Evaluation of the gravity field model EIGEN-6C4 in comparison with EGM2008 by means of various functions of the gravity potential and by GNSS/levelling

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Abstract

The combined gravity field model EIGEN-6C4 (Förste et al., 2014) is the latest combined global gravity field model of GeoForschungsZentrum (GFZ) Potsdam and Groupe Recherches Geodesie Spatiale (GRGS) Toulouse. EIGEN-6C4 has been generated including the satellite gravity gradiometry data of the entire GOCE (Gravity and Ocean Circulation Experiment mission, November 2009 till October 2013, see i.e. Floberghagen et al., 2011 [4], Rummel et al., 2011 [13]) and is of maximum spherical degree and order 2190. In this study EIGEN-6C4 has been compared with the Earth's gravity field model EGM2008 to its maximum degree and order via gravity disturbances and the Tzz part of the Marussi tensor of the second derivatives of the disturbing potential. The emphasis is put on such areas where GOCE data (complete set of gradiometry measurements after reductions) in EIGEN-6C4 obviously contributes to an improvement of the gravity field description. GNSS/levelling geoid heights are independent data source for the evaluation of gravity field models. Therefore, we use the GNSS/levelling data sets over the territories of Europe, USA, Canada, Brazil, Japan, Czech Republic and Slovakia for the evaluation of EIGEN-6C4 w.r.t. EGM2008.

1. Theory

1.1. Gravitational potential and Marussi tensor

The disturbing static gravitational potential outside the Earth masses in spherical coordinates in spherical expansion reads

$$V\left(r,\varphi,\lambda\right)=\frac{GM}{r}\sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{R}{r}\right)^{l}\left(C'_{l,m}\cos m\lambda+S_{l,m}\sin m\lambda\right)P_{l,m}\left(\sin\varphi\right) \tag{1}$$

where GM is a product of the universal gravity constant and the mass of the Earth (known from satellite analyses as a geocentric gravitational constant), r is the radial distance of an external point where V is computed, the symbol R is for the radius of the Earth (which can be approximated by the semi-major axis of a reference ellipsoid), $P_{l,m}(sin\varphi)$ are Legendre associated functions, l and m are the degree and order of the harmonic expansion, (φ, λ) are geocentric latitude and longitude, $C'_{l,m}$ and $S_{l,m}$ are harmonic geopotential coefficients (Stokes parameters), fully normalized, $C'_{l,m} = C_{l,m} - C^{el}_{l,m}$, where $C^{el}_{l,m}$ belongs to the reference ellipsoid.

In our comparisons we will use spherically approximated **gravity anomaly** Δg, defined as

$$\Delta g = \frac{-\partial V}{\partial r} - 2\frac{V}{r} \tag{2}$$

and **Marussi tensor**.

The gravity gradient tensor Γ (the Marussi tensor) is a tensor of the second derivatives of the disturbing potential V and is computed by means of C'_{lm}, S_{lm} of the particular gravity field model known to the maximum degree $l_{\max}$

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 V}{\partial x^2} & \frac{\partial^2 V}{\partial x \partial y} & \frac{\partial^2 V}{\partial x \partial z} \\ \frac{\partial^2 V}{\partial y \partial x} & \frac{\partial^2 V}{\partial y^2} & \frac{\partial^2 V}{\partial y \partial z} \\ \frac{\partial^2 V}{\partial z \partial x} & \frac{\partial^2 V}{\partial z \partial y} & \frac{\partial^2 V}{\partial z^2} \end{bmatrix} \tag{3}$$

If we use a local coordinate system in point P and basis vectors are oriented to south-north-radial direction, where r means radial direction, we can write

$$\Gamma_{33} = \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2 V}{\partial r^2} = T_{zz}. \tag{4}$$

where T_{zz} is the second derivatives of the disturbing potential in radial direction. All these values were computed by software developed by Bucha and Janák, 2013 [2].

1.2. GNSS/levelling – method

A network of geodetic points with levelling heights and observable with GNSS techniques has been established on the different territories of the world. It enables a direct computation of the geoid (if we use thru ortometric heights) or quasigeoid (if we use normal Molodensky heights) undulation ξ according to a simple formula

$$\xi = h_{GNSS} - H_n + \{2\}, \tag{5}$$

where h_{GNSS} is the ellipsoidal height with respect to a reference ellipsoid (here we used GRS80 in all cases) derived from GNSS measurements, H_n is the physical (sea-level) height derived from the levelling, here the normal height according to Molodensky in the Baltic vertical datum, and $\{2\}$ are small terms of the second order, accounting for the curvature of the plumb line. The method is described in Kostelecký et al., 2012 [8].

2. The Gravity Field Model EIGEN-6C4

EIGEN-6C4 (European Improved Gravity model of the Earth by New techniques) is a static global combined gravity field model up to degree and order 2190. It has been elaborated jointly by GFZ Potsdam and GRGS Toulouse and contains the following satellite and ground data:

- LAGEOS-1/2 (deg. 2 - 30): Satellite Laser Ranging data 1985 - 2010
- GRACE, GNSS-SST and K-band range-rate data, processing according to RL03 GRGS (deg. 2 - 130): ten years 2003 - 2012
- GOCE, Satellite Gravity Gradiometry (SGG) data, processed by the direct approach including the gravity gradient components Txx, Tyy, Tzz and Txz out of the following time spans: 837 days out of the nominal mission time span 20091101 – 20120801, 422 days out of the lower orbit phase between 20120801 – 20131020. These GOCE data as well as the LAGEOS and GRACE data are the same as used for the 5th release of ESA's satellite-only gravity field model via the direct approach GO_CONS_GCF_2_DIR_R5 (Pail et al., 2011 [10] and Bruinsma et al., 2014 [1]). For EIGEN-6C4, the GOCE polar gaps were stabilized by the Spherical Cap Regularization using the combined gravity field model EIGEN-6C3stat
- terrestrial data (max degree 370): DTU12 ocean geoid data and an EGM2008 geoid height grid for the continents.

The combination of these different satellite and surface data sets has been done by a band-limited combination of normal equations (to maximum degree/order 370), which are generated from observation equations for the spherical harmonic coefficients (e. g. Shako et al., 2013 [14]). The resulted solution to degree/order 370 has been extended to degree/order 2190 by a block diagonal solution using the DTU10 global gravity anomaly data grid.

3. Models tested and differences between them

The topic and the method are well-established. We continue the work of many other authors. Here we report about our recent tests with EGM2008 (Pavlis et al., 2008, 2012 [11, 12]) and EIGEN-6C4 (Förste et al., 2014 [5]).

The models are compared by means of gravity disturbances, part of Marussi tensor elements, (for theory and examples of various applications see Kalvoda et al., 2013 [6]; Klokočník et al., 2014 [7]) and via GNSS/levelling.

Regional differences for Himalaya, Ethiopia, Egypt, Europe, and specifically for the Czech Republic are presented. Much more examples are available but cannot be presented due to the lack of space. We selected the following examples: first for an area in the Himalaya (Figs. 1 a,b,c,d) with a low quality of terrestrial data in both models (fill-in data) in a remote area with mountains, Ethiopia (Figs. 2 a,b,c,d) - one in the area with better data, outside Europe with very rich river system and erosion, Egypt – old and present Nil river (Figs. 3 a,b,c,d), for Europe (Figs. 4 a,b,c,d) and for the Czech Republic (Figs. 5 a,b,c,d). GNSS/levelling data for Europe (Figs. 6 a,b) comes from EVRS 1997 campaign, and for the Czech Republic (Figs. 7 a,b,c) and Slovakia (Figs. 8 a,b) the high quality terrestrial data

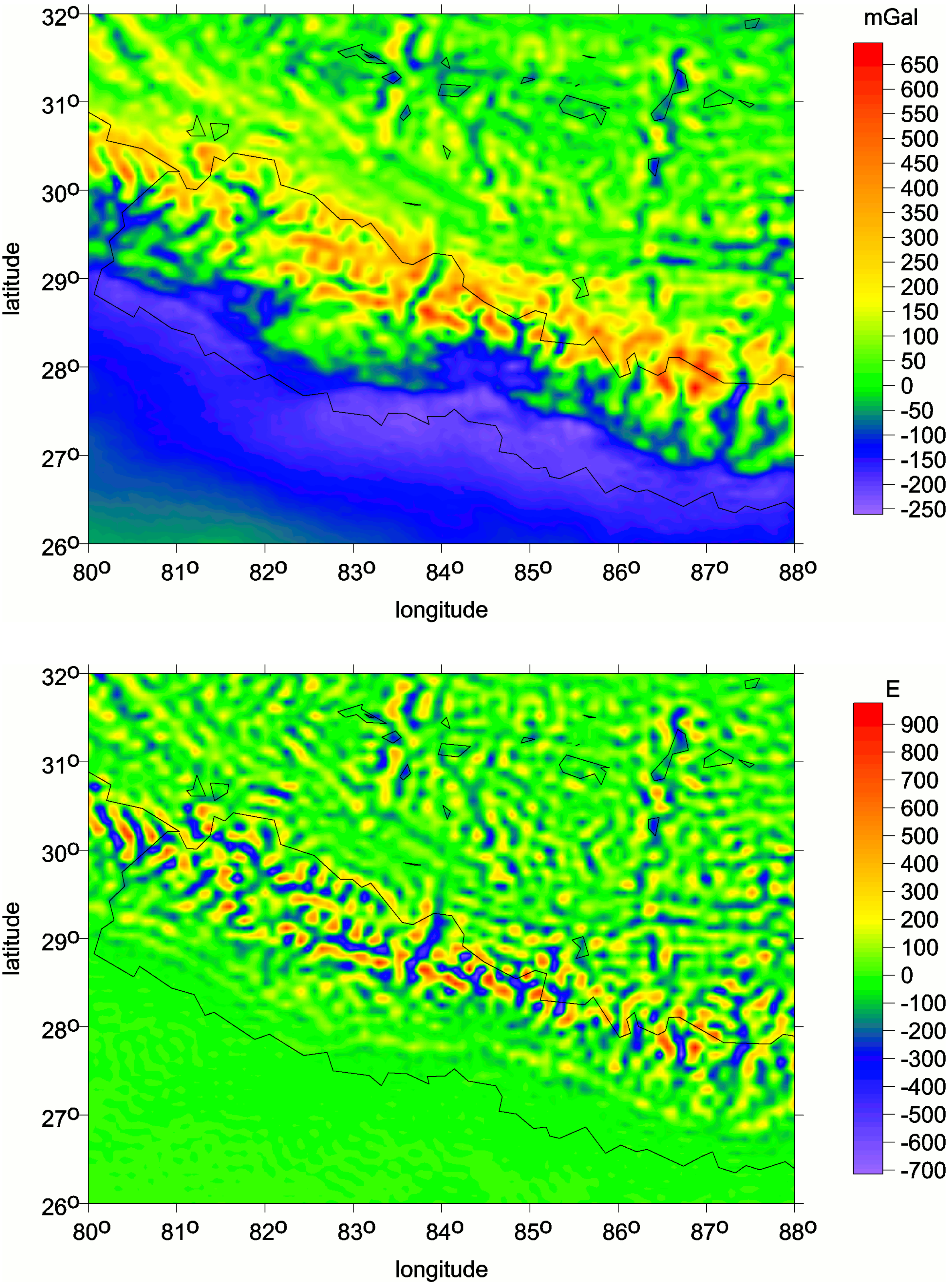

Figure 1 a, b: EIGEN-6C4, Himalaya Δg and T_{zz}

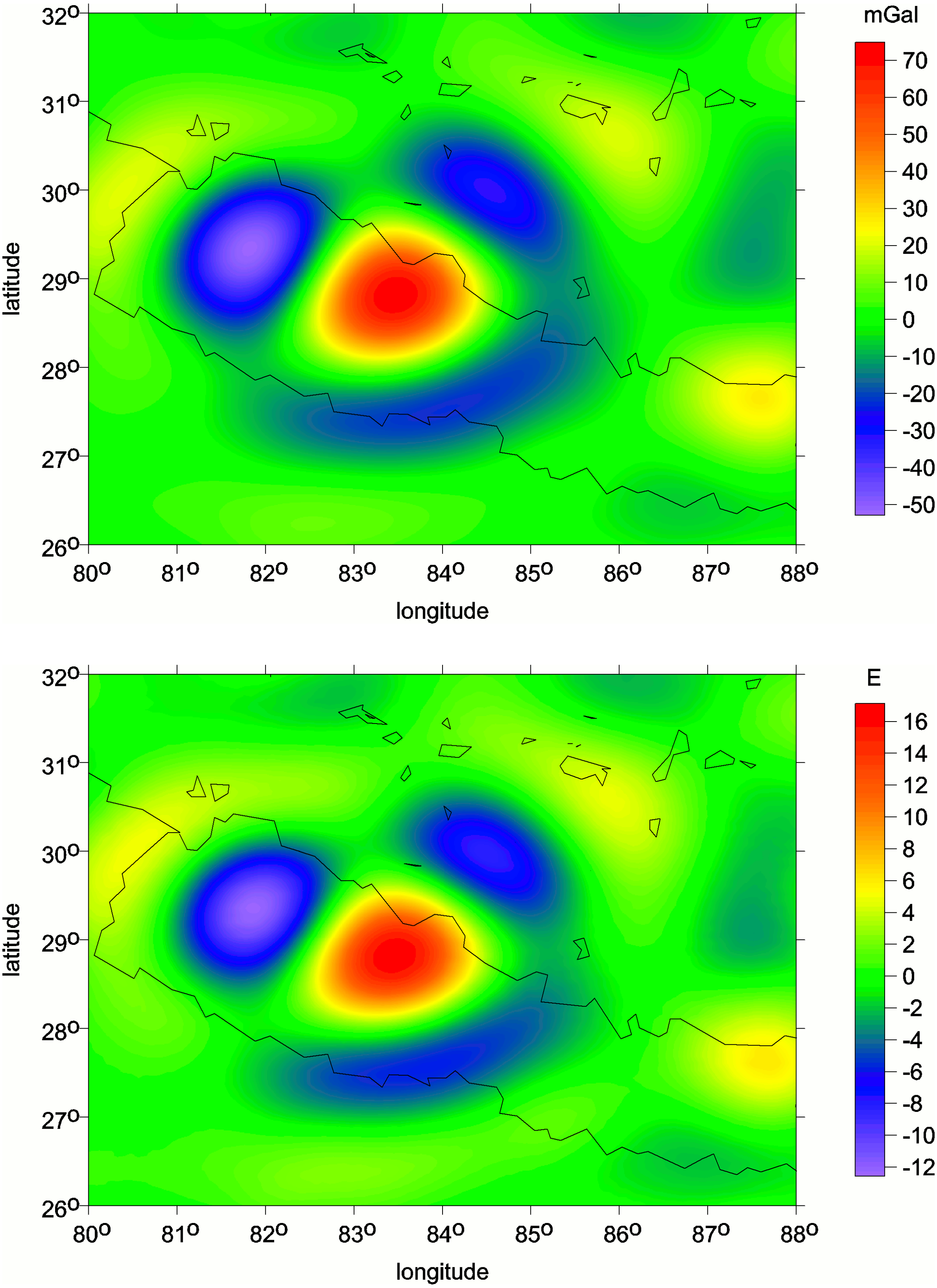

Figure 1 c, d: EIGEN-6C4 minus EGM2008, Himalaya Δg and T_{zz}

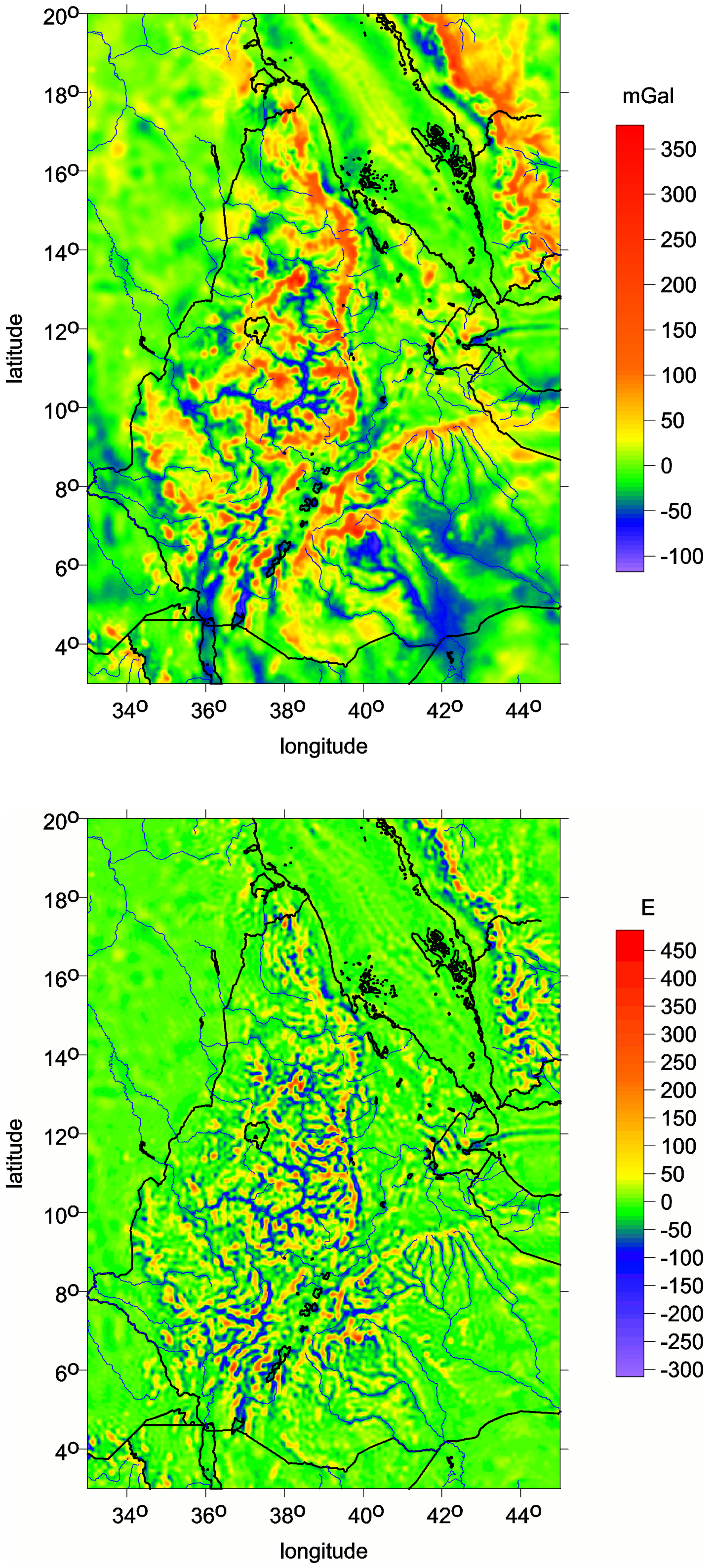

Figure 2 a, b: EIGEN-6C4, Etiopie Δg and T_{zz}

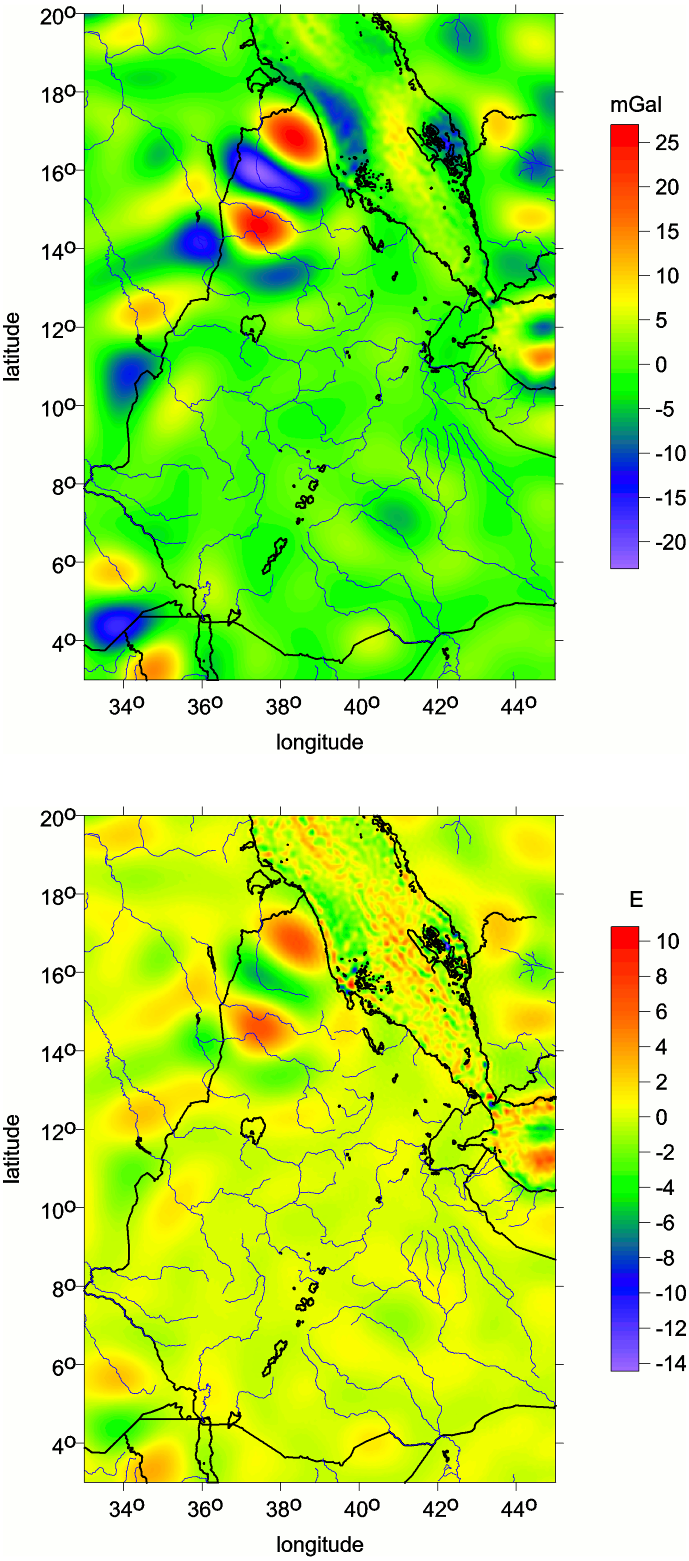

Figure 2 c, d: EIGEN-6C4 minus EGM2008, Etiopie Δg and T_{zz}

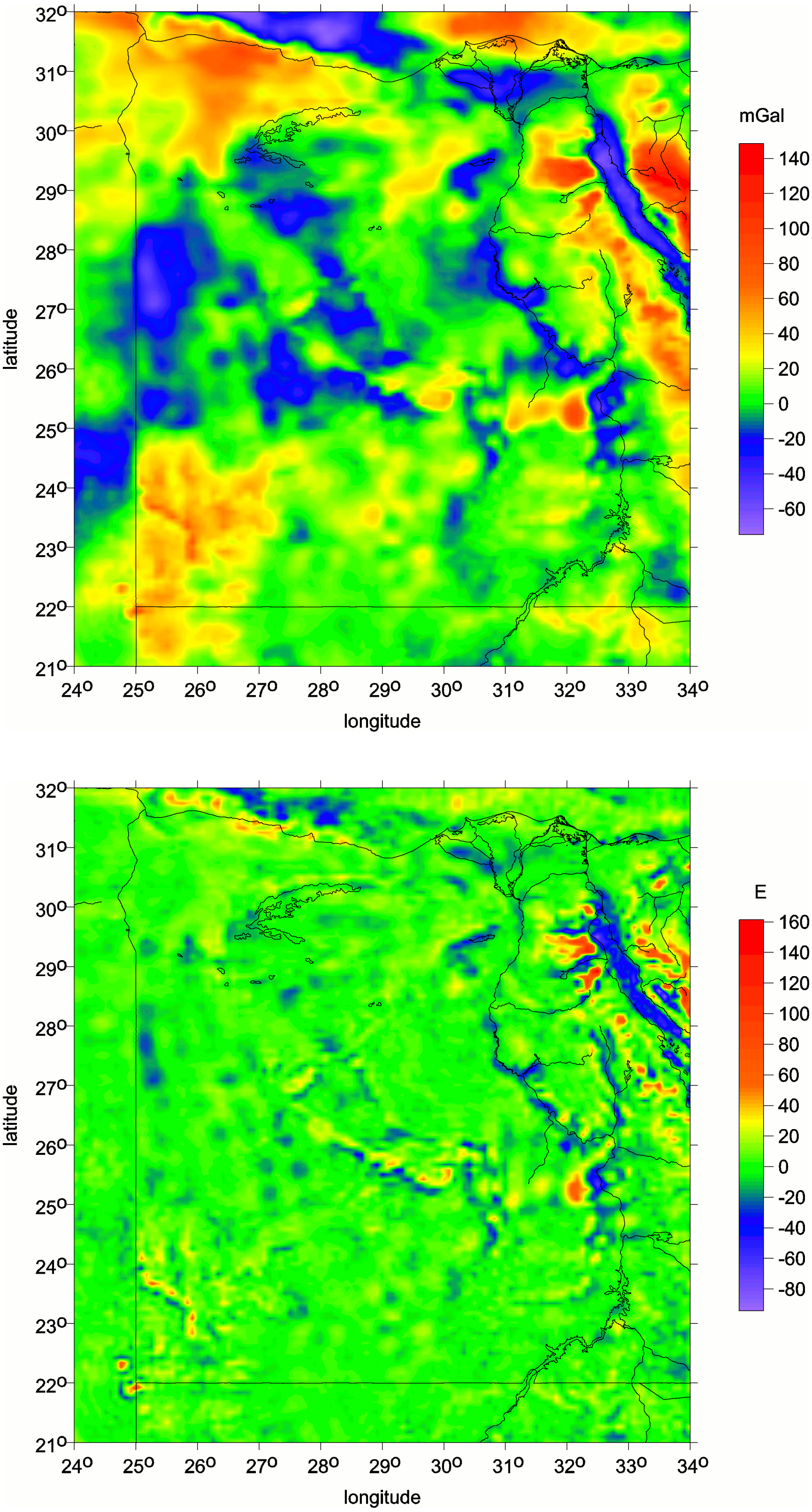

Figure 3 a, b: EIGEN-6C4, Egypt Δg and T_{zz}

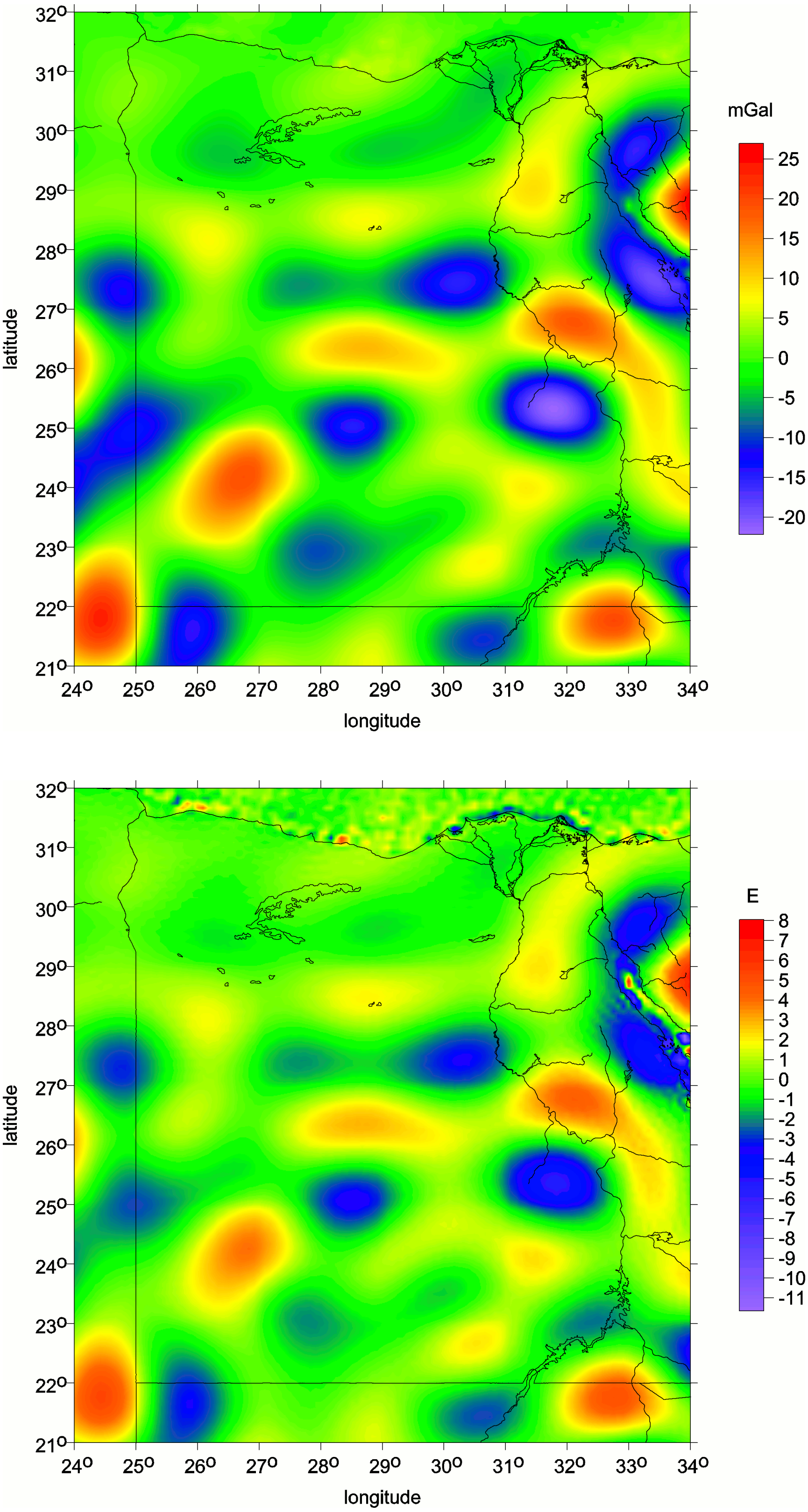

Figure 3 c, d: EIGEN-6C4 minus EGM2008, Egypt Δg and T_{zz}

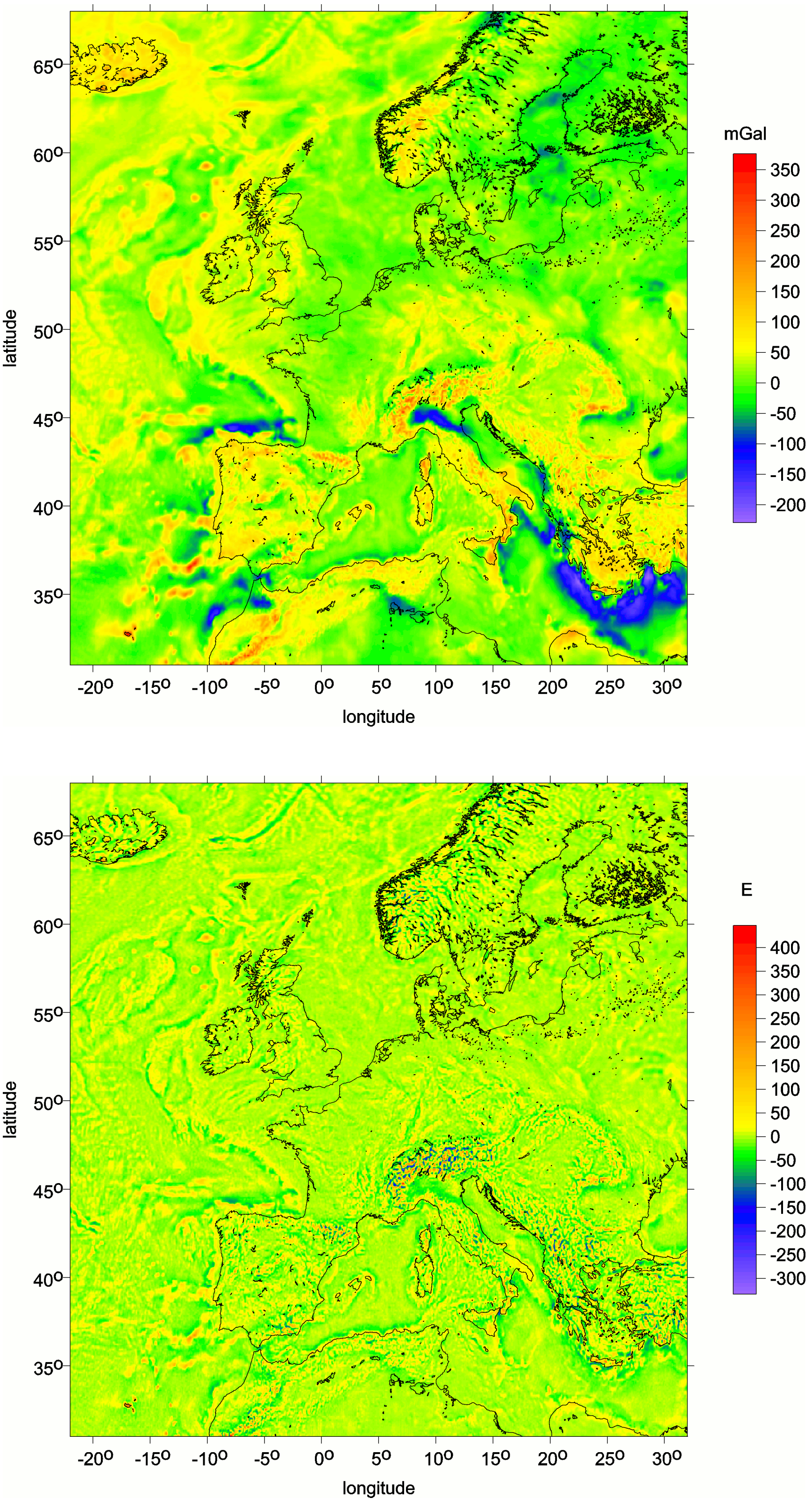

Figure 4 a, b: EIGEN-6C4, Europe Δg and T_{zz}

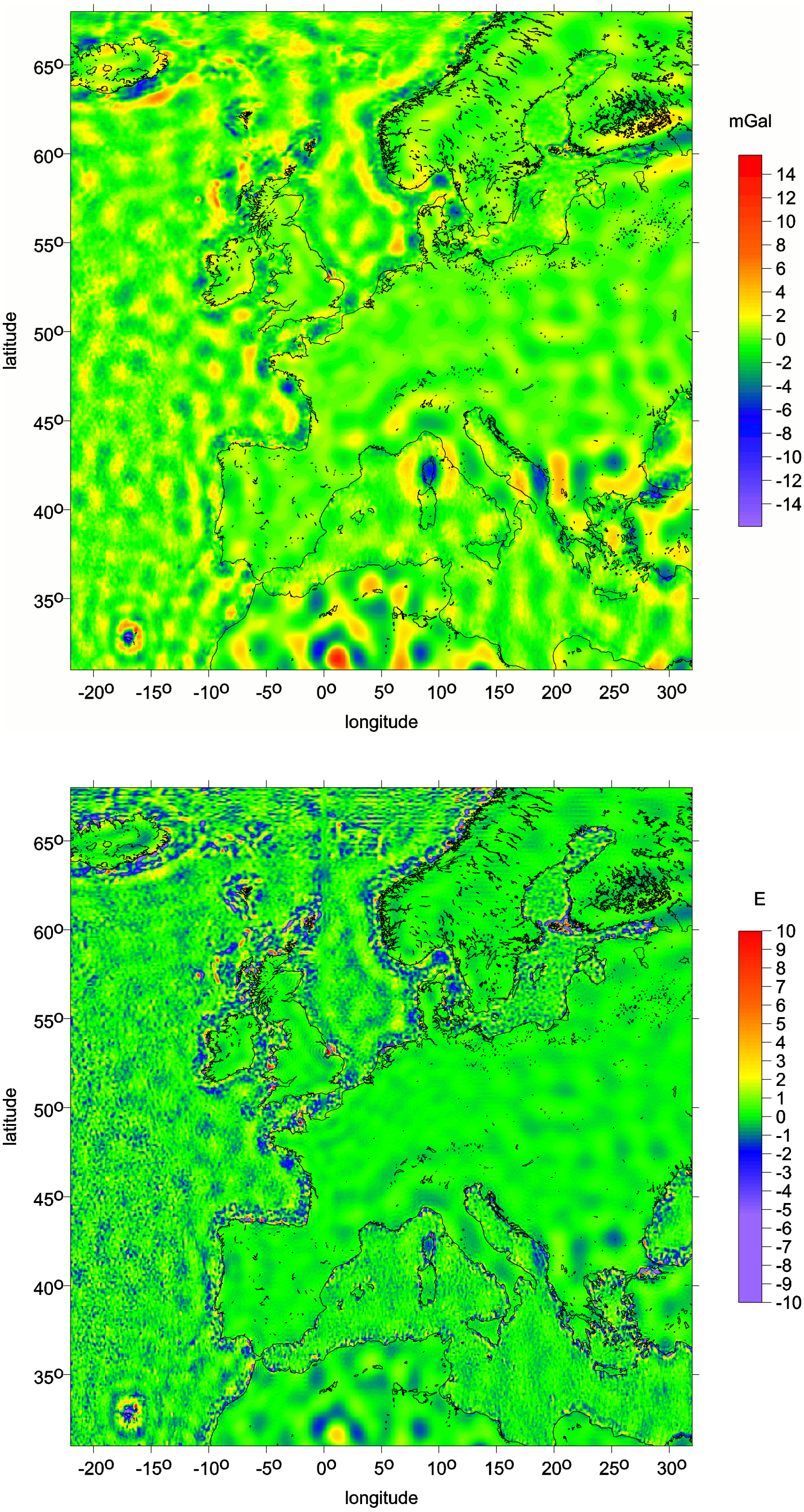

Figure 4 c, d: EIGEN-6C4 minus EGM2008, Europe Δg and T_{zz}

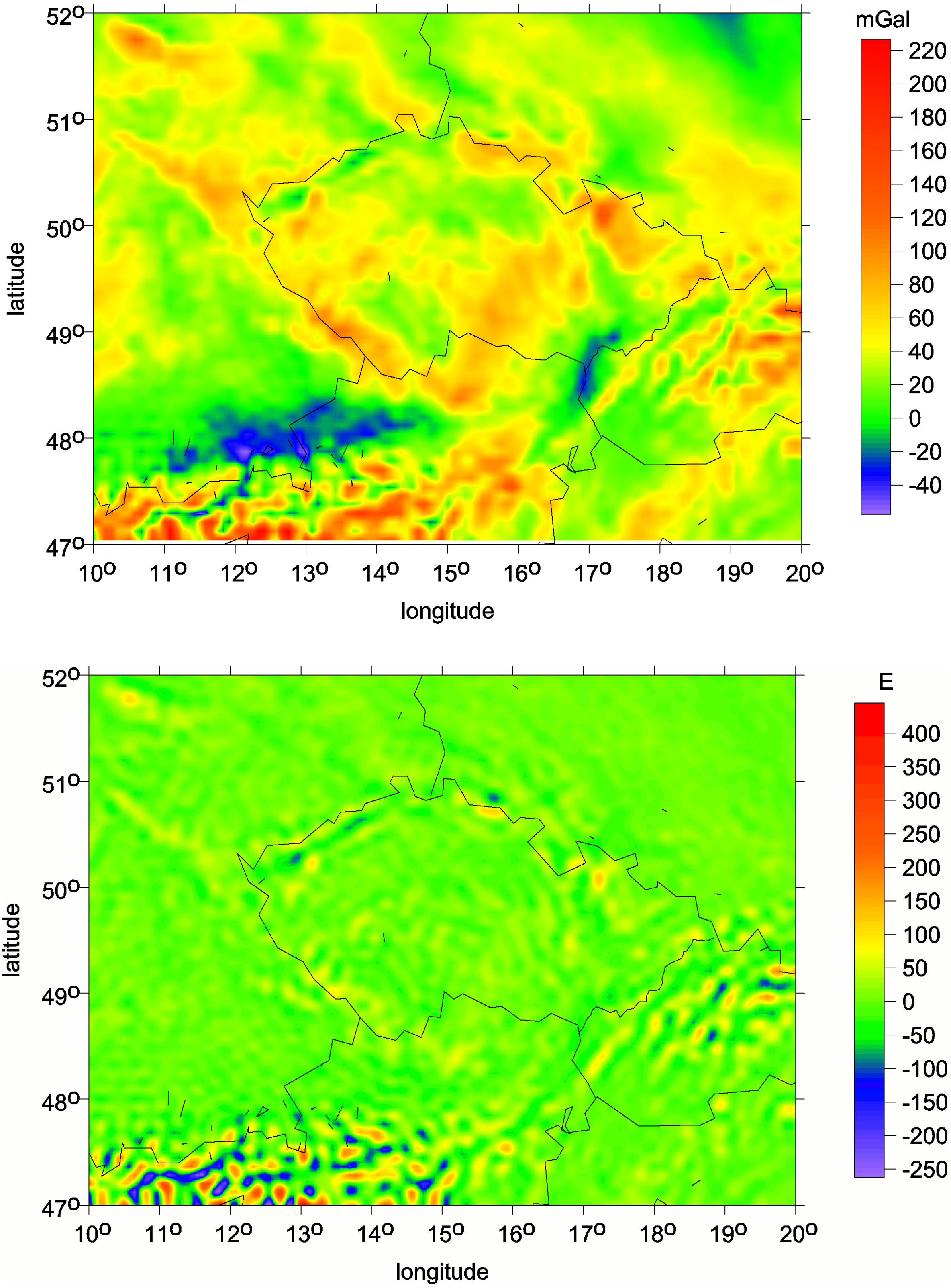

Figure 5 a, b: EIGEN-6C4, Czech Republic Δg and T_{zz}

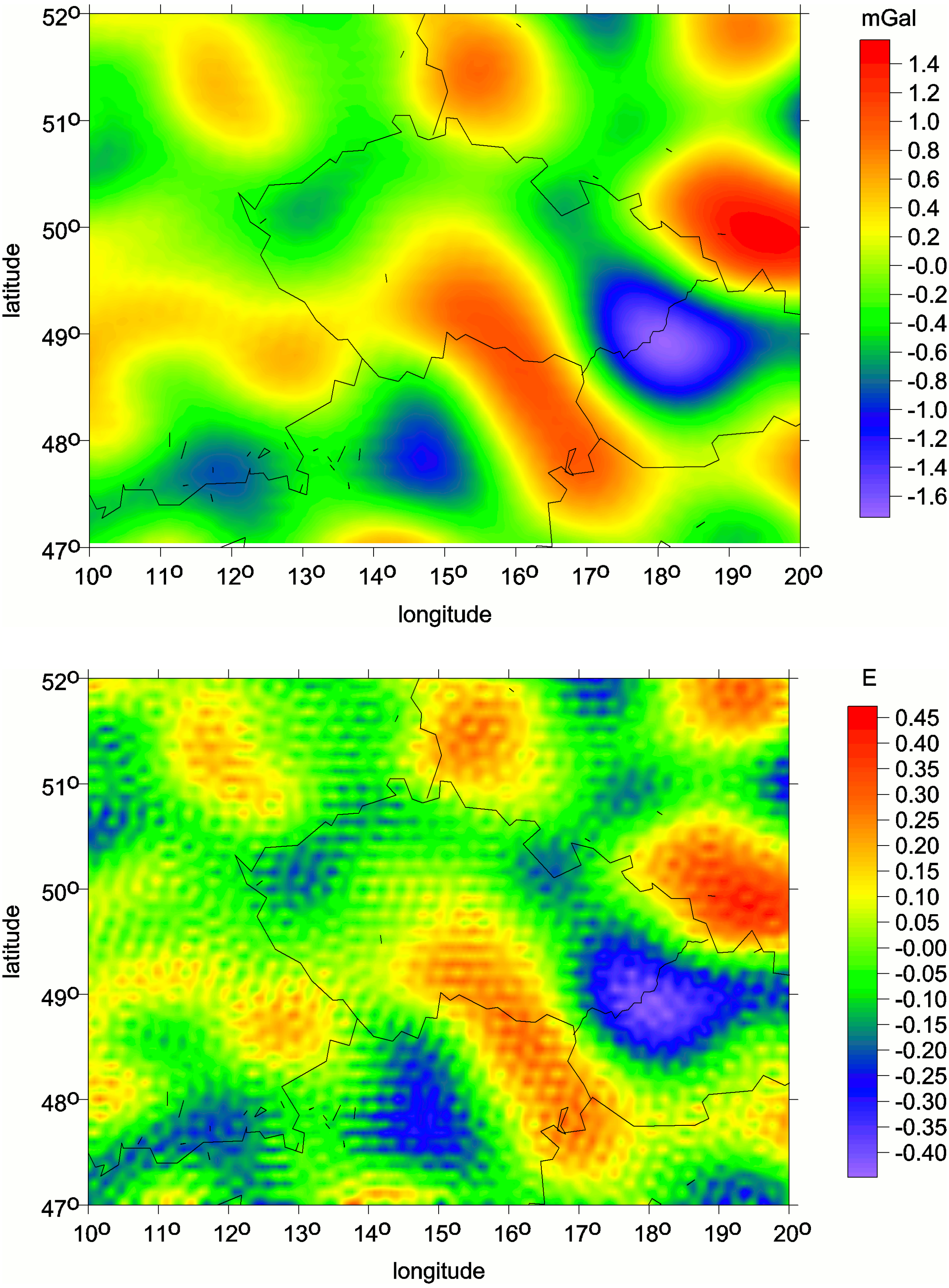

Figure 5 c, d: EIGEN-6C4 minus EGM2008, Czech Republic Δg and T_{zz}

comes from national GNSS/levelling campaigns. In the differences shown by our figures we see long wavelength discrepancies between the two models, where we anticipate a role of the GOCE data. RMS values of differences in Δg and T_{zz} are collected in Table 1, minimal and maximal values of these are visible in the figures. The differences over Europe (for Δg as well as T_{zz}) show significant values for the oceans while the differences over land are small. This is caused by the fact, that EIGEN-6C4 still contains EGM2008 over the continents while recent altimetry-based gravity data were taken for the ocean and sea area. The differences between both models for T_{zz} also show a pixelated pattern (see the figure for Czech Republic 5 c,d). We think this phenomenon is caused by noise of the GOCE data which appears more in the second radial derivative of the potential than in the gravity anomaly.

Obviously, these differences give no quantitative measure about the accuracy of the compared models. For such a purpose, independent, sufficiently precise and as spacious as possible data sets are needed. Here in our study we used GNSS/levelling data, which are independent of the models. These data are very precise but not of global coverage.

Table 1: Root mean square values of differences between EIGEN-6C4 and EGM2008

Territory	Δg (mGal)	T_{zz} (Eötvös)
Himalaya	15.4	3.5
Ethiopia	4.4	1.3
Egypt	6.6	1.6
Europe	1.3	1.0
Czech Republic	0.5	0.1

4. Tests with GNSS/levelling results

4.1. GNSS/levelling network in the Czech Republic

The GNSS/levelling network, containing 1024 points regularly covering the territory of the Czech Republic, has been surveyed by Land Survey Office with the aim to improve the gravimetric quasigeoid. Distribution of these points is visible from Figs. 7. The GNSS coordinates were measured on selected trigonometric points of the Czech Geodetic Control. The height of these points was only known from trigonometry with accuracy of decimeters; thus, the more precise heights of these points were determined by "precise geometric levelling" method using the nearest points of the Czech State Levelling Network. The accuracy of the physical heights is better than 0.5 cm with respect to the nearest points of the State Levelling Network. For all 1024 points we can then compute the geoid (quasigeoid) undulations using inversion of Eq. (1). The accuracy of the GNSS ellipsoidal heights is 1.5 – 2.0 cm. The total error of the height anomaly varies between 1.6 and 2.1 cm. see i.e. Kostelecký et al., 2012 [8].

4.2. GNSS/levelling network in Slovakia and the European gravimetric quasigeoid (EGG97)

For the territory of Slovakia the network consists of 64 points, measured by Geodetic and Cartographic Institute in Bratislava at the SLOVGERENET (SLOVak GEOdynamic REference NETtwork). The distribution of these points is visible from Figs. 8. The accuracy is approximately the same as in the Czech Republic. Since 1997, for whole Europe exists

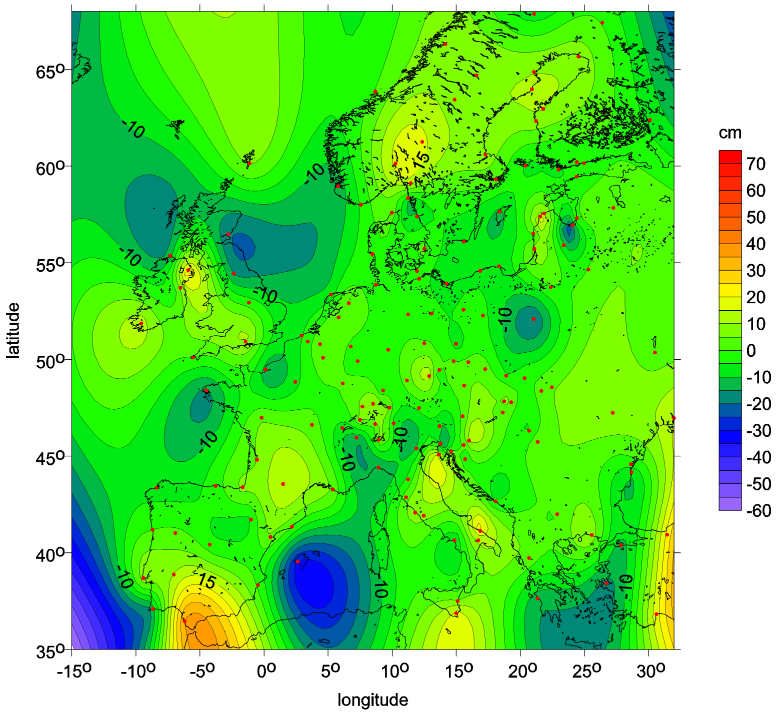

Figure 6 a: Test of EMG2008 by GPS/levelling on the territory of Europe (GPS/levelling geoid minus EGM2008 geoid; constant shift 0 cm, rmse 9.0 cm)

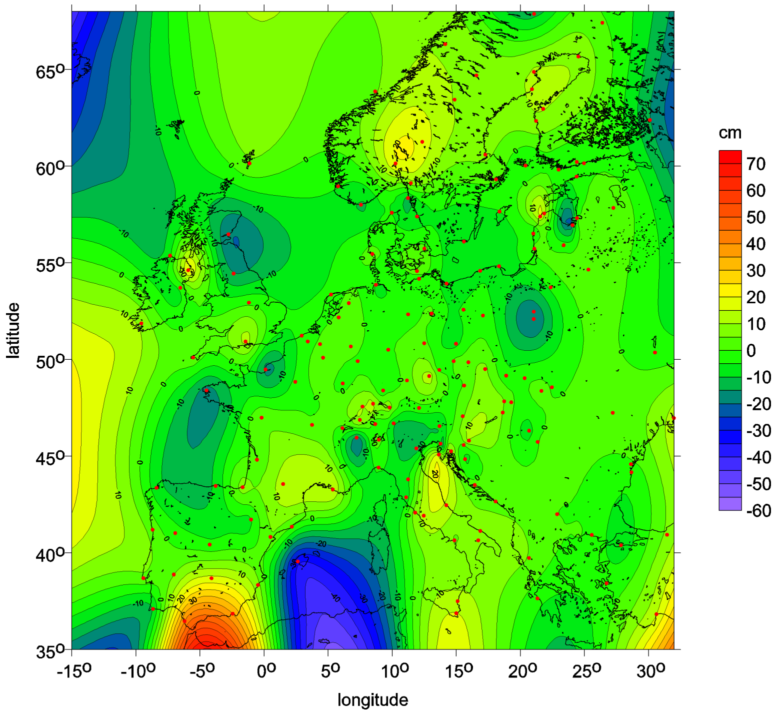

Figure 6 b: Test of EIGEN-C64 by GPS/levelling on the territory of Europe (GPS/levelling geoid minus EGM2008 geoid; constant shift 0 cm, rmse 8.6 cm)

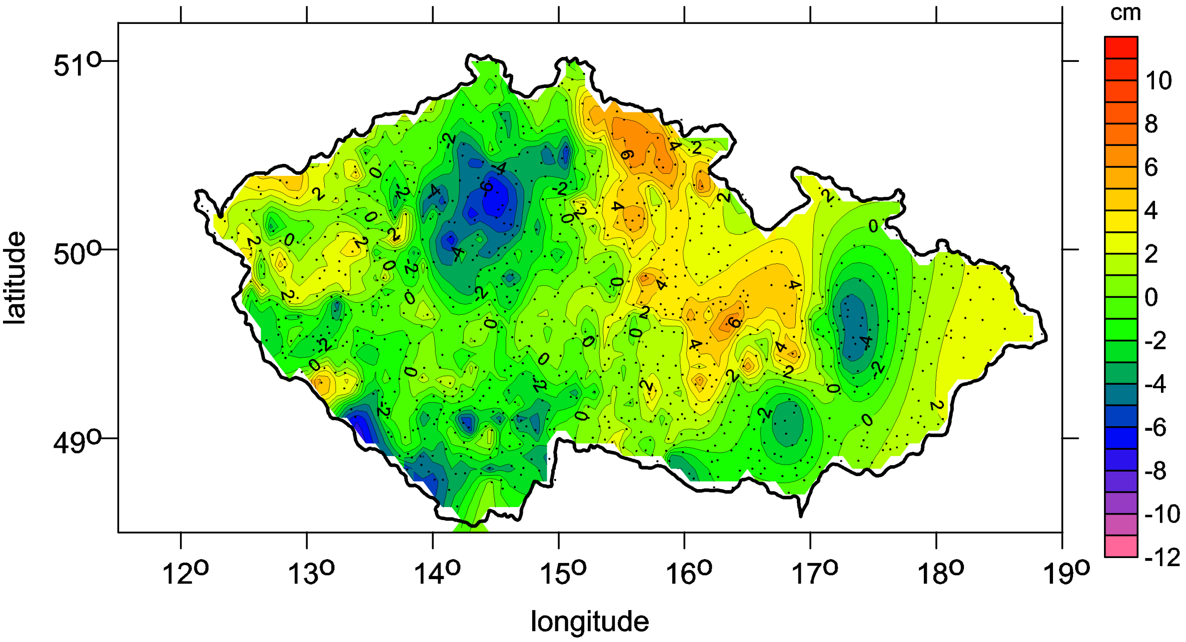

Figure 7 a: Test of EGG97 quasigeoid by GPS/levelling on the territory of the Czech Republic (GPS/levelling quasigeoid minus EGG97, substracted -14 cm, rmse 2.8 cm)

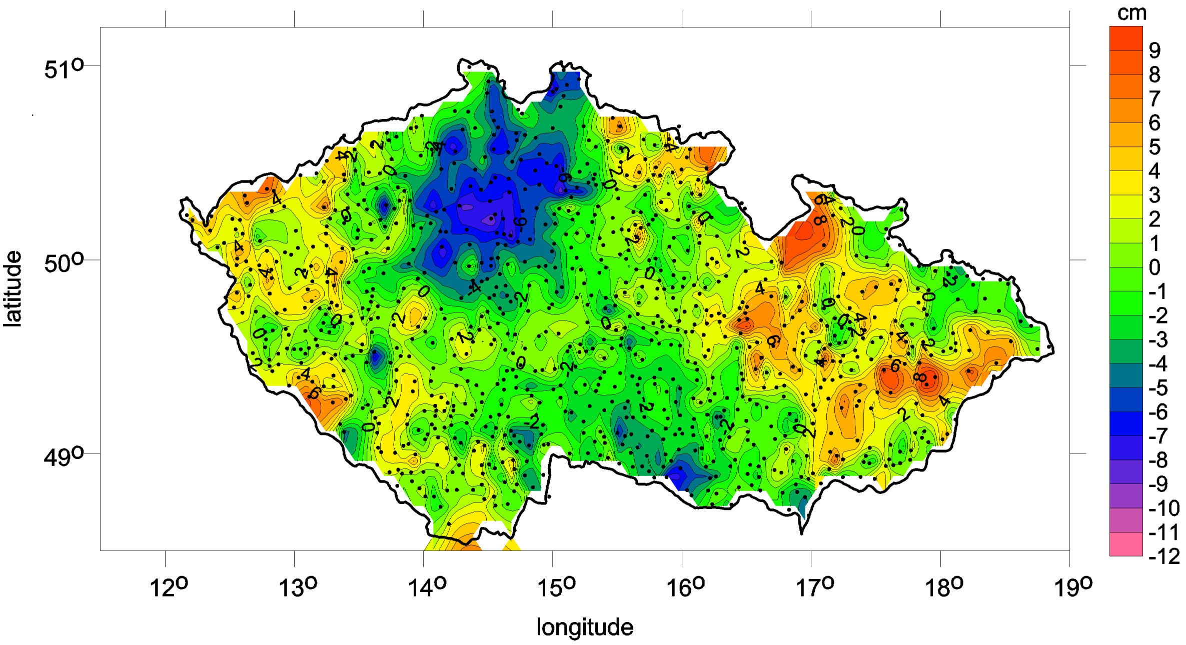

Figure 7 b: Test of EGM2008 by GPS/levelling on the territory of the Czech Republic (GPS/levelling quasigeoid minus EGM2008 geoid; substracted mean -43 cm, rmse 3.3 cm, GPS levelling point accuracy about 2 cm)

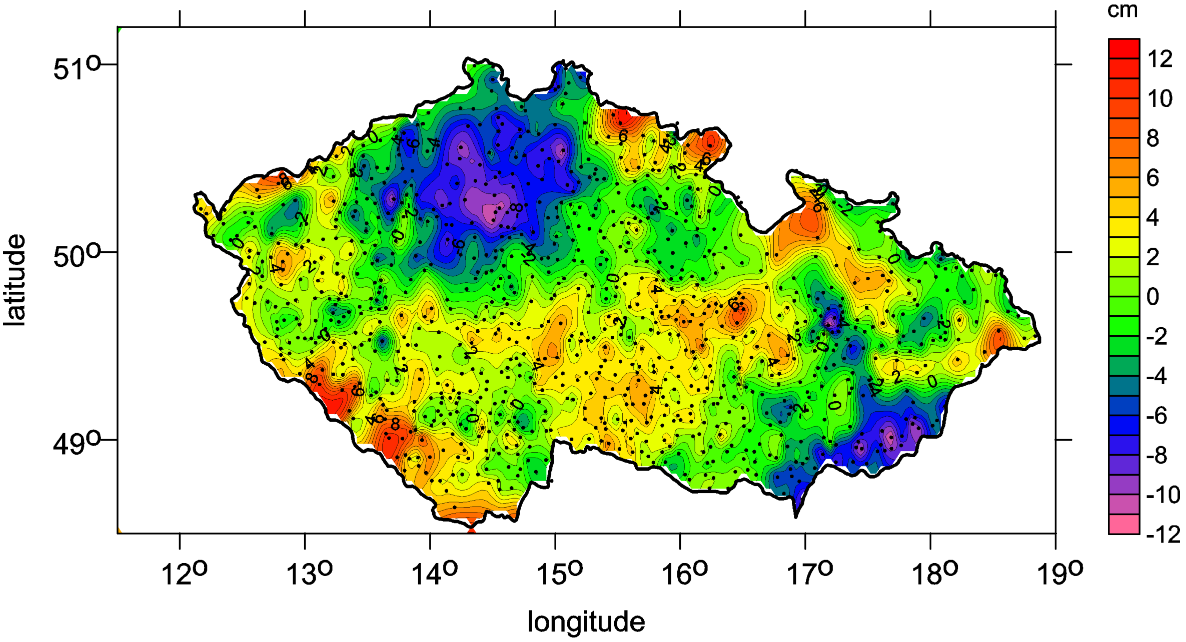

Figure 7 c: Test of EIGEN-6C4 by GPS/levelling on the territpry of the Czech Republic (GPS/levelling quasigeoid minus EIGEN-6C4 geoid; subtracted mean -46 cm, rmse 4.0 cm; GPS/levelling point accuracy about 2 cm)

a model of the European gravimetric quasigeoid (EGG97), based on gravimetric data from different sources of European countries. The accuracy of the EGG97 quasigeoid is about 3 to 5 cm per 100 km - see Denker et al., 2008 [3].

4.3. Evaluation of GNSS/levelling results

GNSS/levelling data sets (with the accuracy about 2 cm) have been used to evaluate EGM2008 and EIGEN-6C4 on the territories of the Czech Republic (CZ) and Slovakia (SK) as well as over Europe. While the Czech Republic (CZ) and Slovakia have very dense GNSS/levelling points, for Europe only data with much less density were available for us.

For GNSS/levelling heights minus geoid values from EGM2008 and EIGEN-6C4 we obtained: RMS of the differences of 3.3/4.0 cm (CZ – Figs. 7 b,c), and 5.0/4.2 cm (SK – Figs. 8 a,b) respectively. The semi-major axis of the ellipsoid used to compute the geoid from respective models is 6378136.3 m for both models. The offset (subtracted mean in the figures) about 40 cm for CZ and SK is caused by a) using different ellipsoids in the geoid and the GNSS/levelling computations (here it is used ellipsoid GRS80 with semi-major axis 6378137.0 m), b) by using different height systems of levelling heights, and c) we compare „regularized geoid“ determined from models with „quasigeoids“ from CZ, SK and Europe (differences between geoid and quasigeoid is in flat parts of Europe approximately 1 cm, in mountains it is 3 – 5 cm). For the Czech Republic (Figs. 7 a,b,c) our figures show a significant «hollow» in the northern part. We are sure that this finding is not an artifact of method or computations. To clear up this phenomenon we added Figure 7a with differences between the GNSS/levelling heights and the quasigeoid model EGG97 (Denker et al., 2008 [3]). This figure shows a systematic trend, but the differences between GNSS/levelling and the quasigeoid EGG97 are

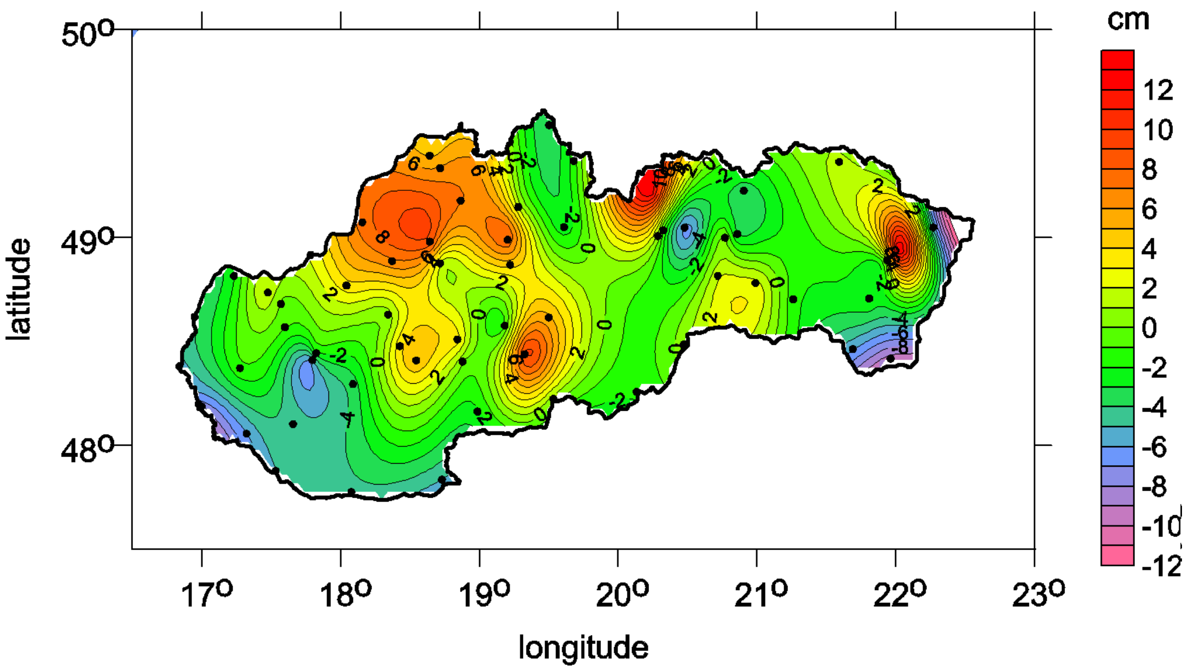

Figure 8 a: Test of EGM2008 by GPS/levelling on the territpru of Slovakia (GPS/levelling geoid minus EGM2008 geoid; subtructed mean -44 cm, rmse 5.0 cm; GPS/levelling point accuracy about 2 cm)

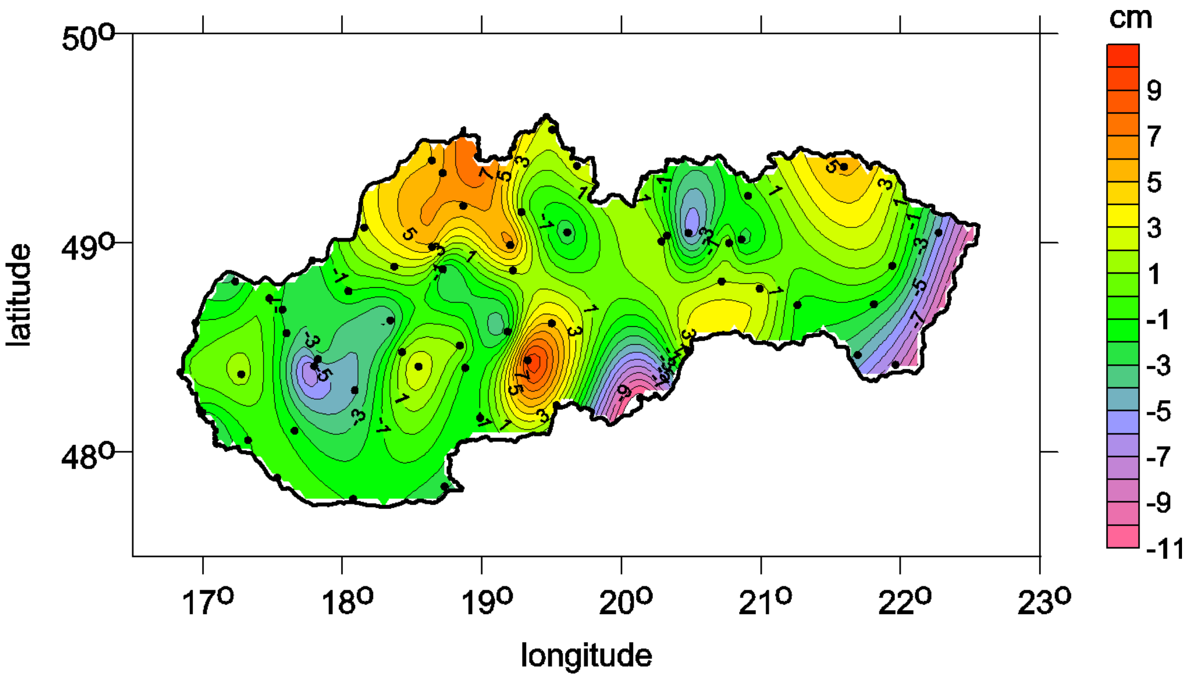

Figure 8 b: Test of EIGEN-6C4 by GPS/levelling on the territory of Slovakia (GPS/levelling geoid minus EIGEN-6C4 geoid; subtracted mean -43 cm, rmse 4.2 cm; GPS/levelling point accuracy about 2 cm) slightly a bit lower than those for the two gravity models. This depression could be caused by a) an inaccuracy of the terrain gravity measurements and/or b) by local recent deformations (GNSS/levelling observations are from the epoch 2003 and the levelling heights are from 1986 and older, see also Zeman et al., 2007 [15]). Note also that the authors of both models estimated precision of geoid undulations computed from them in Europe to 0.10 – 0.15 m RMSE (Förste et al., 2014 [5]).

For Europe the RMS differences between GNSS/levelling height and EGM2008 or EIGEN-6C4 respectively are 9.0/8.4 cm – see Figs. 4 a,b – which means a little improvement of EIGEN-6C4 vs. EGM2008. This finding is confirmed by additional tests computed for GPS/levelling data sets for further, larger regions of Canada (M. Véronneau, Natural Resources Canada , personal communication 2003), USA (Milbert, 1998 [9]), Japan (courtesy Tokuro Kodama, Geospatial Information Authority of Japan), Australia (G. Johnston, Geoscience Australia and W. Featherstone, Curtin University of Technology, personal communication 2007) and Brazil (courtesy D. Blitzkow and A. C. O. Cancoro de Matos, Centro de Estudos de Geodesia Brazil). Again, EGM2008 and EIGEN-6C4 were taken to their maximum degree and order and RMS of differences were computed. The obtained RMS values of the geoid heigth differences after subtraction of the mean are given in Table 2. The results show an improvement of EIGEN-6C4 vs. EGM2008 for most of the tested regions which should be caused by the inclusion of the novel GOCE satellite data. Somewhat worse result for EIGEN-6C4 with comparison of EGM2008 for the Czech Republic should be statistically insignificant since the spatial extend of this region is not very large compared to the spatial resolution of GOCE and the obtained RMS values are already very small.

Table 2: Root mean square (cm) about the mean of GNSS/levelling minus model-derived geoid heights (number of points in brackets) for EGM2008 and EIGEN-6C4.

GNSS/levelling data set	EGM2008	EIGEN-6C4
Canada (1930)	12.6	12.4
USA (6169)	24.6	24.5
Australia (201)	21.5	21.1
Japan (816)	8.2	7.8
Brazil (672)	36.6	30.6
Europe (166)	9.0	8.6
Czech Rep. (1020)	3.3	4.0
Slovakia (64)	5.0	4.2

5. Conclusions

The global combined high-resolution gravity field models EGM2008 and EIGEN-6C4 have been compared by means of two functions of the disturbing gravitational potential. The main difference between both models is adding of GOCE mission data in the EIGEN-6C4 model. Here we show examples of differences in Δg and in T_{zz} for Himalaya, Ethiopia, Egypt, Europe, and namely for the Czech Republic. In our evaluation we see long wave differences between the two models in remote areas of worse terrestrial data. For these regions we assume a positive effect of the GOCE data.

GNSS/levelling data over the territories of Europe, the Czech Republic and Slovakia as well as for further larger regions has been used as an independent data source to evaluate EGM2008 (without GOCE SGG measurements data) and EIGEN-6C4 (with GOCE SGG data). These tests show an improvement for EIGEN-6C4 compared to EGM2008 for most of the included GPS/levelling data sets. The tests confirmed the declared accuracy of both models at the 10 cm level.

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