.36%
2	13.327	17.937	18.151	0.988	134.59%
3	13.327	18.303	18.184	1.007	137.34%
4	13.327	17.688	18.217	0.971	132.72%
5	13.327	17.991	18.244	0.986	134.99%

The predicted deflection behaviour of the prestressed specimen under cyclic loading shows that the initial values of the static deflection can increase up to 1.4 times without damage leading to the failure of an element. The prestressed specimen was not exposed to cyclic loading up to failure, so an even greater increase can be assumed before the slab collapses (i.e. enters the third phase of strain development under cyclic loading).

Conclusions from experimental verification of the fatigue damage function on prestressed slab specimens

Despite the limitations of the experiments, which were designed for a different purpose, the comparison between the measured deflection values of the prestressed specimen and the values calculated using the fatigue damage function shows very good agreement.

The deflections were measured from assumed $n/N = 0.27$ to $n/N = 0.49$, i.e. in the middle of the fatigue endurance of the specimen - the second phase of strain development under cyclic loading.

The principle presented here for evaluating the deflection, taking into consideration the cross-section in tension and neglecting the deteriorative influence of the infrequent stress distribution on a cracked cross-section, has proved to be a possible approach.

According to the experimental results, partial integration is a possible tool for evaluating the increased deflections due to cyclic loading for the prestressed specimens.

CONCLUSIONS

This paper has presented the fatigue damage function, a mathematical function for describing the strain development in concrete under cyclic loading, together with an experimental verification. The fatigue damage function produces a decreasing multiplier of the original modulus of elasticity at the start of cyclic loading, which represents the deteriorative effect of cyclic loading on a concrete structural element. With the help of the fatigue damage function, the increase in the deformations of cyclically loaded structural elements can be assessed and the total fatigue endurance and/or the remaining fatigue endurance can be predicted. This tool can be useful for example for the evaluation of the remaining useful fatigue life of bridges on which a load test was performed at the start of the operation.

The paper has presented an experimental verification of the fatigue damage function on reinforced concrete specimens and on a prestressed concrete slab. For calculating the increase in the deformations, a newly-developed method of partial integration over specimen height has been used to capture the real behaviour and the stress distribution of concrete specimens. This method can represent an improvement of the standard approaches, which appear to be very conservative. The method of partial integration has proved itself to be a useful tool for the evaluating the increase in deformations due to fatigue.

The measured deflection values and the values calculated using the fatigue damage function show very good agreement. A detailed analysis, and also the experimental measurements, have shown that the deflections of a cyclically loaded concrete structural element can reach as much as 140% (for reinforced concrete and also for prestressed concrete) of the initial static deflection without significantly reducing the load-bearing capacity and/or without any danger of the element failing due to fatigue failure of the concrete. Based on the performance of the specimens at the end of the testing, it can be assumed that an even higher increase in deflection is possible before the element fails.

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