

# STATISTICAL ASSESSMENT OF QUANTITATIVE QUALITY PERFORMANCE OF BUILDING PRODUCTS DURING CONFORMITY ASSESSMENT AND CERTIFICATION IN THE FIRE PROTECTION FIELD

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## ABSTRACT

This paper proposes statistical methods for evaluation of quantitative performance of building products determined by tests for objective decision during conformity assessment/certification in the fire protection field. The procedure is applicable even for other types of products, and for research / development of new material products.

## KEYWORDS

Building products, quantitative quality performance, statistical assessment, confidence interval, confidence coefficient, certification

## INTRODUCTION

On the Czech market the construction products are subject to Law No. 22/1997 Coll. [1] and the Government regulation (GR) No. 163/2002 [2] among other things. GR also obligates to examine the so-called "specified products". Every manufacturer shall demonstrate their compliance with the relevant standards. The reason is consumer protection from the point of view of health protection, fire safety, hygiene, energy saving and environmental protection. Annex 2 of the above-mentioned regulation shows the following main groups of products specified for conformity assessment:

- ✓ construction products for concrete and reinforced concrete construction parts,
- ✓ construction products for masonry constructions,
- ✓ building wood products and wood constructions,
- ✓ construction products for metal constructions,
- ✓ protective, thermal insulation materials and products, waterproof materials, roof covering and adhesives,
- ✓ glass building products,
- ✓ construction products for sewer systems and distribution of liquids and gases,
- ✓ construction products for hole fillings,
- ✓ special materials, products, structures and equipment,
- ✓ the technical equipment of buildings,
- ✓ construction products for internal and external finishes of walls, ceilings, floors,
- ✓ construction products for sanitary facilities and other special products.

The European Parliament and Council Regulation (EU) No. 305/2011 (CPR) [3] also apply to the European market of construction products. According to this regulation the assessment obligation occurs when the products are determined by the requirements of the harmonized Czech (European) technical standards and/or European Technical Assessments (ETA). Manufacturers are entitled to label their products with the CE mark (CCZ is the Czech mark of conformity) after fulfilling the prescribed requirements.

The statistical assessment of the product's prescribed quantitative quality performance test values determined according to the prescribed test standards is an important tool for the objective determination whether the product meets/does not meet it.

### STATISTICAL ASSESSMENT OF THE QUANTITATIVE RESULTS OF TEST DETERMINATIONS

The quality of the material construction products may be characterized by the test of a quality feature e.g.  $M$ , test-determined which has a specific value e.g.  $M_0$  in a quality product. If the value  $M = M_0 \pm U$  is found in the supplied sample of product in verifying the property  $M$  during certification/conformity assessment, its quality is confirmed and a certificate can be granted.  $U$  is permissible tolerance's result.

Otherwise, if  $M < M_0 - \Delta$ , or if  $M > M_0 + \Delta$ , and  $\Delta$  is  $> U$  its quality is not confirmed and a certificate cannot be granted to it.

Standardized test method usually requires measurements of  $M$  product's property repeatedly  $n$  times with particular numerical results of  $X_1, X_2, \dots, X_n$ .

Results of  $X_i$  ( $i = 1, 2, \dots, n$ ) are not equal to  $M$ , but they have values of  $X_i = M + u_i$  where  $u_i$  are the individual measurement uncertainties. We assume that the measured results were/are statistically evaluated to remoteness and outliers and that the remote values were /are excluded [5]. If these uncertainties are mutually independent and have normal distribution with the mean value  $\Theta$  and the variance  $\sigma^2$  or more precisely the standard deviation  $\sigma$ , then the following procedure can be used to evaluate the results of tests:

- the so-called confidence interval (CI) is determined for the  $M$  value so that there applies:

$$P(\underline{M} \leq M) \cong 1 - \alpha/2, \text{ that is } P(\underline{M} > M) \cong \alpha/2 \quad (1)$$

$$P(\overline{M} \geq M) \cong 1 - \alpha/2, \text{ that is } P(\overline{M} < M) \cong \alpha/2 \quad (2)$$

$$\text{Then } P(\underline{M} \leq M \leq \overline{M}) \cong (1 - \alpha), \quad (3)$$

The probability that the interval does not contain the actual value is approximately  $\alpha$ .

Number  $(1 - \alpha)$ , selected according to the severity of the consequences of errors (usually 0.90 or 0.95 or 0.99) is the so-called coefficient of reliability (confidence coefficient). Its complement to 1, equal to  $\alpha$ , is the so-called Risk of mistake.

Under the assumptions (normality of errors distribution) the confidence interval has the following formula

$$\underline{M} = \bar{x} - \mu_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}, \quad (4)$$

$$\overline{M} = \bar{x} + \mu_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}, \quad (5)$$

Where  $\bar{x}$  is the sample average of the results

$$\bar{x} = \frac{1}{n} (x_1 + x_2 + \dots + x_n) \quad (6)$$

and  $\mu_{1-\alpha/2}$  is the so-called quantile of the standard normal distribution, defined by the formula

$$\Phi(\mu_{1-\alpha/2}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\mu_{1-\alpha/2}} e^{-\frac{z^2}{2}} dz = 1 - \frac{\alpha}{2} \quad (7)$$

For the most common values of  $\alpha$  (0.1 or 0.05 or 0.01) the quantiles' sizes are listed in the following table [6]

Tab. 1: Quantiles of standard normal distribution for the selected values of errors' risks  $\alpha$

$\alpha$	$1 - \alpha/2$		$M_{1-\alpha/2}$
0.01	0.995		2.576
0.05	0.975		1.960
0.1	0.950		1.645
0.2	0.900		1.282

Decision-making about the product's ability to the given purpose/ compliance could be as follows:

$$\bar{M} > M_H = M_0 + \Delta, \quad (8)$$

this is the possibility that  $M > M_H$  is not excluded, therefore the product does not meet the quality criterion and cannot be approved,

$$\underline{M} < M_D = M_0 - \Delta, \quad (9)$$

this is the possibility  $M < M_D$  is not excluded. Again, the product does not meet quality criterion and cannot be approved,

$$M_D < \underline{M} < \bar{M} < M_H, \quad (10)$$

indicates with a high probability that  $M$  is among the permissible limits  $M_D$  and  $M_H$  and so the product can be approved.

Degree of protection against possible error can be expressed as follows:

- If in fact

$$M > M_H = M_0 + \Delta, \quad (11)$$

then with the preselected high probability of  $(1 - \alpha)/2$  will be  $\bar{M} > M_H$  and if  $M > M_H$ , the product is needed to be disapproved with a high probability ,

- If  $M < M_D = M_0 - \Delta,$  (12)

then with the probability of  $(1 - \alpha)/2$  will be  $\underline{M} < M_D$  and the product must also be disapproved,

- If in fact,  $M \cong M_0$  (13)

( $M$  equals to the correct value  $M_0$ ), then with a high probability  $(1 - \alpha)$  will be

$M_D < \underline{M} < M_0 < \bar{M} < M_H$  and the product can be approved (justified correct decision). An unpleasant/indecisive situation can occur when confidence interval  $(\underline{M}, \bar{M})$  contains both the value of  $M_0$ , and one of the values  $M_D, M_H$ . In this case, the possibility  $M = M_0$  (correct value) cannot be excluded, nor  $M < M_D$  ( $M$  is too low) possibly

$M > M_H$  ( $M$  is too high). This situation can be avoided if the number of measurements  $n$  is chosen so that the length of the confidence interval

$d = \underline{M} - \bar{M}$  is smaller than  $\Delta$  number, that is when

$$n = \min \{k, k_{integer}, k \geq \frac{4\mu_{1-\alpha/2}^2 \cdot \sigma^2}{\Delta^2}\} \quad (14)$$

A simple calculation can convince us that in this case it is not possible that the interval  $(\underline{M}, \bar{M})$  contains the correct value  $M_0$ , and simultaneously one of already unacceptable levels of  $M_D$  or  $M_H$ .

However, if the value of  $\sigma$ , characterizing the variability of the results of errors of measurement is not known, it should be estimated from the measurement results of

$x_1, x_2, \dots, x_n$  according to the formula

$$s = \frac{1}{\sqrt{n-1}} \cdot \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}, \quad (15)$$

where

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^n x_i \quad (16)$$

Then the confidence interval is

$$\underline{M} = \bar{x} - t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}} \quad (17)$$

$$\bar{M} = \bar{x} + t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}} \quad (18)$$

where  $t_{n-1, \alpha/2}$  is the so-called 100  $\alpha/2$  %-critical value of the Student's t-distribution with  $(n-1)$  degrees of freedom [6]. The length of the confidence interval

$$(\bar{M} - \underline{M}) = 2 t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}} \quad (19)$$

of course, is not known. It depends on the value  $s$  that is determined by measuring/test. Therefore we cannot determine the required number  $n$  of repeated measurements.

The way out of this situation is the following two-step procedure for determining a confidence interval of predetermined length:

1.  $n_1$  repeated measurements of monitored variable  $X$  will be realized and then basic statistical indicators of the result will be calculated:

- sample arithmetic average

$$\bar{x}_1 = \frac{1}{n_1} (x_1 + x_2 + \dots + x_{n_1}) \quad (20)$$

and sample standard deviation

$$s_1 = \frac{1}{\sqrt{n_1 - 1}} \cdot \sqrt{\sum_{i=1}^{n_1} (x_i - \bar{x}_1)^2}, \quad (21)$$

2. the length of the confidence interval will be calculated at these values

$$d_1 = \bar{M} - \underline{M} = 2 s_1 \cdot t_{n-1, \alpha/2} \cdot \frac{1}{\sqrt{n_1}} \quad (22)$$

If  $d_1 \leq \Delta$ , one can proceed as in the case where  $\sigma$  is known with the value  $s_1$  instead of the  $\sigma$  value.

3. If in the second step  $d_1 > \Delta$  further  $n_2$  measurements

$x_{n_1+1}, x_{n_1+2}, \dots, x_{n_1+n_2}$  will be realized and one will calculate the new arithmetic mean

$$\bar{x} = \frac{1}{n_1 + n_2} (x_1 + x_2 + \dots + x_{n_1} + x_{n_1+1} + x_{n_1+2}), \quad (23)$$

$\bar{x}$  is the sample arithmetic mean of all  $n = n_1 + n_2$  measurements.

Confidence interval ( $\underline{M}$ ,  $\overline{M}$ ) shall be determined as

$$\underline{M} = \bar{x} - \Delta, \quad (24)$$

$$\overline{M} = \bar{x} + \Delta, \quad (25)$$

Number of additional needed  $n_2$  measurements is determined so that

$$\sqrt{n_1 + n_2} \geq 2 t_{n_1 - 1, \alpha/2} \cdot \frac{s_1}{\Delta} \quad (26)$$

## CONCLUSION

The current century is predicted to be "The century of quality". Without question the products' quality improvement demands not only the correct, accurate and fair testing according to the respected international test methods, but also proper using the statistical methods during the assessment of measured results. It is valid fully apart from other things for the products' assessment of conformity/certification, and in the research and development of new material products.

## REFERENCES

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